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Short Note

The double-elliptic approximation in the group and phase domains

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INTRODUCTION

Elliptical anisotropy has found wide use as a simple approximation to transverse isotropy because of a unique symmetry property (an elliptical dispersion relation corresponds to an elliptical impulse response) and a simple relationship to standard geophysical techniques (hyperbolic moveout corresponds to elliptical wavefronts; NMO measures horizontal velocity, and time-to-depth conversion depends on vertical velocity). However, elliptical anisotropy is only useful as an approximation in certain restricted cases, such as when the underlying true anisotropy does not depart too far from ellipticity or the observed angular aperture is small. This limitation is fundamental, because there are only two parameters needed to define an ellipse: the horizontal and vertical velocities. (Sometimes the orientation of the principle axes is also included as a free parameter, but usually not.)

In a previous SEP report Muir (1990) showed how to extend the standard elliptical approximation to a so-called *double*-elliptic form. (The relation between the elastic constants of a TI medium and the coefficients of the corresponding double-elliptic approximation is developed in a companion paper, (Muir, 1991).) The aim of this new approximation is to preserve the useful properties of elliptical anisotropy while doubling the number of free parameters, thus allowing a much wider range of transversely isotropic media to be adequately fit. At first glance this goal seems unattainable: elliptical anisotropy is the most complex form of anisotropy possible with a simple analytical form in both the dispersion relation and impulse response domains. Muir's approximation is useful because it *nearly* satisfies both incompatible goals at once: it has a simple relationship to NMO and true vertical and horizontal velocity, and to a good approximation it has the same simple analytical form in both domains of interest.

The purpose of this short note is to test by example how well the double-elliptic

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approximation comes to meeting these goals:

1. Simple relationships to NMO and true velocities on principle axes.
2. Simple analytical form for both the dispersion relation and impulse response.
3. Approximates general transversely isotropic media well.

The results indicate that the method should work well in practice.

REVIEW OF SYMMETRIC PROPERTIES OF ELLIPSES

Figure **??** shows a transversely isotropic medium and its vertical paraxial elliptic approximation represented four different ways. Note that the dashed elliptical-approximation curve plots as an ellipse in both the group-velocity and phase-slowness domains. The vertical and horizontal velocities of the elliptical impulse response define its principle axes; the corresponding vertical and horizontal slownesses give the principle axes of the elliptical dispersion relation.

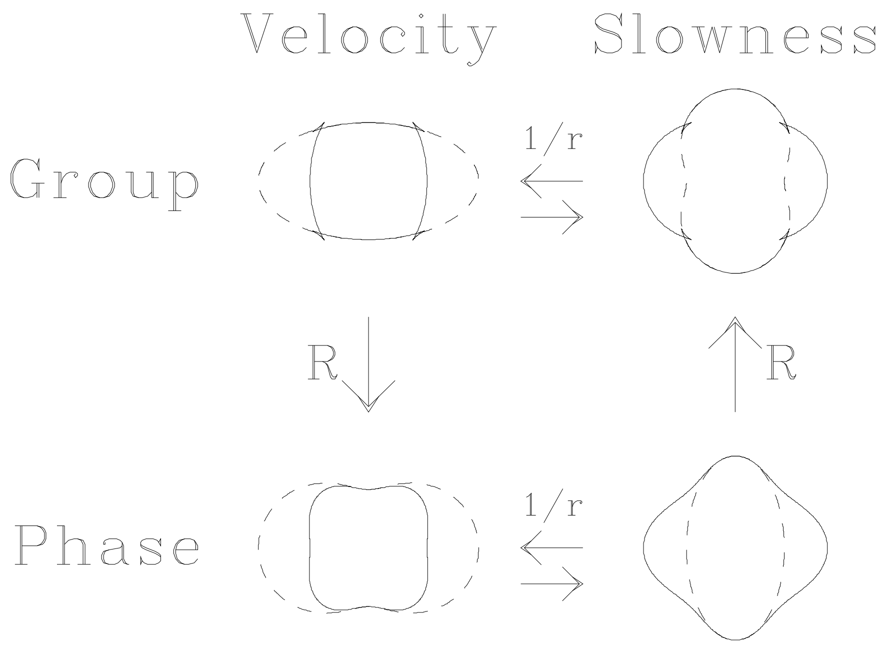

Figure 1: Group-four $\boxed{\text{joe1-Group-four}}$ [ER]

Group velocity (= impulse response), group slowness, phase velocity, and phase slowness (= dispersion relation) plots for the qSV mode of Greenhorn Shale (thick solid line) and an elliptically anisotropic paraxial approximation to it (thin dashed line). The “R” and “1/r” symbols indicate the recurring transformations linking the four representations.

These vertical and horizontal velocities have simple geophysical interpretations. The vertical velocity of the paraxial elliptic approximation (equals the true vertical velocity) is what we need to do time-to-depth conversion. The horizontal velocity of the paraxial elliptic approximation (*not* the same thing as the true horizontal velocity) is what is required for NMO. (In a surface survey the sources and receivers are laid out along a horizontal line; if the vertical velocity were changed, it would effectively change the vertical scale of the survey, but the traveltime field recorded along the horizontal surface would remain unchanged. So it is horizontal velocity that matters. But it isn't the true horizontal velocity, because we're only considering NMO for near-vertical propagation. It is the horizontal velocity of the vertical paraxial elliptic approximation.)

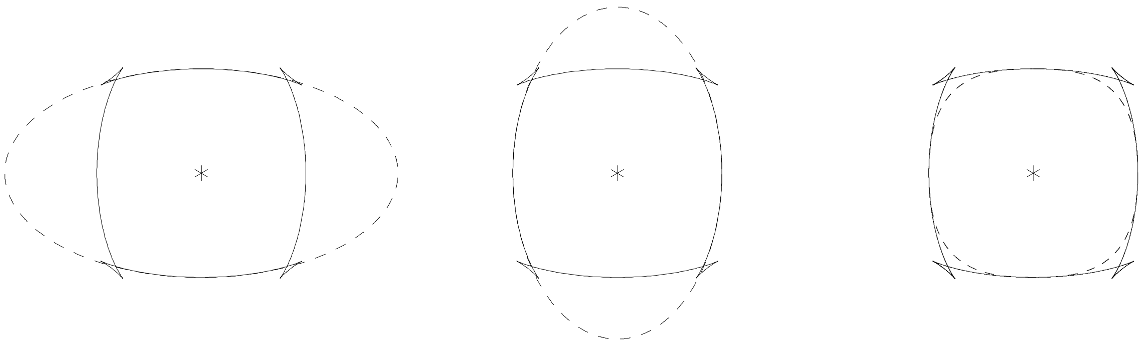

Figure 2: verthoriz joe1-verthoriz [ER]

Three different approximations (dashed curves) to the qSV impulse-response surface of Greenhorn Shale (bold curves). On the left is the standard vertical paraxial elliptic approximation. In the center is the horizontal paraxial elliptic approximation. On the right is Muir's double-elliptic approximation. Three different approximations (dashed curves) to the qSV dispersion-relation surface of Greenhorn Shale (bold curves). On the left is the standard vertical paraxial elliptic approximation. In the center is the horizontal paraxial elliptic approximation. On the right is Muir's double-elliptic approximation; the fit is so close the dashed curve is hard to see.

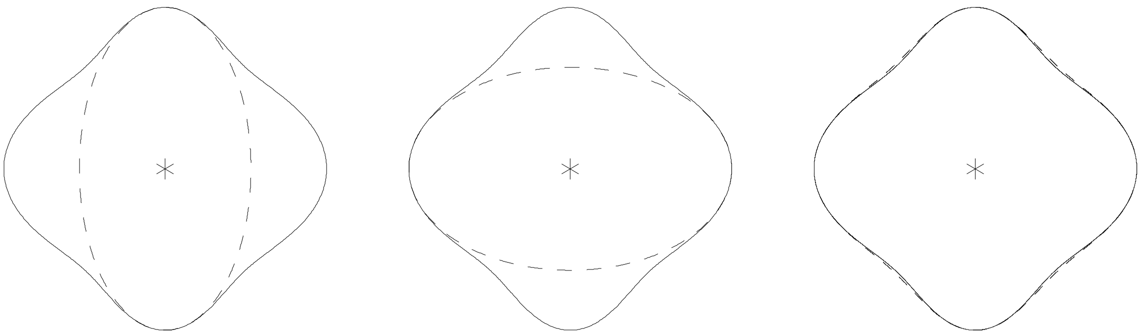

Figure 3: verthoriz2 joe1-verthoriz2 [ER]

THE DOUBLE ELLIPTIC APPROXIMATION...

in the impulse-response domain

Muir's trick is to turn the problem on its side and also look at the *horizontal* paraxial approximating ellipse. This has all the useful properties of the vertical ellipse, with the difference that it fits the true horizontal velocity and the vertical NMO velocity (not the same thing as the true vertical velocity). Figure **??** shows the vertical and horizontal paraxial approximating ellipses, along with Muir's double-elliptic approximation. Muir's approximation fits four parameters, the vertical and horizontal true and NMO velocities. Since it is single-valued, it cannot follow the qSV triplication, but fits well elsewhere.

in the dispersion-relation domain

There is no such problem in the dispersion-relation domain, since triplications are impossible there. Figure **??** shows how the three approximations shown in Figure **??** work in the dispersion-relation domain. Just like elliptical anisotropy, Muir's four-parameter approximation is calculated in the dispersion-relation domain by simply using slownesses instead of velocities in the same approximation formula. Note in Figure **??** that the double-elliptic approximation in the dispersion-relation domain is nearly exact for this example.

but are they consistent?

Of course there is one technicality: the elliptical approximations in the previous two figures were perfectly consistent, but the double-elliptic approximations were not. Muir's approximation did a much better job in the dispersion-relation domain than it did in the impulse-response domain. Figure **??** shows how the double-elliptic approximations fit in the two different domains compare in the impulse-response domain. Should we be concerned by this discrepancy? Probably not, because this is a rather extreme example. If there are no triplications the approximations as fit in the two domains are much more nearly consistent, as is demonstrated in Figure **??**.

CONCLUSIONS

The double-elliptic approximation introduced by Muir (1990) can approximate the kinematics of TI media quite accurately, although it is forced to cut off triplications when fit in the impulse-response domain. In the dispersion-relation domain it remains accurate even for triplicating media.

Our analysis in this paper was limited to simple kinematics. Before pronouncing the double-elliptic approximation a success, we need to also demonstrate how it works when used as the basis for an imaging technique; i.e, how accurately does it model the *dynamics* of the wave equation? Karrenbach (1991) examines this question in a companion paper.

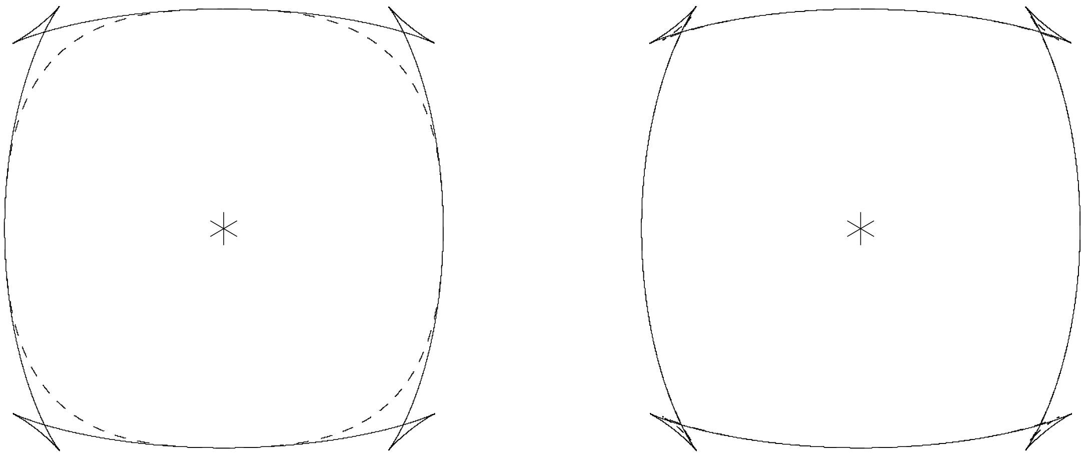

Figure 4: Compare joe1-Compare [ER]

Two different double-elliptic approximations (dashed curves) fit to the qSV mode of Greenhorn Shale (solid curves). Left: the approximation is fit in the impulse-response domain, and so the dashed curve has a simple analytic form. Right: the approximation is fit in the dispersion-relation domain, and so is able to closely follow the triplication. This approximating curve can only be calculated parametrically, however, and so is less useful. Two different double-elliptic approximations (dashed

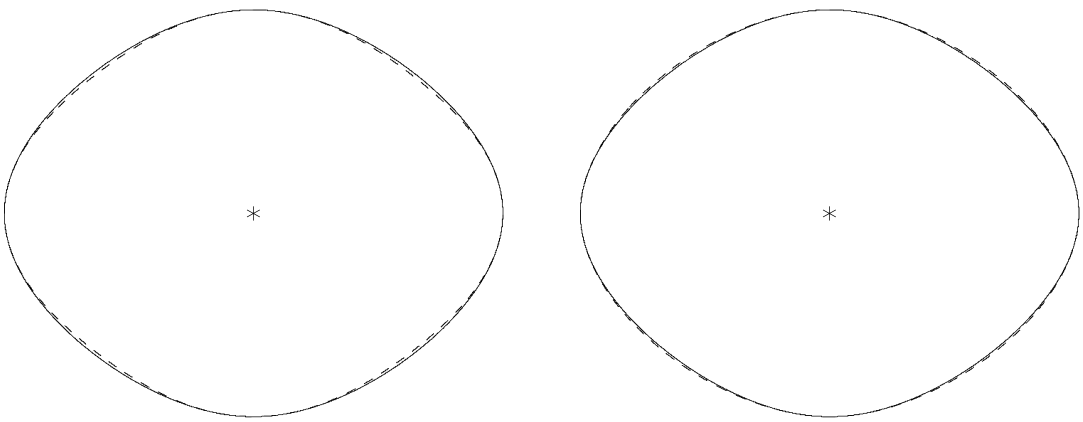

Figure 5: Compare2 joe1-Compare2 [ER]

curves) corresponding to those in Figure **??**, but this time fit to the qP mode of Greenhorn Shale (solid curves). (The size of the "$*$" in the middle shows the relative scales.) The discrepancy is much less since there are no troublesome triplications.

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