Stanford Exploration Project, Report 70, December 18, 1997, pages 1–385

Short Note

Subtracting sources in the frequency domain

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keywords: *sources, algorithm, Fourier transform*

INTRODUCTION

When using passive seismic data to detect and locate sources of seismic energy related to various endogenous processes the typical situation is that these sources have to be determined against a strong background, usually of surface origin. Vanyan and Cole (1990) tried to cope with this problem using a least squares method for subtracting unwanted signals in frequency domain. Here I give some formal basis for applying this method to passive data, describe a more general multichannel approach, and show the results of applying both methods to synthetic datasets.

A detailed description of the least squares method and its application to conventional seismic data can be found, for example, in Jenkins and Watts (1968) and White (1973). The algorithm used to locate sources was computing the semblance coefficient along moveout trajectories for all possible source locations in a 3-D grid (Nikolaev & Troitskiy,1987). Synthetic datasets were created for the seismic array used in the SEP drill-bit source experiment (Cole, 1991), so the synthetics model a realistic case.

DESCRIPTION OF THE SUBTRACTION ALGORITHMS

Let us consider a simple model of seismic traces recorded by an N-channel seismic array:

$$X_i(f) = U_i(f) + S_i(f), \tag{1}$$

where $X_i(f)$ is the Fourier component of the trace i for frequency f, $U_i(f)$ is signal from the source we want to locate, and $S_i(f)$ is the unwanted (interfering) signal. An important assumption is that signals from different sources are uncorrelated with one another, but signals from the same source at different traces are well-correlated.

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Single-channel approach

If we have an estimate S_0 of the interfering signal the following subtraction can be applied to the traces (Jenkins & Watts,1968):

$$X_{ir} = X_i - \frac{\overline{X_i S_0^*}}{\overline{S_0 S_0^*}} \approx U_i, \tag{2}$$

where X_{ir} is the residual, $\overline{X_i S_0^*}$ is the smoothed cross-spectrum between the i-th trace and the estimate, and $\overline{S_0 S_0^*}$ is the smoothed power spectrum of the estimate.

The problem with (2) is that often it is difficult to obtain a good estimate S_0. Now assume $S_i >> U_i$ and substitute into (2) instead of S_0 X_k–just one trace. Remembering that $X_k = U_k + S_k$ we get:

$$X_{ir} \approx S_i - \frac{\overline{S_i S_k^*}}{\overline{S_k S_k^*}} S_k + U_i - \frac{\overline{S_i S_k^*}}{\overline{S_k S_k^*}} U_k - \frac{\overline{U_i U_k^*}}{\overline{U_k U_k^*}} S_k \approx U_i - \frac{\overline{S_i S_k^*}}{\overline{S_k S_k^*}} U_k \approx U_i - \frac{S_i}{S_k} U_k, \tag{3}$$

and, rewriting the result:

$$X_{ir} \approx U_i - S_i \frac{U_k}{S_k}. \tag{4}$$

Here U_k/S_k is a complex constant. The contribution of the interfering source to each trace has been reduced by the absolute value of this constant (which is $<<$ 1 under our assumption), but the phase shift between channels is preserved. To summarize, the magnitude of the interfering source was reduced significantly without adding major artifacts, and location of the weaker source is now more feasible.

Multichannel approach

This approach can be viewed as a generalization of the preceding method. Instead of single variables, vectors and matrices are used:

$$X_{ir} = X_i - \mathbf{X}_{N-1}^T \overline{\mathbf{X}}_{N-1,N-1}^{-1} \overline{\mathbf{X}}_{N-1,i}, \tag{5}$$

Here $\mathbf{X}_{N-1}$ is a vector whose components are Fourier transforms of all $N-1$ traces beyond the i-th trace, $\overline{\mathbf{X}}_{N-1,N-1}$ is the spectral matrix of these traces, and $\overline{\mathbf{X}}_{N-1,i}$ is the vector of cross-spectra between the i-th trace and all others. All spectra are for some particular frequency.

I don't know yet how to prove that algorithm (5) does a good job of subtracting interfering signals, but I will now try to demonstrate that this is so using some synthetic examples.

APPLICATION TO SYNTHETIC DATA

Synthetic data were created for a 240 channel array. The array has twelve lines of 20 receivers each, with an inline spacing of 100 feet and a crossline spacing of 150 feet. This is essentially the array used in the SEP drill-bit experiment (Cole, 1991). The synthetic dataset contains a source at depth of 1000 feet. The figures below show semblance as a function of x and y for this depth. In other words what is shown are horizontal slices through the 3-D grid of possible source locations. All computations were made using a frequency of 50 Hz.

Figure **??** shows the semblance for a dataset containing only one source. Figure **??** is after adding another source of 10 times larger in amplitude at depth of 300 feet. This is the unwanted or interfering source. The first source is totally hidden by this stronger source. Figure **??** was obtained after removing the stronger source using the single-channel algorithm.

The next dataset also contains two sources, but the ratio of amplitudes is 50. Figure **??** shows the result of applying the single-channel algorithm to this dataset. The weaker source can not be seen after the subtraction. The result of applying the multi-channel algorithm is shown at Figure **??**. Obviously, here subtraction did a very good job of suppressing the unwanted source. Semblance for depth 1000 meters, one

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Figure 1: 1

source. Semblance for depth 1000 meters, two sources, ratio 1:10. Semblance after

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Figure 2: 2a

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Figure 3: 2a.rs

applying single-channel subtraction, two sources, ratio 1:10. Semblance after applying

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Figure 4: 2.rs

single-channel subtraction, two sources, ratio 1:50. Semblance after applying multichannel subtraction, two sources, ratio 1:50.

CONCLUSIONS

Frequency-domain methods can be used to attack unwanted sources in array data if we have an estimate of the signal of the unwanted source and if energy from this source is uncorrelated with that from other sources. If the interfering source is not too strong, a single-channel least squares method can do an effective job of removing the interfering source. For more difficult cases, I have demonstrated a multichannel method that looks promising. More work is needed to understand the theoretical basis of the multichannel method and to better understand when to use it.

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Figure 5: 2.mr

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