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Migration to zero-offset in variable velocity media

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ABSTRACT

Migration to Zero-Offset (MZO) or Prestack Partial Migration (PSPM) transforms prestack data into zero-offset data and is equivalent in a constant velocity medium to normal moveout correction (NMO) followed by the dip moveout correction (DMO) applied as a single step process. I present an algorithm to construct the MZO operator in variable velocity media as a potentially efficient alternative to full prestack migration. The MZO impulse response is computed using finite-difference travel-time maps, by considering the MZO process as the combination of full prestack migration followed by zero-offset modeling. The generalized impulse response can be applied to the data as an integral operator. I apply the operator to different synthetic models with depth variable velocity and show the improvements in stacking alignment over conventional DMO and NMO processing.

INTRODUCTION

Conventional seismic data processing most commonly comprises a processing sequence composed of normal moveout correction (NMO), dip-moveout correction (DMO) and stacking followed by post-stack migration. This sequence is correct when velocity is constant. Indeed, for constant velocity the sequence NMO, DMO, stacking and zero-offset migration is equivalent to full prestack migration and stacking (Hale, 1983).

Different algorithms have been proposed to perform DMO assuming a constant velocity medium. Yilmaz and Claerbout (1980) proposed a processing sequence called "Prestack Partial Migration" (PSPM) which represents the difference between the migration before stack and the conventional data processing scheme of NMO correction, stack and zero-offset migration.

Deregowski and Rocca (1981) proposed an integral formulation for DMO by considering the impulse response of the NMO+DMO operator as a transformation from constant-offset to zero-offset following raytracing arguments. This opened the road

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toward the development of the most flexible DMO method: DMO as an integral method. The kinematics of the impulse response are correctly determined by this method though the amplitude of the operator continues to be an area of research where different schools of geophysics do not reach an agreement.

Hale (1983) proposed a method to perform DMO via Fourier transforms which is kinematically accurate for all dips and offsets in constant velocity media. He also derived extensions to DMO processing to approximate slow velocity variations with depth. Hale's DMO method became the standard to which most DMO publications compare their. Zhang (1988) modified the Jacobian of Hale's DMO to obtain a true amplitude DMO operator.

Jakubowitz (1990) proposed an efficient version of DMO via Fourier transform by decomposing the data into a discrete number of dips, each processed separately with a dip-dependent stacking velocity. These dips are then added together after dip filtering so that each dip contributes only once to the final CMP stack. He also proposed in principle an extension to his algorithm for post-stack DMO.

Biondi and Ronen (1987) proposed an algorithm to perform DMO directly in shot profiles by multiplying the data in the Fourier domain by a time and space invariant shot-DMO operator.

These conventional DMO operators are kinematically correct when velocity is constant and they are approximately correct when velocity increases linearly with depth (Hale, 1983), but they cannot be applied when velocity varies laterally or when velocity varies relatively fast with depth and depth prestack migration is required to image the structure. In this case the migration to zero-offset (MZO) process cannot be split into NMO followed by DMO, but must be applied as a single process.

MIGRATION TO ZERO-OFFSET

I compute the impulse response of MZO by considering the process as the combination of full prestack migration followed by zero-offset modeling (Deregowski and Rocca, 1981; Deregowski, 1985). I use a fast algorithm to compute the travel-time map necessary for modeling wave propagation in a general 2-D medium (Van Trier and Symes, 1990). I applied the proposed algorithm for computing the kinematics of the impulse response when velocity is constant and for different velocity functions of depth. The impulse response in varying velocity can differ substantially from the constant velocity one (Popovici, 1990). I use the impulse response in a Kirchoff type algorithm applied to a series of synthetic models. As a result there is a better alignment of prestack data migrated to zero-offset than in the case of conventional algorithms, which imply better stacking for a range of different offsets.

The proposed algorithm for computing the impulse response of MZO using finite-difference travel-time maps is based on the principle that MZO is the combination of two processes: full prestack migration and zero-offset modeling. In a constant velocity medium this definition allows for an analytical formulation of the MZO operator which is identical with the DMO after NMO formulation (Popovici and Biondi, 1989). The algorithm used to investigate the MZO operator in variable velocity media follows the definition of the MZO method and can be divided in two parts:

- Construct the full prestack migration depth model. The depth model is the position in space of all points that can generate a given impulse in a constant-offset section. For a constant velocity medium this is equivalent to constructing the migration ellipse for a constant-offset section. For a variable velocity medium, the loci of points with equal travel-time from source to receiver form a curve resembling an ellipse or a superposition of several ellipses.

- Zero-offset modeling. Given the depth model, raytrace back at 90 degrees from the reflector, to model the zero-offset data. The intersection of the ray with the surface will give the x-coordinate of the MZO operator, while the travel-time along the raypath will provide the zero-offset time-coordinate.

The two conjugate MZO operators

Spreading data along the MZO impulse response is identical to summing along a conjugate curve. For zero-offset migration in a constant velocity medium, this is equivalent to saying: spreading the data over circles produces a result identical to summing along hyperbolas. From a computational point of view, the spreading operator (ellipse for DMO, circle for constant velocity migration) is a "**PUT**" operator, meaning that the operator takes a time sample and puts it in all the adjacent traces. The conjugate summation operator (hyperbola for constant velocity migration) is a "**GET**" operator because it gets the input from the adjacent traces.

By using finite-difference travel-time maps, both the "GET" and "PUT" operators can be calculated in very similar ways:

1. Compute a travel-time map for the source given a velocity model $v(x, z)$. Figure 1a is an example of a travel-time field calculated with the source located at $X = -1000$.

2. Compute a travel-time map with the source in the receiver location, given the same velocity model. Figure 1b is an example of a travel-time field calculated with the receiver located at $X = 1000$.

3. Sum the two travel-time maps. In each location of the grid we have the sum of the travel-time from source to receiver $T_{co}(x, z)$. Figure 1c is an example of a constant-offset travel-time field calculated by summing the source and receiver travel-time fields.

4. The gradient of the constant-offset travel-time field $T_{co}(x,z)$ is a vector which has the same direction as the zero-offset ray. In other words, the vector perpendicular to a constant-offset isochron bisects the angle between the ray coming from the source and the one returning to the receiver. This is obvious for some people, however, I demonstrate this proposition in the Appendix. Therefore, by calculating the gradient of the constant-offset travel-time map in each point of the grid, we can find the ray direction for the zero-offset case for each grid point.

5. Using the gradient table we can compute for each grid location the surface coordinate of the zero-offset $X_0(x,z)$, which is equivalent to finding the intersection of the surface with the zero-offset ray.

6. Using the gradient table we can compute for each grid location the zero-offset travel-time $T_0(x,z)$ along each zero-offset ray.

The algorithm is identical up to this point for the two conjugate operators. At this time there are three tables for our grid velocity model:

$$\begin{cases} T_{co}(x,z) \text{ the Constant-Offset Travel-Time Field.} \\ X_0(x,z) \text{ the Zero-Offset Surface Coordinate Field.} \\ T_0(x,z) \text{ the Zero-Offset Time Field.} \end{cases}$$

The PUT operator is obtained by associating for each value of $T_{co}(x,z)$ a pair of coordinates $(T_0(x,z), X_0(x,z))$. In other words, for a given value of the Constant-Offset Travel-Time, we find an isochron $T_{co}(x,z) = t_h = constant$ of coordinates (x,z) and the values of the $X_0(x,z)$ and $T_0(x,z)$ associated with that curve. The resulting operator is a function of $X_0(x,z)$ and $T_0(x,z)$ and a parameter $t_h = T_{co}(x,z) = constant$. Therefore, we can write it in the form $P(T_0(x,z), X_0(x,z); t_h)$ where the parameter t_h fixes the curve $T_{co}(x,z) = constant$.

The GET operator can be defined in a similar way by associating for each value of $T_0(x,z)$ a pair of coordinates $(T_{co}(x,z), X_0(x,z))$. In other words, for a given value of the Zero-Offset Time Field, we find an isochron $T_0(x,z) = t_0 = constant$ of coordinates (x,z) and the values of the $X_0(x,z)$ and $T_{co}(x,z)$ associated with that curve. The resulting operator is a function of $X_0(x,z)$ and $T_{co}(x,z)$ and a parameter $t_0 = T_0(x,z) = constant$. Therefore, we can write it in the form $G(T_{co}(x,z), X_0(x,z); t_0)$ where the parameter t_0 fixes the curve $T_0(x,z) = constant$. From the above formulation one can see the two operators (PUT and GET) are conjugate (adjoint) operators.

In practice, I obtain the PUT operator using a binary search algorithm in the table $T_{co}(x,z)$ (the binary search is possible because the velocity model is depth variable $v(z)$) and I store all pairs of values $(T_0(x,z), X_0(x,z))$ corresponding to an input time sample, which allows for all possible triplications in the operator. For a laterally varying velocity the binary search can be replaced with a linear search. The

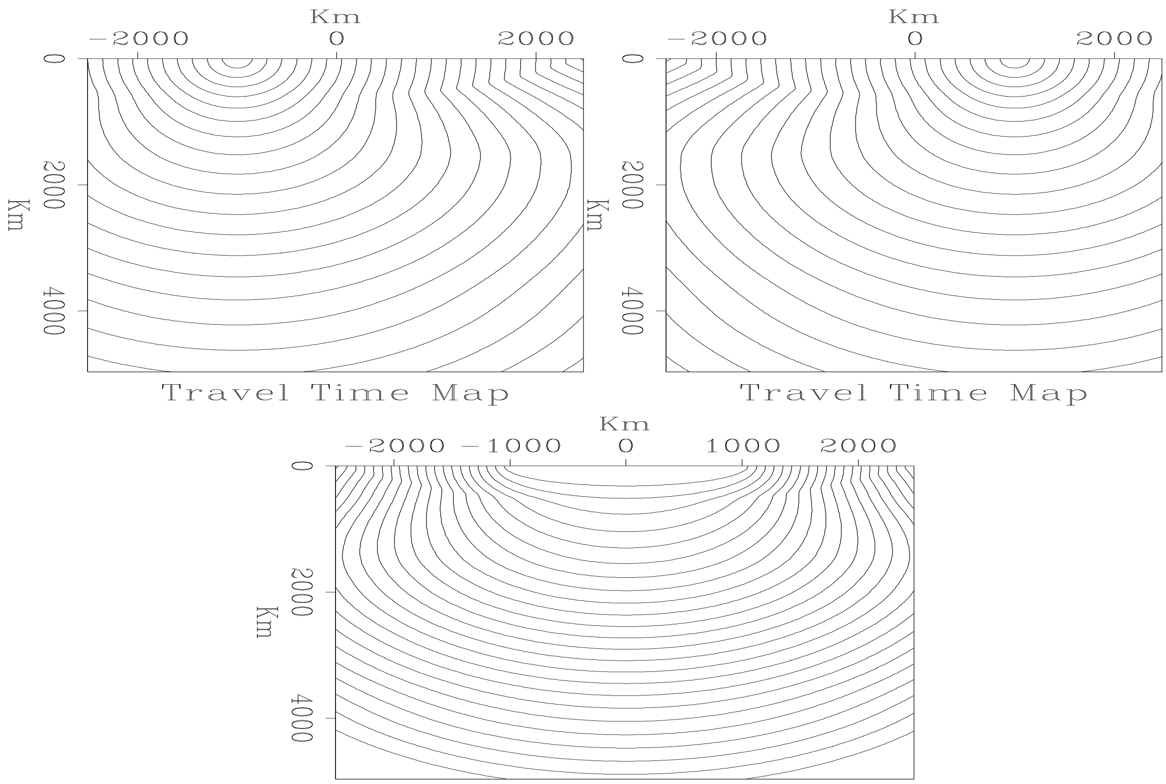

Figure 1: (a) Travel-time from a source located at $X = -1000$. (b) Travel-time from a source located at $X = 1000$. (c) Isochrons for a constant-offset model ($h = 1000$). mihai1-Rayabc [NR]

GET operator is obtained using a binary search algorithm in the table $T_0(x, z)$ and by storing all the pairs of values $(T_{co}(x, z), X_0(x, z))$ corresponding to an output time sample.

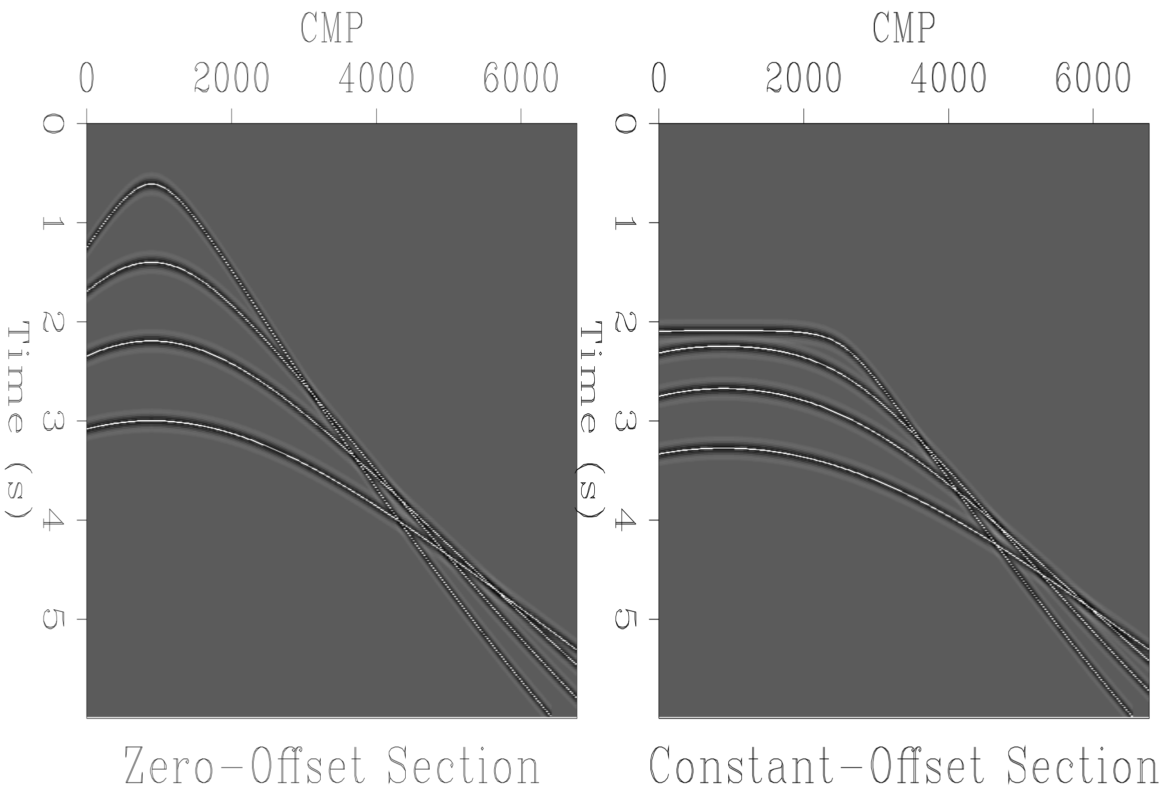

Figure 2: Zero-offset section and maximum constant-offset section for the constant gradient velocity. mihai1-model16 [NR]

MZO and DMO in variable velocity media

I applied the MZO operator to a series of depth variable velocity models. Each model consists of four diffractors placed at equal time depths. In Figure 2, I show the zero-offset section and the maximum constant-offset section for a velocity model with a linear increase in velocity. In Figure 3, I show the zero-offset section and the maximum constant-offset section for a velocity model with a more general depth variable velocity. The two velocity models are shown in Figure 4. The maximum constant-offset is 3325 m. The normal moveout correction and the dip moveout correction are, in essence, an approximation to the migration to zero-offset in a variable velocity medium. Ideally, after each process the flattened hyperbola generated by a diffractor in a constant-offset section should be migrated to a zero-offset hyperbola. The alignment of the events in Figures 5 and 6 should be a measure of success in migrating the constant-offset sections to zero-offset. Kinematically, the MZO operator shows better alignment, but the amplitude of the events is attenuated, due to the unsophisticated amplitude scheme I used for the MZO operator.

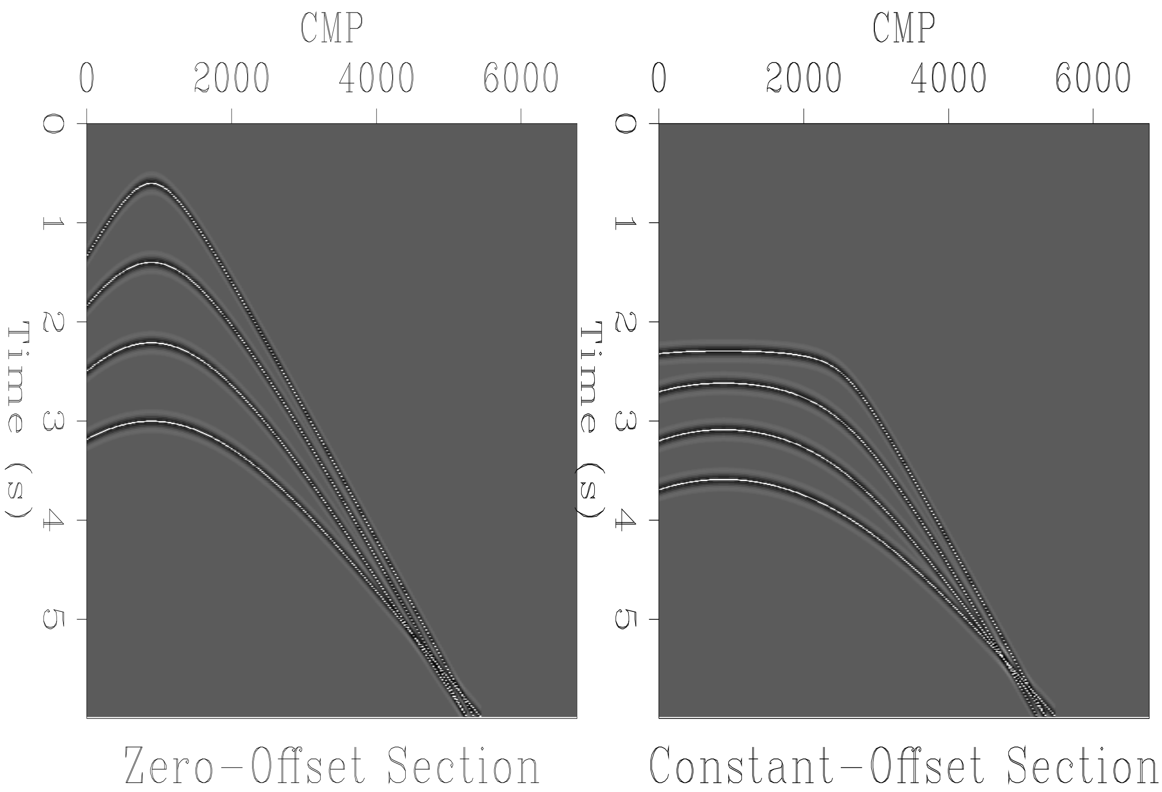

Figure 3: Zero-offset section and maximum constant-offset section for the depth variable velocity. mihai1-model46 [NR]

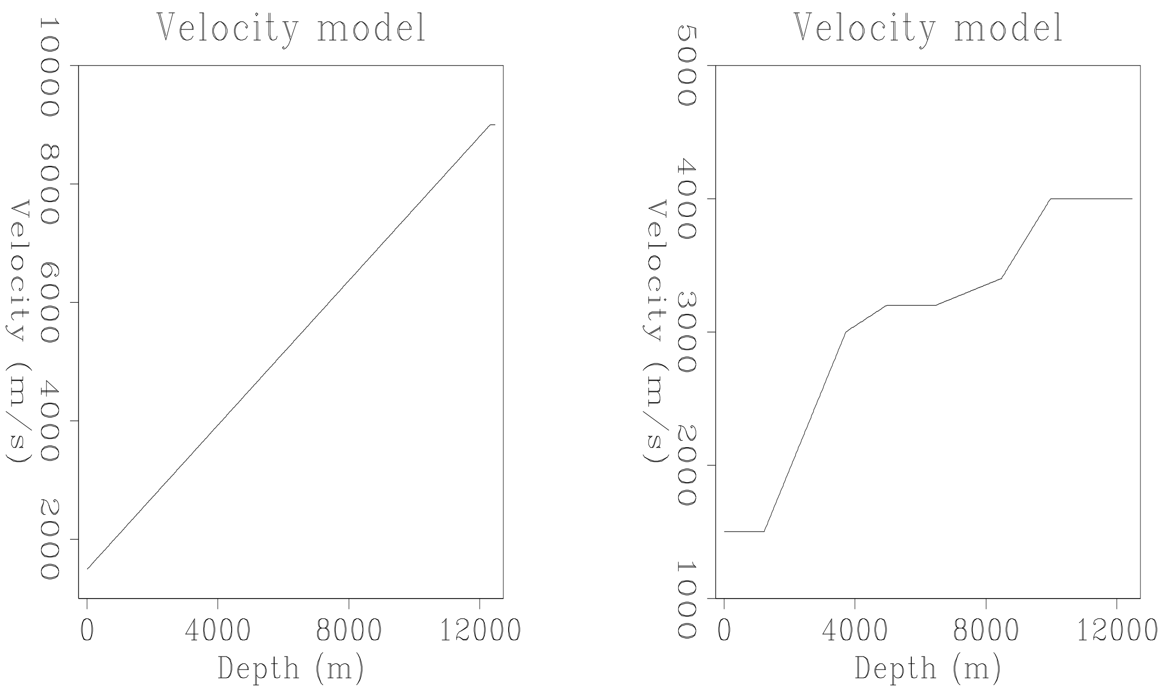

Figure 4: Velocity models for the two cases considered. mihai1-Velm1m4 [NR]

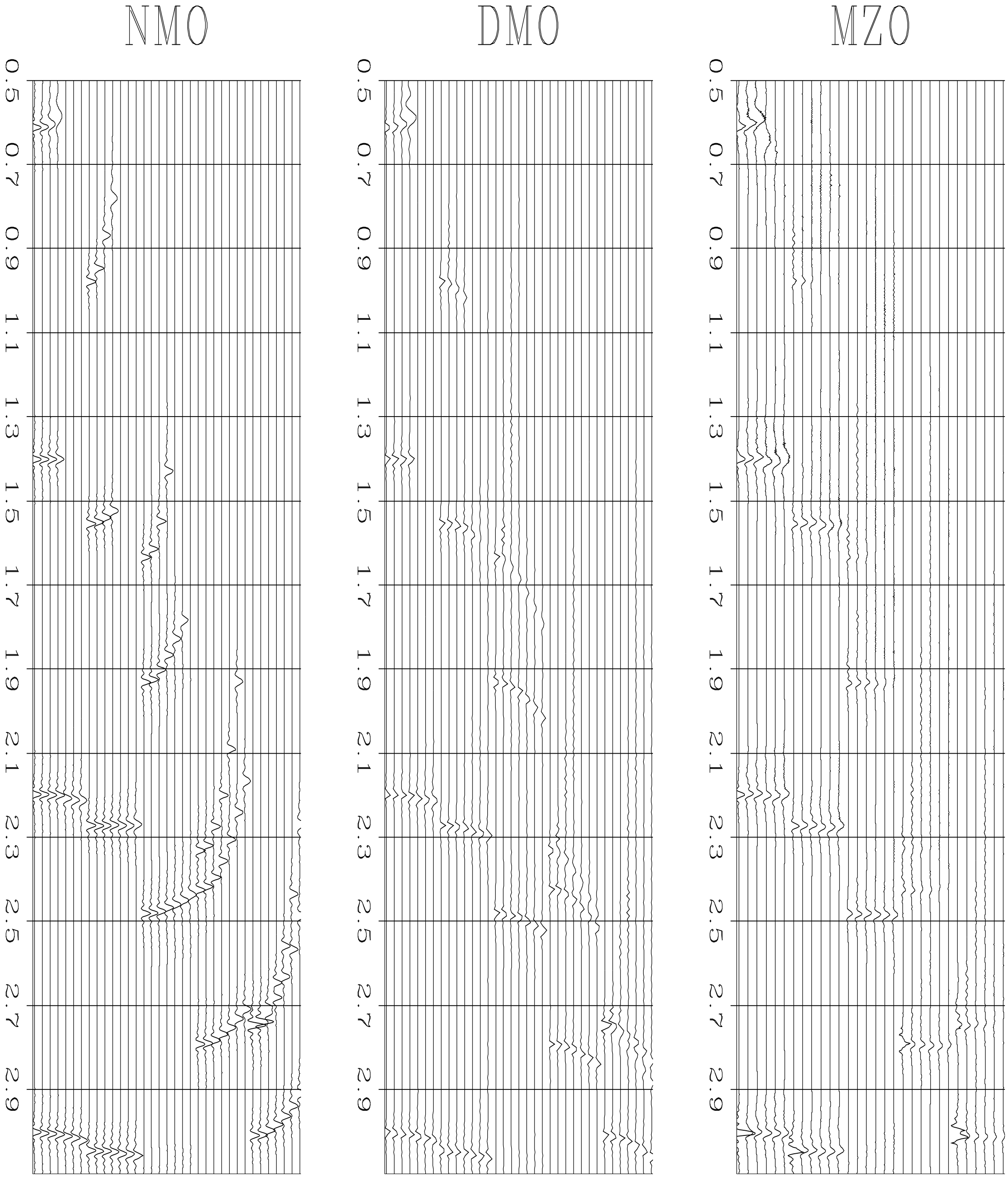

Figure 5: NMO, DMO and MZO applied to four diffractors in a medium with linear velocity increase. Migration to Zero-Offset should align all the events. There are five groups of seven traces in each plot, each group representing the same event at increasing offsets. The MZO operator shows better alignment but inferior amplitude preservation. mihai1-allm1 [NR]

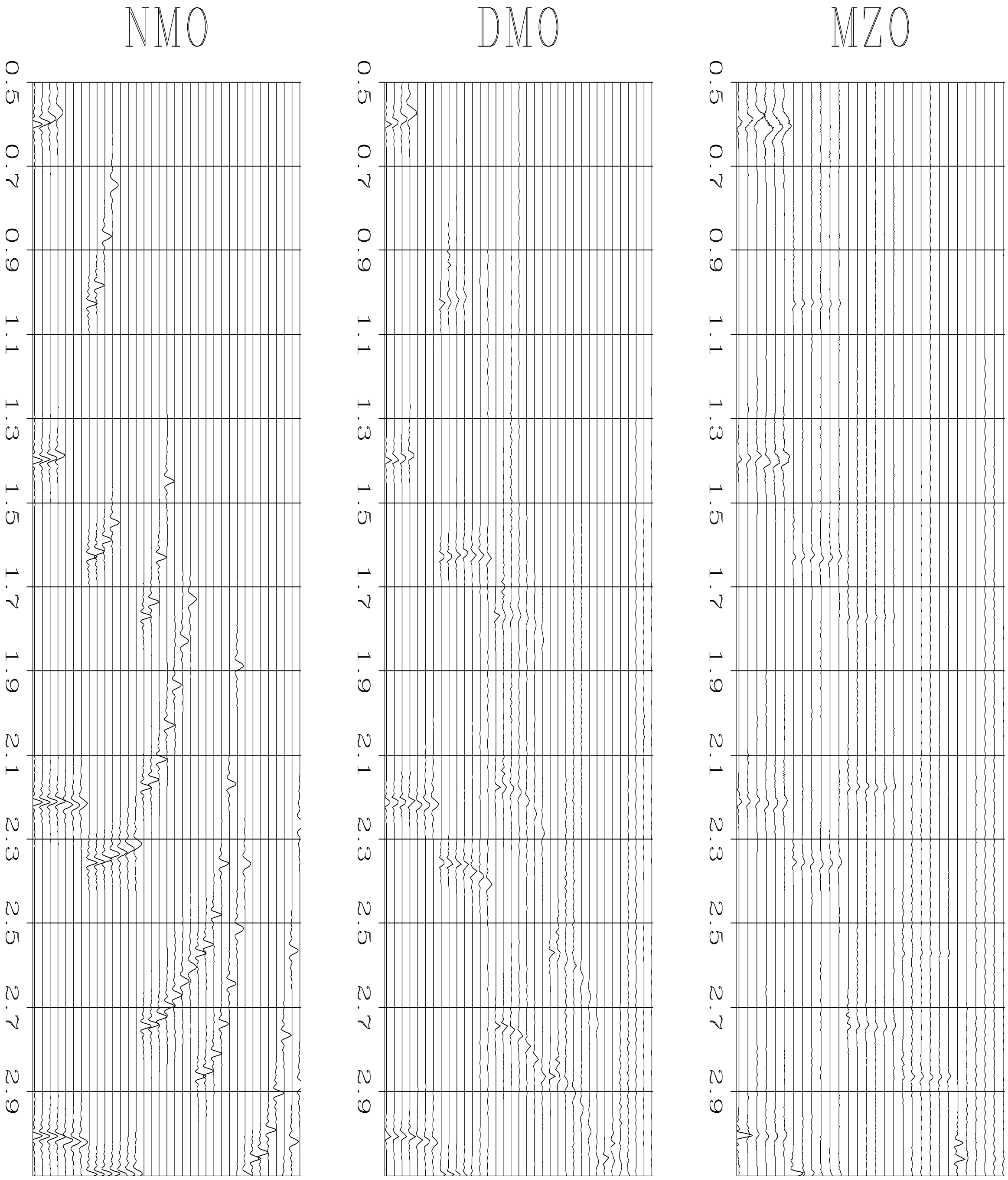

Figure 6: NMO, DMO and MZO applied to four diffractors in a medium where velocity varies in depth v(z). Migration to Zero-Offset should align all the events. There are five groups of seven traces in each plot, each group representing the same event at increasing offsets. The MZO operator shows better alignment but inferior amplitude preservation. mihai1-allm4 [NR]

How to get the amplitudes along the MZO operator

The amplitude scheme that I use for the MZO operator is naive. It is roughly equal to Zhang's (1988) true amplitude DMO operator minus the phase correction. I plan to develop a fast algorithm to calculate the amplitudes along the MZO operator. The principle of the algorithm is based on the definition of the MZO operator, which is an operator that transforms constant-offset data to zero-offset data. Given a single diffractor in a velocity model, we can model the zero-offset diffraction, convolving the kinematic curve with a particular wavelet. We do the same for a constant-offset section and obtain the flat top pseudo-hyperbola. The MZO operator applied to the flat top pseudo-hyperbola should transform the section into a zero-offset section. We know the kinematics of the MZO operator, which implies we know the input samples from the constant-offset section which are summed together in an output point in the zero-offset section. For each point in the zero-offset section, we have a number of input values along a particular impulse response of the MZO operator. To determine the amplitude for the input points along the MZO operator, we have to solve a linear system of equations, or a least-squares problem by taking as many output values as necessary from the zero-offset section and the corresponding input values along a particular MZO operator in the constant-offset section. We don't have to solve the system of equations for each output time sample, as we can later interpolate for the full operator.

CONCLUSIONS

I have presented an algorithm for computing the impulse response of the MZO operator for any velocity model. The MZO operator is obtained using travel-time maps. I examined several synthetic models in varying velocity media and compared the effect of the MZO operator to the more conventional sequence NMO and DMO. For the models shown, the MZO operator migrates the data to zero-offset better than the NMO+DMO sequence. However, the amplitudes along the MZO operator have to be computed with a better algorithm, which is a subject of ongoing research.

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APPENDIX

The proposition I want to prove is that the vector perpendicular to the constant-offset isochron bisects the angle between the ray coming from the source and the one returning to the receiver. This is the direction of the zero-offset ray, as the incident angle is equal to the reflected angle and the zero-offset ray bisects both angles. The gradient to a constant-offset isochron is a vector

$$\bar{R} = T_x \bar{i} + T_z \bar{j}$$

where

$$\begin{cases} T = \text{constant-offset travel-time field} \\ T_x = \frac{\partial T}{\partial x} \\ T_z = \frac{\partial T}{\partial z} \\ \bar{i} = \text{versor of } x \\ \bar{j} = \text{versor of } z \end{cases}$$

Figure 7: The gradient of the constant-offset travel-time field bisects the angle between the ray from the source and the ray to the receiver.

The constant-offset travel-time field can be decomposed into the sum of the travel-time from the source (T_s) and the travel-time to the receiver (T_r). The vector $\bar{R}$ can be therefore as the sum of vectors

$$\bar{R} = (T_{sx} + T_{rx})\bar{i} + (T_{sz} + T_{rz})\bar{j}$$

where $T_{sx}, T_{sz}, T_{rx}, T_{rz}$ are the partial derivatives of the travel-time from source and receiver, respectively. If we note

$$\begin{cases} \bar{R}_s = T_{sx}\bar{i} + T_{sz}\bar{j} \\ \bar{R}_r = T_{rx}\bar{i} + T_{rz}\bar{j} \end{cases}$$

then

$$\bar{R} = \bar{R}_s + \bar{R}_r$$

which are two vectors: $\bar{R}_s$ along the ray from the source and $\bar{R}_r$ along the ray from the receiver. The gradient of the constant-offset travel-time field is a sum of the two vectors $\bar{R}_s$ and $\bar{R}_r$, and because the two vectors have equal length $\|\bar{R}_s\| = \|\bar{R}_r\| = slowness$ (from the eikonal equation), the vector $\bar{R}$ bisects the angle between the vectors $\bar{R}_s$ and $\bar{R}_r$.