Stanford Exploration Project, Report 72, December 18, 1997, pages 1–244

Stanford Exploration Project, Report 72, December 18, 1997, pages 1–244

Traveltime picking through phase spectrum calculation

Lin Zhang and Jon Claerbout[1]

keywords: *phase, wavelet, Fourier transform, algorithm*

ABSTRACT

Traveltime tomographic inversions of seismic data usually require picking the arrival times of certain events recorded by receivers. We have developed an algorithm that picks the traveltime of an event according to the phase properties of the wavelet of the event. If the event has a minimum phase wavelet, the algorithm picks the first break of the wavelet. If the wavelet of the event has a zero phase spectrum, the central point of the wavelet is picked. For any mixed phase wavelet, the algorithm picks the first break of the causal, minimum phase part of the wavelet. Numerical experiments show that, for isolated, noise-free events, the algorithm performs well. However, for field data with strong, spiky noises, the algorithm fails to pick the correct traveltimes.

INTRODUCTION

Picking the traveltimes of the events on seismic sections plays an important role in many velocity inversion methods. For example, in traveltime tomography using cross-well data, the first step is to pick traveltimes of the events on recorded sections. There are many factors that can cause errors in traveltime picking. First, the wavelets of the events to be picked are often band-limited. Determining the exact picking positions of the finite duration wavelets is rather difficult. And precursors introduced in the preprocessing and noises in the data make this picking task even harder to accomplish.

In practice, a tentative rule for traveltime picking is to identify the first breaks of the wavelets. However, this rule cannot be applied to the cases where precursors are introduced by preprocessing. An extreme example is the data collected with a vibrator-type source. After we cross-correlate data with the known source signature, the wavelets in the data have a zero phase spectrum; hence the central points of the wavelets should be picked. Such situations suggest that the determination of the picking position should depend on the type of wavelet.

[1]**email:** not available

In this paper, we describe a picking algorithm that chooses the picking position according to the phase spectrum of the wavelets. The paper is organized in the following way. First, we relate the traveltime picking problem to the estimation of the time delay of the source wavelet in the recorded data, and define our convention for determining the picking position by factorizing the spectrum of the source wavelet. A formula is then derived to calculate the time delay from the phase spectrum of the data. We describe two methods for calculating the phase spectrum of data (Oppenheim and Schafer, 1975). The first method uses the Fourier transform to find the principal phase spectrum and performs phase-unwrap by adding appropriate multiples of 2π. The second method finds the phase spectrum by averaging the group delay over the frequencies. Then, taking into consideration the presence of noises in field data, we modify the second method by including a weighting function in the integration. We use both synthetic wavelets and wavelets recorded in the field to demonstrate the picking results. We also show two picking results for Walk-Cross data.

THEORY

Let $w(n)$ be a source wavelet of length N. Its Z-transform can be factorized into a minimum phase part and a maximum phase part as follows:

$$W(Z) = W_{min}(Z) W_{max}(Z), \tag{1}$$

if none of its zeros are exactly on the unit circle. The received signal can be modeled as a delayed source wavelet as follows:

$$s(n) = \alpha w(n - n_0), \tag{2}$$

or in the frequency domain,

$$S(Z) = C W(Z) Z^{n_0}, \tag{3}$$

where C is a constant. The amount of delay n_0 is equal to the traveltime it takes for the wavelet to propagate from the source location to the receiver location. This equation shows that traveltime picking can be equivalently accomplished by estimating n_0.

Determining the picking position

We choose the convention that the picking position of a wavelet is the first break of the minimum phase part and the ending point of the maximum phase part. The minimum phase part is causal and the maximum phase part is anti-causal:

$$\begin{aligned} W_{min}(Z) &= \prod_{k=1}^{N_{min}} (1 - \alpha_k Z) \\ W_{max}(Z) &= \prod_{k=1}^{N_{max}} (1 - \beta_k Z^{-1}), \end{aligned} \tag{4}$$

where α_k and β_k are generally complex with their norm less than one, and $N_{min} + N_{max} = N$. Clearly, this choice ensures that for a minimum phase wavelet, the first break of the wavelet is picked, and that for a zero phase wavelet, the center point of the wavelet is picked.

To estimate n_0, we need to examine the phase response of $S(Z)$. Let $\Phi_s(\omega)$ be the phase spectrum of $S(e^{j\omega})$, and $\Phi_w(\omega)$ the phase spectrum of $W(e^{j\omega})$. Equating the the phase spectra of the two sides of equation (3) yields

$$\Phi_s(\omega) = \Phi_s(0) + \omega n_0 + \Phi_w(\omega). \tag{5}$$

Because $w(n)$ is a real sequence and has causal minimum phase and anti-causal maximum phase parts, its phase spectrum is anti-symmetric with respect to $\omega = \pi$:

$$\Phi_w(2\pi - \omega) = -\Phi_w(\omega) \quad 0 \leq \omega \leq \pi.$$

Using this condition, we can solve equation (5) for n_0 as follows:

$$n_0 = \frac{1}{2\pi}[\Phi_s(2\pi - \omega) + \Phi_s(\omega) - 2\Phi_s(0)]. \tag{6}$$

By setting $\omega = \pi$, we see that n_0 is the phase delay of the received signal at the Nyquist frequency:

$$n_0 = \frac{\Phi_s(\pi) - \Phi_s(0)}{\pi}.$$

Calculating the phase spectrum

The phase spectrum is usually calculated by taking the arctangent of the ratio of imaginary to real parts of the Fourier transform. Because the arctangent function is a multivalue function, its principal value has to be determined as being, for example, between $-\pi$ and π. However, the phase spectrum calculated from the principal-value range is not a continuous function. It does not satisfy equation (5). One way to solve this problem is to add appropriate multiples of 2π to the samples of the principal value. The appropriate multiple of 2π can be determined by assuming that the sampling interval is sufficiently small so that the discontinuities between adjacent samples of the phase spectrum correspond to 2π phase-wrap-around. If the phase spectrum varies rapidly, zero padding should be done in the time domain to ensure that there is a sufficiently small sampling interval in frequency.

Now let us look at some examples. Figure **??** shows a minimum wavelet with a delay of 0.08 second. After the spectrum factorization, the minimum phase part is the original wavelet and the maximum phase part is a spike. We see that the algorithm picks the first break. Figure **??** shows the results in the case of a zero phase wavelet. As expected, the central peak of the zero phase wavelet is picked. In both cases, the correct delays are found. Figure **??** shows the picking result of a trace recorded in a marine survey. The wavelet generated by an air-gun propagates through water and

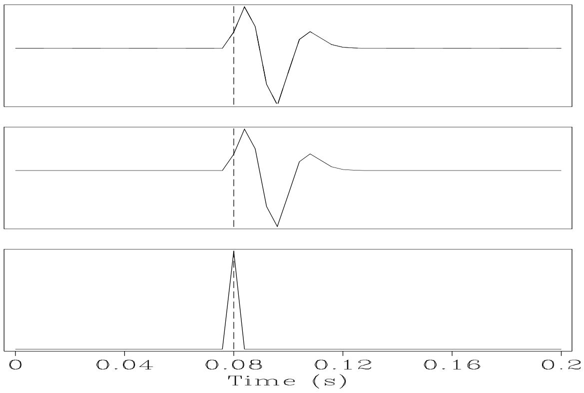

Figure 1: The top panel shows a synthetic, minimum phase wavelet delayed for 80 ms. The middle and bottom panels show the minimum phase and maximum phase parts of this wavelet, respectively. The vertical dashed lines indicate the picking positions. The fat, dashed curve superposed on the curve of the top panel is the signal reconstructed from two factorized parts. lin4-swhfactm [ER]

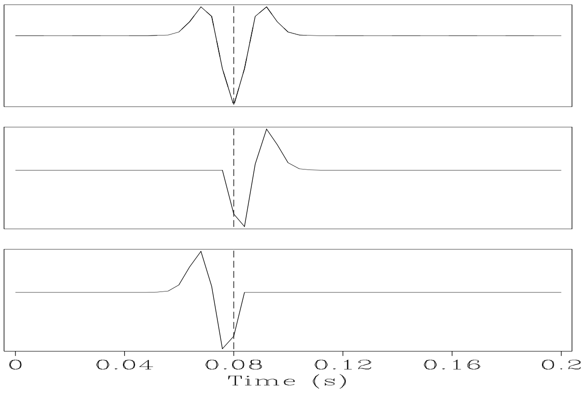

Figure 2: The top panel shows a synthetic, zero phase wavelet delayed for 80 ms. The middle and bottom panels show the minimum phase and maximum phase parts of this wavelet, respectively. The vertical dashed lines indicate the picking positions. The fat, dashed curve superposed on the curve of the top panel is the signal reconstructed from two factorized parts. lin4-swhfactz [ER]

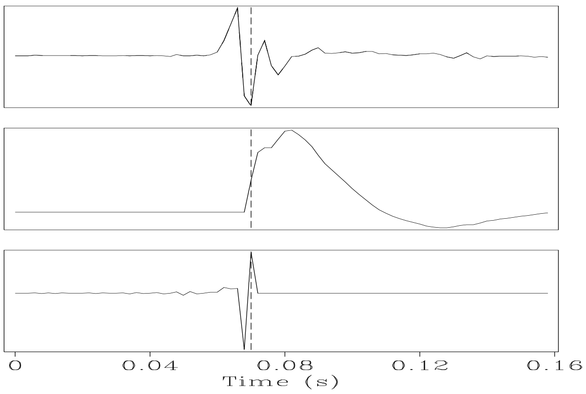

Figure 3: The top panel shows a trace recorded in a marine survey. The middle and bottom panels show the minimum phase and maximum phase parts of this wavelet, respectively. The vertical dashed lines indicate the picking positions. The fat, dashed curve superposed on the curve of the top panel is the signal reconstructed from two factorized parts lin4-vwhfact [ER]

is recorded by a hydrophone. It is apparent that the picking position is not the first break. The minimum phase part contains mainly low frequency components while the maximum phase part contains only high frequency components. The non-overlapping spectra of two parts indicate that the factorization algorithm may become numerically unstable. However, the instability of the spectrum factorization does not affect the results of traveltime picking because the formula for calculating n_0 uses the total phase response. The spectrum factorization is only used to explain the principle of picking.

Factorizing for minimum phase and allpass wavelets

Using the Kolmogoroff spectrum factorization method (Claerbout, 1985), one can find the minimum phase wavelet that has the same amplitude spectrum as a given wavelet. Apply this algorithm to $S(Z)$, we obtain its minimum phase equivalent $W_{min}(Z)W_{max}(Z^{-1})$. Then,

$$\begin{aligned} S(Z) &= \alpha W_{min}(Z)W_{max}(Z)Z^{n_0} \\ &= \alpha Z^{n_0}\frac{W_{max}(Z)}{W_{max}(Z^{-1})}W_{min}(Z)W_{max}(Z^{-1}) \\ &= \alpha P(Z)Q(Z), \end{aligned} \tag{7}$$

where $Q(Z) = W_{min}(Z)W_{max}(Z^{-1})$ is a new minimum phase wavelet and

$$P(Z) = Z^{n_0}\frac{W_{max}(Z)}{W_{max}(Z^{-1})}$$

is an allpass wavelet. To estimate n_0, we can apply the method explained in the last section to $P(Z)$. Figure **??** shows an example with the same field data as in Figure **??**. We see that two factors of this factorization process have similar frequency content. As expected, the traveltimes picked by two methods are equal. Because the phase spectrum of an allpass filter is always a nondecreasing function, the procedure for unwrapping the phase of $P(Z)$ is easier than that for $S(Z)$.

Averaging group delay

Another way to overcome the problem of phase-wrap-around is to compute the phase spectrum by integrating its derivative. The phase spectrum of a signal is related to its Fourier transform as follows:

$$\Phi_s(\omega) = \Im[\ln S(\omega)]. \tag{8}$$

Taking the derivative with respect to ω on both sides gives

$$\frac{d\Phi(\omega)}{d\omega} = \Im[\frac{S'(\omega)}{S(\omega)}]. \tag{9}$$

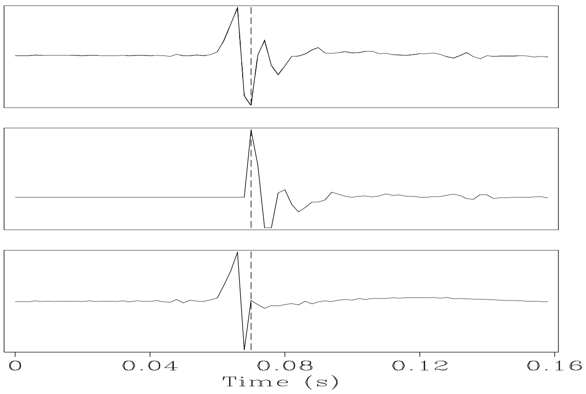

Figure 4: The top panel shows the same trace as one in Figure ??. The middle and bottom panel show the minimum phase and allpass parts of this wavelet, respectively. The vertical dashed lines indicate the picking positions. The fat, dashed curve superposed on the curve of the top panel is the signal reconstructed from two factorized parts. lin4-vwkfact [ER]

If we assume $\Phi_s(0) = 0$, then

$$\Phi_s(\omega) = \Im \int_0^{\omega} \frac{S'(\omega)}{S(\omega)} d\omega. \tag{10}$$

The phase spectrum computed in this way is continuous. Furthermore, it can be shown (Oppenheim and Schafer, 1975) that

$$n_0 = \frac{1}{2\pi j} \int_{-\pi}^{\pi} \frac{S'(\omega)}{S(\omega)} d\omega. \tag{11}$$

If we define the group delay as

$$n_g = \frac{d\Phi_s(\omega)}{d\omega}, \tag{12}$$

then equation (11) tells us that n_0 is an average group delay over frequency range $[-\pi, \pi]$. Considering the limited band width of the seismic signals and the presence of noises, we should include uneven weights in the averaging step of equation (11) as follows:

$$n_0 = \frac{1}{j} \int_{-\pi}^{\pi} M(\omega) \frac{S'(\omega)}{S(\omega)} d\omega, \tag{13}$$

where $M(\omega)$ is a weighting function. One choice of the weighting function is

$$M(\omega) = \frac{|S(e^{j\omega})|^{\gamma}}{\displaystyle\int_{-\pi}^{\pi} |S(e^{j\omega})|^{\gamma} d\omega}.$$

If $\gamma = 0$, the result is the evenly weighted average of the group delay. If $\gamma = \infty$, the result is equal to the group delay of the frequency corresponding to the peak amplitude spectrum.

APPLICATIONS

We have applied the algorithm to two types of dataset. The first is a zero-offset section, and the second is a common receiver gather in a Walk-cross survey. In both examples, equation (13) with $\gamma = 1$ is used to compute traveltimes.

Picking sea bottom reflections

Figure **??** shows a zero-offset section recorded in a marine survey. Each trace was recorded by a hydrophone below the sea surface, with an air-gun above it. On the section, the first flat event is the direct arrival of down-going waves from sources to receivers. The picked traveltimes are superposed on the event. Because the event is isolated, and noises and wavelet distortions are small, the curve that indicates the picked traveltimes follows the event quite well. The second event is the primary reflection from the sea bottom. The event bends upwards at the left end. The curve indicating the picked traveltimes is with the event at most locations. However, at those locations where strong, spiky noises appear, the algorithm fails to pick the correct traveltimes. The algorithm is sensitive to spiky noise because adding a large-amplitude spike to a wavelet considerably changes the phase property of the wavelet.

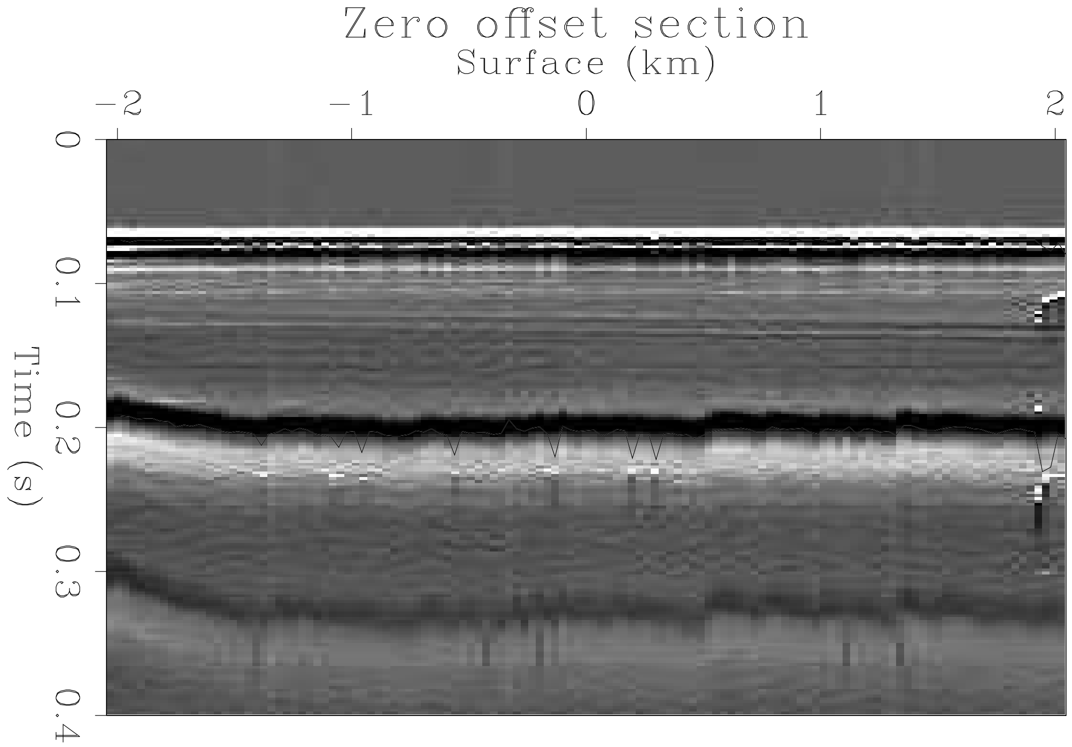

Figure 5: Picked traveltimes for the direct arrival and the primary sea bottom reflection. The picks of the primary see bottom reflection are attracted to large spiky additive noises of unknown origin. lin4-zospick [ER]

Traveltime picking of the first arrival

The common receiver gather used in this example was recorded in a marine Walk-cross survey, in which, a receiver was loaded down into a bore-hole. The receiver records the waves generated by the shots along the surface. If the medium above the receiver were homogeneous, the first arrival on the common receiver gather would follow a hyperbola. We fitted a hyperbola to the first arrival on this gather and performed the moveout corrections. Figure **??** shows the first arrival after the moveout corrections. The event is not flattened because the actual medium is not homogeneous. The residual time shifts of the event can be used to estimate the velocity anomalies of the media. We then applied our algorithm to estimate the residual shifts. The result is superposed on the event. Figure **??** compares the wavelets of the event before and after residual shift corrections. The wavelets do not line up perfectly at a point because their waveforms are distorted from one another.

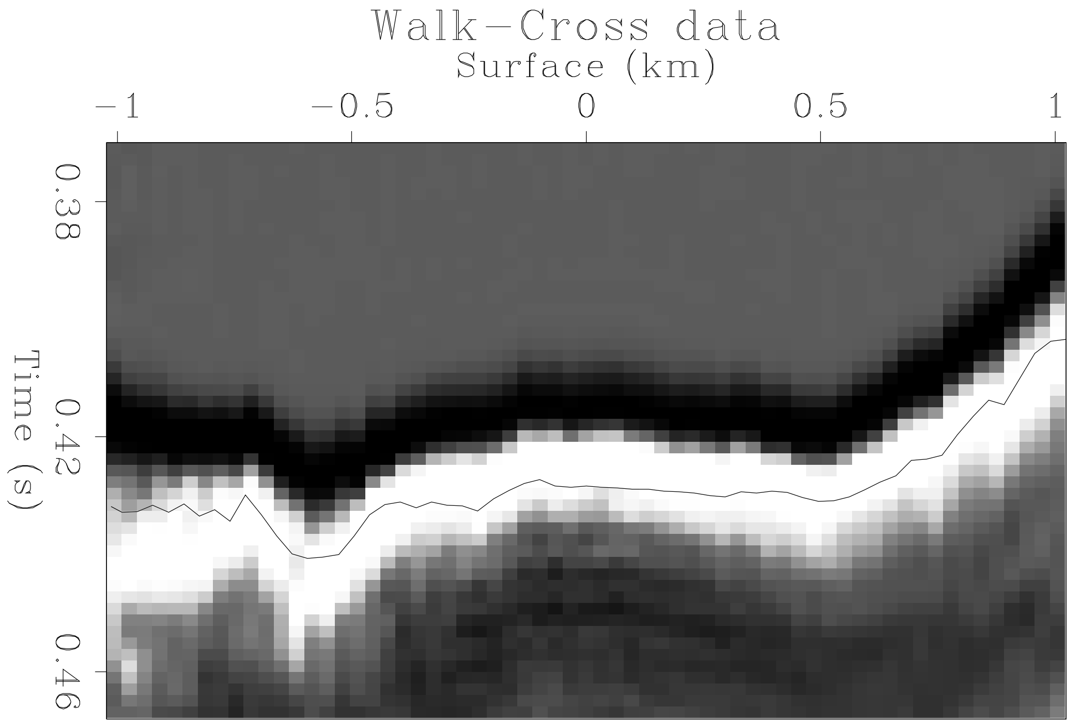

Figure 6: Picked residual shifts of the first arrival. lin4-vsppick [ER]

CONCLUSIONS

This paper presents an algorithm for picking the traveltimes of seismic events. We model seismic events as delayed wavelets and define the picking position as the first break of the minimum phase part of a wavelet. This definition accords with usual practice. For minimum phase wavelets, the first breaks are picked. For zero phase wavelets, the central peaks are picked. Two implementations of the algorithm are

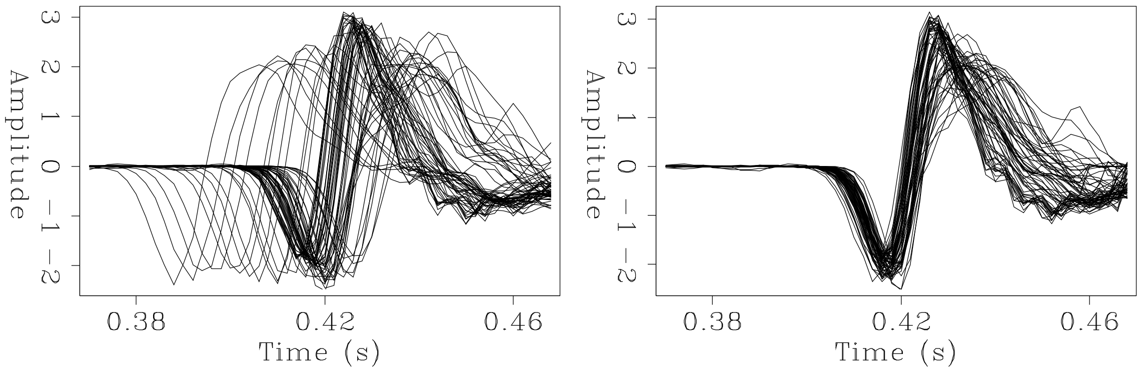

Figure 7: Wavelets before (left) and after (right) the residual shift corrections. lin4-vspcor [ER]

given. For isolated, noise-free events, the algorithm works well. But examples with field data reveal that the algorithm fails to pick the correct traveltimes when spiky noises are present.

REFERENCES

Claerbout, J.F., 1985, Fundamentals of Geophysical Data Processing: McGraw-Hill Inc., 59–62.

Oppenheim, A.V., and Schafer, R.W., 1975, Digital Signal Processing: Prentice-Hall Inc., 500–510.