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Spatial aliasing

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ABSTRACT

The plane wave model links an axis that is not aliased with one that might be.

INTRODUCTION

Spatial aliasing is often described as being like temporal aliasing but on another axis. Mathematically, aliasing on the two axes is independent, however, the plane-wave model links them in our day-to-day experience. Here we examine spatial aliasing as it manifests itself on plane waves.

CHIRP EXAMPLE

A function generally useful with seismic vibrator sources is the "gated-chirp function". Within a specified time gate this function is a sinusoid with a time-variable frequency. Here we will take this signal to be defined by

$$f(\tau) \quad = \quad \begin{cases} \sin(0.5\,\omega_{\max}\tau^2/\tau_{\max}) & 0 \leq \tau \leq \tau_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The phase ϕ and instantaneous frequency ω of this chirp signal is

$$\phi \quad = \quad 0.5\ \omega_{\max}\ \tau^2/\tau_{\max} \tag{2}$$

$$\omega \quad = \quad \frac{d\phi}{d\tau} \quad = \quad \omega_{\max}\ \tau/\tau_{\max} \tag{3}$$

which shows that as τ ranges from 0 to $\tau_{\max}$, the frequency ω ranges from 0 to $\omega_{\max}$.

Now consider the chirp signal seen on a wave, say $f(t - px)$. For constant x we see a chirp signal, possibly shifted on the time axis. For constant t we see a chirp function along the x axis, but the $-p$ causes the chirp to be reversed and stretched. Figure 1 shows the chirp wave function $f(t - px)$ for $p = 0.45$ and for $\omega_{\max} = 2\pi$.

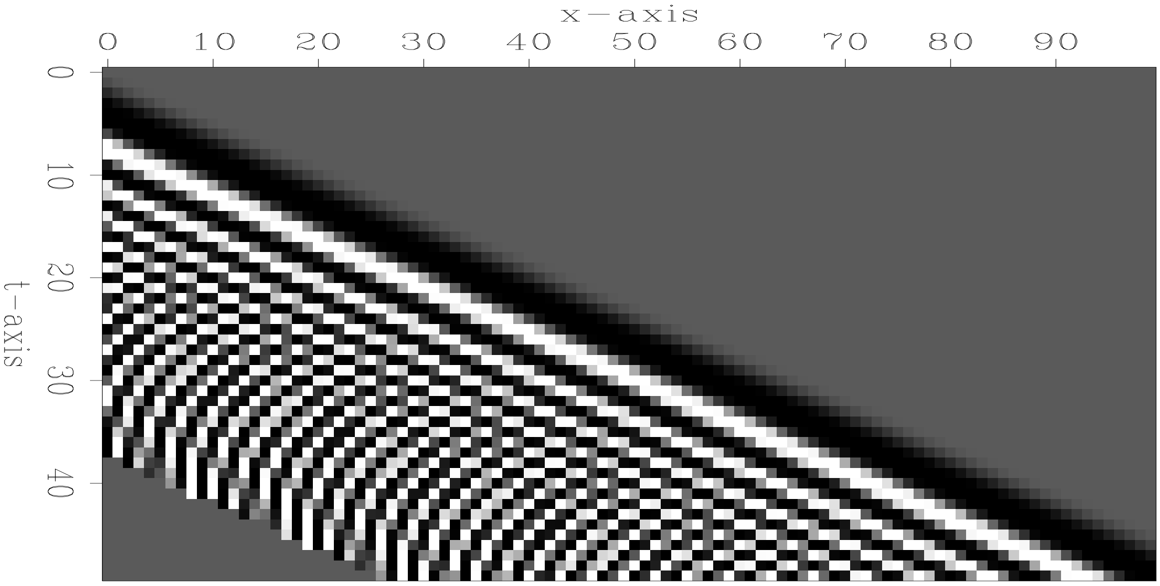

Figure 1: A coarsely sampled chirp function on a wavefront. jon2.alias-alias2d [ER]

Since $\omega_{\max} = 2\pi$ is double the Nyquist frequency, you see aliasing on the time axis in Figure 1. In other words, along the left edge, the frequency first increases with time until it reaches the Nyquist frequency and then, as frequency goes beyond Nyquist, the apparant frequency decreases. Something else happens on the space axis which is best seen along the bottom edge of the plot. There the frequency sweeps steadily from zero to Nyquist.

Thus, we see that the continuum is different from the sampled world. In a continuum, the function $f(t - px)$ at constant t looks like the same function at constant x except for shifting, reversal, and stretching. In the sampled world we see different functions on each axis. The sampled world matches the continuum only at low frequencies.

In real life, figures like Figure 1 do not arise because it is easy to sample densely enough in time, or to high-cut filter the data before sampling it. Figure 1 might be a starting point, however, for studying whether and why travel times of waves can be mapped more precisely than the time sampling interval, Δt.

To convert Figure 1 to something closer to real life we need only change the given figure from $|p| < 1$ to $|p| > 1$. There is no need for another figure though because we need only interchange the time axis and the space axis in Figure 1. This leads to the most elementary description of spatial aliasing: Spatial aliasing is when a spatial frequency looks lower than it would if the data were more densely sampled in space. What we seek now are more analytic handles on spatial aliasing.

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ORTHOGONALITY OF CROSSING PLANE WAVES

As an alternative to the customary approach of defining and analyzing aliasing by Fourier analysis, I suggest that spatial aliasing may be defined and analyzed with reference to plane waves. Consider two sinusoidal signals of different frequencies. Since they have different frequencies they should be orthogonal, but they need not be orthogonal if there is aliasing, because aliasing can make a high frequency look like a low one. Now let us see if we can substitute the word "dip" for the word "frequency." Can we say that plane waves with different dips should be orthogonal?

Normally, waves do not contain zero frequency. Thus the time integral of a waveform normally vanishes. Likewise, for a dipping plane wave, the time integral vanishes. Likewise, a line integral across the (t, x)-plane along a straight line that crosses a plane wave or a dipping plane wave vanishes. Likewise, two plane waves with different slopes should be orthogonal if one of them has zero mean. The other wave, however, need not have zero mean. This "other wave" need not be a wave at all, but could be an impulse function, say $\delta(t-px)$, spread over a mesh. Once it is spread out, it will look like an impulsive plane wave. The purpose of this impulsive plane wave is to multiply onto data, thereby enabling us to do a line integral of the data. What is important is how we represent this impulse function on the mesh. Theoretically, we have line integrals that should vanish when they cross a zero-mean plane wave. When in practice they do not, it means that we have not done a good job of representing the line integral.

Fourier analysis and sinc functions play an important role in integrals along straight lines. Production reflection seismic data processing involves much effort with line integrals along hyperbolic curves and other shapes—rarely straight lines. Hopefully, the theoretical concepts above will be suggestive of further theoretical and practical developments.