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Crossline regridding by inversion

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ABSTRACT

I sketch the regression strategy I am planning for crossline regridding and interpolation by iterative migration.

INTRODUCTION

The ultimate goal of seismic inversion is rock parameter estimation and mapping. Should a successful inversion be achieved, a byproduct is regridding on a mesh of arbitrary refinement. In an attempt to bridge the gap between theory and practice here I set a goal lower than full inversion: regridding post-stack crossline marine reflection data. The problem has real economic significance and is shared by seismology in all its scales, from earthquake seismology to engineering seismology. Indeed, this problem transcends seismology and permeates all of reconnaissance geophysics. "Geophysical inversion" purports to offer a solution to the regridding problem. Since it is so rarely used successfully for that purpose, we see a measure of the departure of theory from practice that tells us that there is much work to do and many remaining opportunities. I chose crossline marine data because inline stacks typically give data of high quality, principally marred by the irregularity of the crossline spacing.

The top two frames of Figure 1 show the synthetic data I made for this study. The synthetic data in Figure 1 is on a 200×200 mesh. The right frame shows the data with 150 of 200 channels deleted by an algorithm that considers traces 4 at a time and then randomly selects one to be kept while the other 3 are deleted. The middle row of the figure shows that the method of Claerbout (1992) is not powerful enough to do a fully satisfactory job of filling in the missing traces because of the many smiles in migration shown in the bottom row. My hypothesis is that I should be able to improve the data interpolation based on the concept that the migration on the bottom right should have smiles removed by the principle that the migration should leave a local monoplane as described in Claerbout (1992).

The number of missing data points is $150 \times 200 = 30,000$. The migration-diffraction program for the synthetic data runs both ways in a few seconds. Thus,

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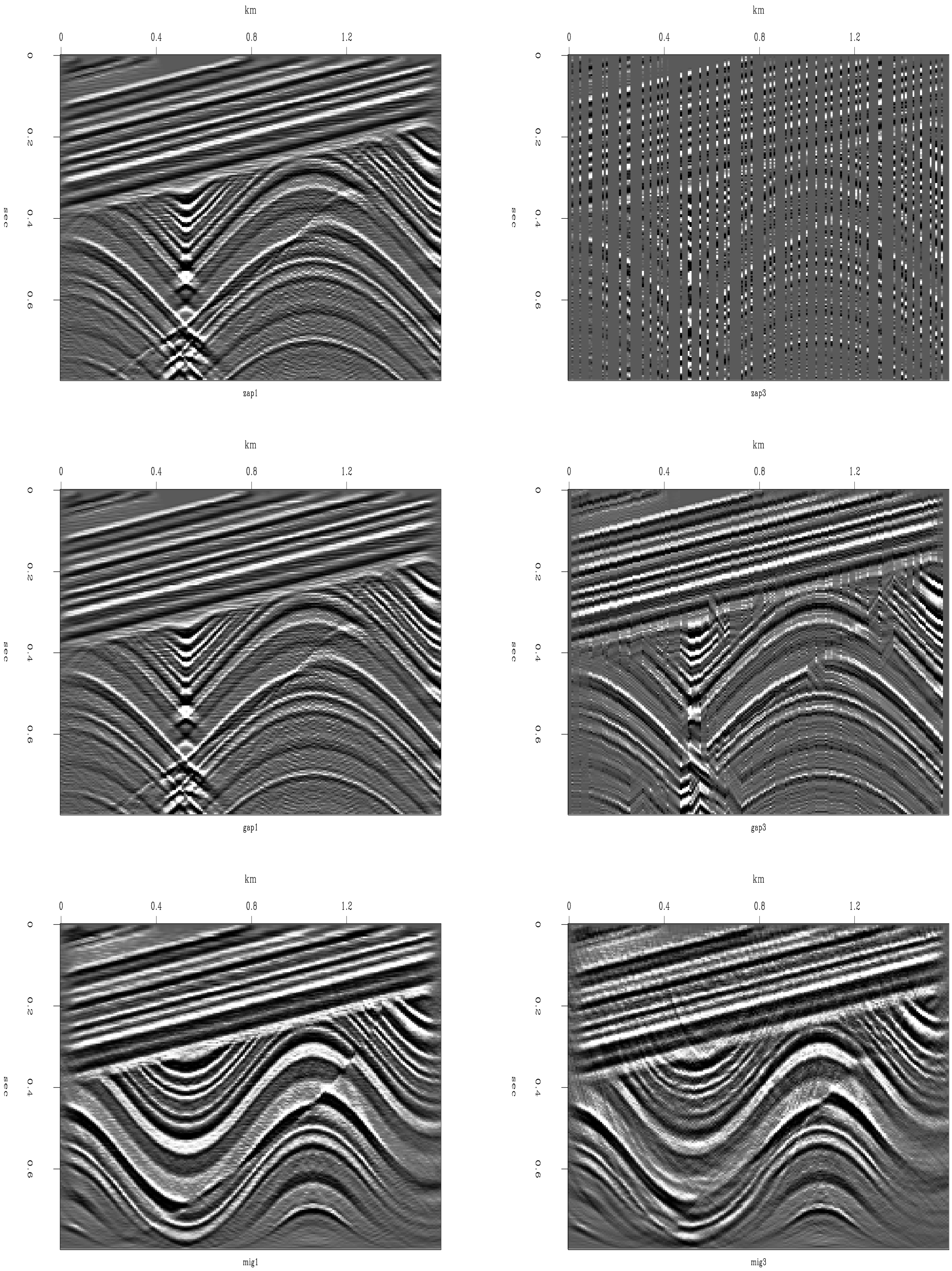

Figure 1: Top is data. Left is without gaps. Right is with gaps. Middle is linear interpolation over gaps. Bottom is migration. $\boxed{\text{jon7.mismig-gapmig}}$ [ER]

theoretically, a linear least squares solution for the unknowns should run in a day or two. On the discouraging side, the problem is not one of *linear* least squares, and the code has not yet been written. Nonlinear least squares arises because we must simultaneously estimate the model covariance matrix. On the encouraging side, SEP has a parallel computer and the times quoted above are for my desktop workstation. Also, satisfactory results may be found long before the theoretically required 30,000 iterations.

To be a success, I believe I should achieve useful results in 100 or fewer iterations, preferably in a handful of iterations. Fundamentally, many issues arise. Is a preconditioning strategy essential? Damping is an important part of any inversion formulation. Does damping speed the arrival of a useful solution? Figure **??** suggests that nearest-neighbor interpolations could frustrate iterative migrations. Will it? Is a conversion to the antialias method of (Claerbout, 1992a) mandatory or advisable? (That would require vectorizing that code over the midpoint axis, a non-trivial undertaking.) How can/should the covariance matrices be bootstrapped? Because of this bewildering array of imponderables, and because of my many frustrating experiences with iterating least-squares problems of high dimensionality (Claerbout, 1990) and others, unpublished, I chose not to attack the problem frontally, but begin by examining the ingredients to any inversion, the gradient, the gradient after filtering with the inverse-covariance filters, etc.

DEFINITIONS

Ultimately I hope to do this problem exactly the way theory says to do inverse problems. Meanwhile, I need short cuts to convince me that the iterations I hope to do will head in the right direction.

The traditional formulation of geophysical inverse problems requires a noise covariance matrix and a model covariance matrix. In my experience these are hard to define. I have recently made considerable progress defining the model covariance matrix from a model or a model estimate. (See my companion paper: Information from smiles, monoplane-annihilator weighted regression.)

For starters, I reorganize inverse theory to define the image as the migration operator applied to the complete data set and I take the unknowns to be the missing data. This formulation enables me to work with data and model covariances instead of model and noise covariances. If it shows promise we can always come back and look for the noise covariance.

Start with the definitions:

k	known data
u	unknown values in data space
$d = k + u$	the complete data space.
$d_0 = k + g$	raw data with gaps filled by gapfill program
K	Kirchhoff migration operator (for example)
M	matrix diagonal with 1 where data missing, 0 otherwise
$m = Kd$	model or image, migration of the complete data space.
A_d	array of 2-D prediction-error filters in data space
A_m	array of 2-D prediction-error filters in model space
Λ	gain control function in model space
$r_d = A_d d$	data residual
$r_m = \Lambda A_m m$	model residual
$\Delta d = MK'A'_m A_m K d_0$	perturbation of the unknown data

A regression I might try for the unknown u is

$$0 \quad \approx \quad r_d \quad = \quad A_d(k+u) \tag{1}$$

$$0 \quad \approx \quad r_m \quad = \quad \Lambda A_m m \quad = \quad \Lambda A_m K(k+u) \tag{2}$$

Of these two criteria, I will need to choose a balance. I can solve for u with either alone, or with some weighted combination. Naturally, I'll try all combinations. If I get it to work, I'll reorder it to make m the unknowns.

FINDING A CONVINCING DIRECTION

It remains to be shown that I can bootstrap the covariance matrix. I feel that this is possible because this is effectively what I did in PVI (Claerbout, 1992) figures 8.8 to 8.10.

Given some initial data d_0 with missing traces filled by `gapfill()` on page 426 I use `lomoplan()` on page 441 to solve for the array of prediction-error filters A in

$$0 \quad \approx \quad A_m K d_0 \tag{3}$$

$$0 \quad \approx \quad \lambda A_m \tag{4}$$

I tried this with various values of λ. At first I tried $\lambda = 0$, but when I looked at the gradient $\Delta d = MK'A'_m A_m, K d_0$, and compared it to the desired difference, I despaired of iterative migration ever reaching its goal. I understand the small valued regions of A as controlling the ultimate solution (d can be large where A is small). I fear the large-valued regions of A (particularly if they are at strange frequencies, strange dips, or strange locations in data space) because they are probably impediments to rapid convergence of the iterative migration.

I will not have courage to begin the iterative migrations until I am able to define everything so that the Δd-planes seem to have the general character of the removed data. A problem above is that λ beats down *all* the coefficients in the model A, and I wish it was more selective. Below is the filter I began with.

$$\begin{array}{cc} a_{3,0} & a_{3,1} \\ a_{2,0} & a_{2,1} \\ a_{1,0} & a_{1,1} \\ 1 & a_{0,1} \\ \cdot & a_{-1,1} \\ \cdot & a_{-2,1} \\ \cdot & a_{-3,1} \end{array} \tag{5}$$

There was much too much high frequency in Δd, because, as you see, the inverse covariance matrix appears twice in $\Delta d = MK'A'_m A_m, Kd_0$. In ordinary deconvolution, we may gap the filter to avoid an output dominated by near Nyquist frequencies. Thus I experimented with gapping this filter, and I decided to work with

$$\begin{array}{cc} \cdot & a_{3,1} \\ \cdot & a_{2,1} \\ \cdot & a_{1,1} \\ 1 & a_{0,1} \\ \cdot & a_{-1,1} \\ \cdot & a_{-2,1} \\ \cdot & a_{-3,1} \end{array} \tag{6}$$

Inspecting the Δd-plane I saw too much unrealistically steep dip. First I thought, maybe the semicircular smiles need it. Then I realized, our concern is defects in the data plane, not the model plane. So I decided to cut back on the number of filter coefficients to match that in Claerbout (Claerbout, 1992c). Thus, finally, I am working with the filter

$$\begin{array}{cc} \cdot & a_{2,1} \\ \cdot & a_{1,1} \\ 1 & a_{0,1} \\ \cdot & a_{-1,1} \\ \cdot & a_{-2,1} \end{array} \tag{7}$$

But then I saw little change, leading me to wonder about the interactions of everything I did up till now. Inspecting some more results, I was disappointed that Δd was so small near the the "bow tie" so I turned off AGC and that seemed to help. I am still uncertain that was the right thing to do, and I may go back and turn it on again.

Figures 1 and 2 show the ingredients of Δd. Since figure captions do not seem tolerant of mathematics these days I will explain here that the left column is without missing data. The 6 panels, in vertical order, are Md, $d+g$ (g is unknown data estimated by the gapfill program), $K(d+g)$, $AK(d+g)$, $A'AK(d+g)$ and $\Delta d = K'A'AK(d+g)$.

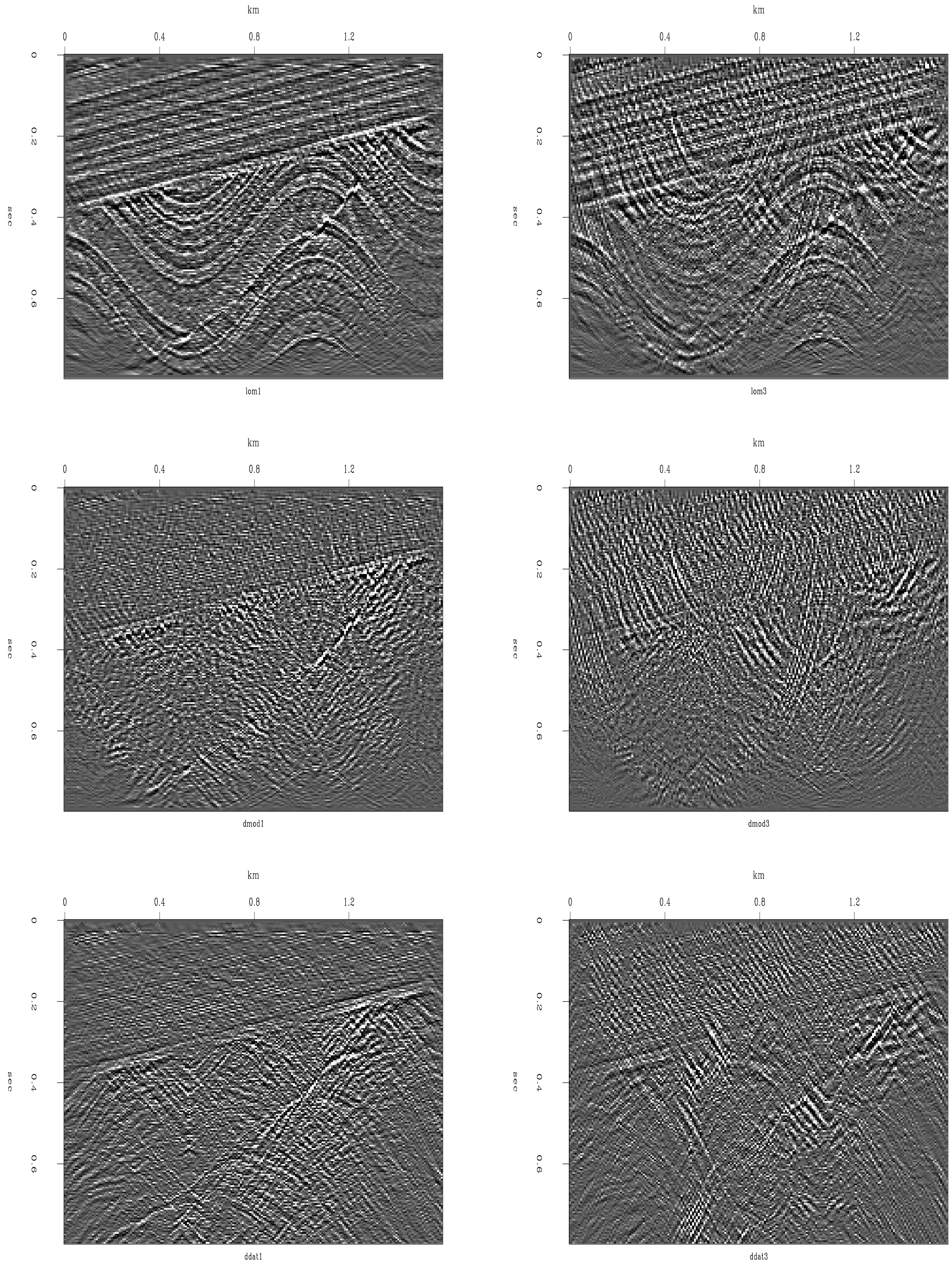

Figure 2: Top res. Middle dmod. Bottom ddat. jon7.mismig-delta [ER]

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