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Short Note

Logarithmic variable transformation

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INTRODUCTION

The problem at hand is stretching a time series $p(t)$. The amount of stretch is to be proportional to time t:

$$p_1(t_1) = p(\alpha t). \tag{1}$$

This task occurs frequently when performing dip moveout correction (DMO) (Forel and Gardner, 1988; Notfors and Godfrey, 1987; Bolondi et al., 1982). Constant velocity DMO stretches each input trace for a range of alpha values. All the stretched traces are added into the final zero offset data. After applying logarithmic resampling and Fourier transform to a time series the stretching expressed by equation (1) reduces to a multiplication. Unfortunately, this frequently used logarithmic transformation requires long traces, in order to prevent data aliasing. We propose a variation of the logarithmic resampling which results in shorter series in the transformed domain.

SOLUTIONS

The standard answers

A straightforward solution to (1) is trace interpolation. But because of the very simple form of stretch, there are elegant alternatives to explicit trace interpolation.

After logarithmic and Fourier transformation of the time function $p(t)$ the original stretch can be performed by a simple multiplication[2]:

$$P_1(\sigma_1) = P(\sigma_1) e^{i\sigma_1 \log(\alpha)} \tag{2}$$

For brevity, we will refer to this **F**ourier transformed **L**ogarithmic domain as **FL** domain. σ_1 denotes the variable in the **FL** domain after stretching.

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[2]See Appendix A

To prevent the time trace from being undersampled, the trace after logarithmic transformation will have to have n_s samples[3], where

$$n_s = 2t_{\max} f_{\max} \log(\frac{t_{\max}}{t_{\min}}). \tag{3}$$

$f_{\max}$ is the maximum frequency of $p(t)$, $t_{\min}$, $t_{\max}$ are the upper and lower limits of the section of $p(t)$, that the logarithmic trace of n_s points resamples correctly. In practice the logarithmic trace is often found to be extremely long.

An alternative method

The alternative method we propose first Fourier transforms the time series $p(t)$. Then it applies the traditional log-Fourier transformation (described above) to the *spectrum* of the original time trace. We call this new domain the Fourier-log-Fourier domain(**FLF**). We denote τ_1 as the variable of the new domain after the original α stretch.

In this domain the intended trace stretch is carried out by a simple scaling of the series' coefficients[4]:

$$P_1(\tau_1) = P(\tau_1) e^{i\tau_1 \log \frac{1}{\alpha}} \tag{4}$$

Comparing (2) and (4), we find that the stretching in the **FL** and in the **FLF** domain differ only in the exponantial scaling factor.

The number of samples n_ω for a well sampled representation of our input data in the **FLF** domain is:

$$n_\omega = 2f_{\max} t_{\max} \log(\frac{f_{\max}}{f_{\min}}). \tag{5}$$

This expression derives directly from equation (3) by interchanging the role of the frequency and time axis. Especially, note the symmetry of the non logarithmic factor of (3) and (5).

[3]See Appendix C
[4]See Appendix B

Logarithmic trace stretch - An overview		
Interpolation	Log-Fourier transformation **FL**	Fourier-Log-Fourier transformation **FLF**
Interpolation according to (1): $p(t) \to p(t_1)$	**L***og* : $p(t) \to p(s)$ **F***ourier* : $p(s) \to p(\sigma)$ α stretch by multiplication: $p(\sigma) \to p(\sigma_1)$	**F***ourier* : $p(t) \to p(\omega)$ **L***og* : $p(\omega) \to p(\nu)$ **F***ourier* : $p(\nu) \to p(\tau)$ α stretch by multiplication: $p(\tau) \to p(\tau_1)$
Disadvantages		
Each stretch requires a new interpolation.	Overhead calculations: **FL**. Long traces after logarithmic stretch.	Comparatively,expensive overhead calculations: **FLF**.
Advantages		
Easy to implement. No overhead calculations.	Each α stretch is a simple multiplication.	Each α stretch is a simple multiplication. *Shorter* traces in the transform domain.

ADVANTAGES

The frequency spectrum of a typical seismic trace is usually bandlimited. $f_{\max}$ and $f_{\min}$ are upper and lower limits of this relatively small, significant part of the spectrum. On the other hand, a seismic time series carries valuable information at early as well as late times. Therefore, $t_{\max}$ and $t_{\min}$ must be close to the extreme ends of the time series. In other words, for seismic traces $t_{\max}/t_{\min}$ is usually large, while $f_{\max}/f_{\min}$ is comparatively small. Equation (3) and (5) show that the proposed method operating on $p(t)$'s bandlimited spectrum results in shorter traces. The length ratio of a trace being processed by the proposed versus the standard logarithmic transformation is:

$$\frac{n_\omega}{n_s} = \frac{\log(f_{\max}/f_{\min})}{\log(t_{\max}t_{\min})}. \tag{6}$$

Having applied logarithmic and Fourier transformation to $p(t)'s$ frequency spectrum or to $p(t)$ directly, the desired trace stretching (1) is performed by simple scaling of the trace.

Stretching the input trace by the proposed method is more efficient than the standard methods, if each input trace gets mapped into many output traces. Then the overhead of the initial Fourier and logarithmic transformations are justified by facilitating a fast stretching step which operates on short traces.

REFERENCES

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APPENDIX A

The time stretch (1) amounts to time stretch proportional to time:

$$\Delta t = t_1 - t = t_1(1 - \frac{1}{\alpha}). \tag{A-1}$$

The logarithmic resampling is defined by:

$$s = \log(\frac{t}{t_{\min}}). \tag{A-2}$$

After logarithmic variable transformation from t to s the time stretch of equation (1) and (A-1) is:

$$s_1 = s + \log(\alpha), \tag{A-3}$$

$$\Delta s = \log(\alpha). \tag{A-4}$$

Having Fourier transformed the time series $p'(s)$ this constant shift translates to a complex scaling:

$$P_1(\sigma_1) = P(\sigma_1) e^{i\sigma_1 \log(\alpha)}. \tag{A-5}$$

σ_1 denotes the α stretched variable of the **FL** domain.

APPENDIX B

After Fourier transformation from variable t to ω the original stretch relation (1) becomes

$$\omega_1 = \frac{1}{\alpha}\omega. \tag{B-1}$$

After logarithmic transformation (A-2) of the frequency trace we find relationships equivalent to equation (A-3) and (A-4):

$$\nu_1 = \nu + \log(\frac{1}{\alpha}), \tag{B-2}$$

$$\Delta\nu = \log(\frac{1}{\alpha}). \tag{B-3}$$

ν describes the new variable in the logarithmic domain. An additional Fourier transformation allows us to express the intended time stretch as a simple scaling (4) of the transformed trace.

APPENDIX C

We want to map our input trace onto a logarithmic mesh. How many samples do we need in the logarithmic domain?

According to equation (A-2) the relation between interval Δs and Δt is:

$$\Delta s = \Delta t \frac{ds}{dt} = \frac{\Delta t}{t}. \tag{C-1}$$

Δt is the sampling interval for which the time trace is properly sampled. $t_{\max}$ is the maximum time of $p(t)$. At $t_{\max}$ Δs is at a minimum:

$$\Delta s = \frac{\Delta t}{t_{\max}} = \frac{1}{2 t_{\max} f_{\max}}, \tag{C-2}$$

where $f_{\max}$ is the Nyquist frequency corresponding to Δt. The number of sample points in the log domain has to be

$$n_s = \frac{s_{\max} - s_{\min}}{\Delta s} = 2 t_{\max} f_{\max} \log(\frac{t_{\max}}{t_{\min}}). \tag{C-3}$$