Stanford Exploration Project, Report 73, November 18, 1997, pages 285–292

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DMO after logarithmic stretching

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keywords: *DMO, velocity, NMO*

ABSTRACT

DMO stacks seismic offset data by honoring the reflector dips. This method can be an attractive compromise between computationally expensive prestack migration and inaccurate NMO. The DMO process implemented here is accurate for any constant velocity medium. The algorithm is velocity independent in the sense that it does not assume a certain velocity value. The DMO step prepares the data for an NMO step. The subsequent NMO facilitates an improved velocity analysis and stack, since it does not require a flat reflector assumption. Logarithmic stretching and Fourier transformation along the time axis reduce the 3D DMO operator to a midpoint and frequency invariant 2D operator. The duality properties of the Fourier transformation and the band limitation of our signal allow an efficient representation of the logarithmically scaled traces. In this article I present the kinematics of the basic DMO operator. I outline a program based on logarithmic resampling in temporal frequency, followed by 2D convolution over constant-frequency planes. The proposed DMO algorithm is highly parallel and should lend itself to a very efficient implementation.

INTRODUCTION

Normal moveout (NMO) in common midpoint (CMP) gathers is based on a subsurface model with constant velocity and flat reflectors. In the presence of dipping reflectors, the NMO process adds events which correspond to different subsurface locations. To first order, constant velocity dip moveout (DMO) corrects for the reflector dip. (Levin, 1971). Ottolini (1982) and Claerbout (?) show that the constant velocity DMO operator varies with time and offset.

Bolondi (1982) and Notfors (1987) introduce a DMO operation which is based on logarithmic resampling and a Fourier transformation of the time axis. In this new domain the DMO operator varies only with offset.

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Forel and Gardner (1988) separate the stacking process into a velocity independent DMO step and a standard NMO step. The DMO step does not change the data cube's dimensions. The following NMO step uses the data redundancy for an unbiased velocity analysis, which now correctly incorporates events from dipping interfaces. The resulting velocity function is used to stack the data. A velocity estimation with possible recursive DMO steps, as described by Hale (1984), is not necessary.

I find that by performing Forel's time stretch using a variation of the logarithmic transformation (Schwab and Biondi, 1992) a highly parallel implementation of the velocity-independent DMO is possible. Like Forel's approach, this implementation improves velocity analysis and NMO stack by removing the flat layer assumption.

RATIONALE

Gardner's approach to constant velocity DMO transforms the nonzero offset data cube into an output cube with an artificial offset k and traveltime t_1. After NMO and poststack migration an input impulse results in an output data cube which depicts the corresponding ellipsoidal surface. The transformation is defined by:

$$k^2 = h^2 - b^2, \tag{1}$$

$$t_1 = \frac{t_n k}{h}, \tag{2}$$

$$t_0 = t_1^2 - \frac{4k^2}{V^2}, \tag{3}$$

where t_n and h are the traveltime and offset of the original data input trace, and b is the distance between the replacement trace's midpoint and the input trace's midpoint. t_0 is the desired zero offset traveltime at b, k is defined by equation (1). Equation (3) shows t_0 to be the NMO corrected time for offset k using the moveout velocity V. Transformations (1) and (2) are velocity independent. The final NMO step (3) permits a standard velocity analysis[2].

Equations (1) and (2) define the DMO operator, which in the input data set is a slanting hyperbola, lying in the radial plane. The DMO operation sums along this hyperbola and stores the sum at the hyperbola's vertex. The impulse responses of the DMO process are slanting semicircles.

Stretching the traces

To realize transformation (2) each input trace, $q(t, m, h)$, has to be stretched by the factor k/h:

$$q_1(t_1, m, h) = q(t_1 \frac{h}{k}, m, h). \tag{4}$$

[2]see Appendix A for a short derivation

The differential time moveout $\Delta t = t - t_1$ of equation (1) is proportional to time:

$$\Delta t = t_1(\frac{h}{k} - 1). \tag{5}$$

Logarithmic resampling is defined by:

$$\tau_1 = \log(\frac{t_1}{T_c}), \tag{6}$$

where τ_1 denotes the frequency variable after Fourier transformation of the input trace. T_c is the start of the unmuted frequency trace. Elsewhere in this report (?), I show that After two Fourier transforms and the intermediate logarithmic resampling, each trace stretching amounts to a simple trace scaling:

$$Q_1(\omega_1, m, h) = Q(\omega_1, m, h)\exp(-i(\frac{h}{k} - 1)\omega_1) \tag{7}$$

Resampling the *Fourier transformed input trace* rather than the *trace* itself takes advantage of the band limitation of typical seismic data (?). The more compact data cube reduces the computation time and the amount of memory required.

Convolution in constant frequency planes

To find the impulse response of the DMO operator in the different transform domains, we keep the impulse location m, h, and t fixed. The impulse response of the DMO operator is spread along the intersection of a cylinder satisfying equation (1) and a radial plane satisfying (2). Logarithmic stretching of the time axis transforms the linear plane of slope h/t into a curved surface with

$$k = h\exp(\tau_1 - \tau) \tag{8}$$

where $\tau = \log(\frac{t}{T_c})$.

The Impulse response after Fourier transform is located on the same semicircles in all frequency planes. The amplitude along the semicircle varies according to the complex factor of equation (7)[3]. I spread out the impulse along the circle defined by equation (1), conducting a 2D convolution. The convolution operator varies only with offset, but is midpoint independent. The convolution is identical on all frequency planes:

$$\hat{Q}_1(\omega_1, m, k) = \int_{-h}^{h} Q_1(\omega_1, m + b, h = \sqrt{k^2 + b^2})db, \tag{9}$$

where $\hat{Q}_1$ and Q_1 denote the output and input data, ω_1 is the frequency variable, m and h denote midpoint and offset, respectively. k represents the DMO offset of equation (1).

[3]Unfortunately, if the data is Fourier transformed along the m, h axes then this factor would result in a 2D convolution. Hence, the 2D convolution in constant ω planes cannot be done by a Stolt type migration algorithm.

ALGORITHM

The algorithm comprises 5 major routines:

- Logarithmic time interpolation of traces.
- Fourier transformation of interpolated traces.
- DMO step: Convolution in constant frequency plane.
- Inverse Fourier transformation of traces over time axis.
- Inverse logarithmic interpolation of traces.

The outer routines are parallel over time, because Fast Fourier Transformation as well as interpolation require communication between different time samples. The inner routine is identical for each frequency plane. The two-dimensional convolution follows the outline proposed by Biondi (1991). The offset k is laid out serial, because the convolution operator is invariant along the h and ω axes, but depends on k according to equation (8)

The data organization results in a very efficient code for parallel machines. But, it also requires a transpose step, in which the data layout has to be changed.

On the CM the intrinsic function 'reshape' transposes small data cubes. For data sets, which cannot reside in the CM's distributed memory the transpose could be implemented in the read and write from and to disk.

DISCUSSION AND SUMMARY

The purpose of Dip Move Out is to map a nonzero offset dataset into a zero offset dataset. After migration both data sets should result in the same subsurface picture.

The introduction of the artificial offset k splits the 'stacking NMO' step into a velocity independent DMO step and a velocity depending standard NMO. The NMO step is unbiased with respect to dipping events. All events regardless of reflector dip line-up and contribute to the stacked image constructively. This improves NMO velocity analysis and the poststack image.

The two-dimensional convolution is the only cross-trace calculation in the proposed algorithm. Due to its strongly parallel features the DMO procedure is inexpensive, when performed on a parallel machine.

Forel (1988) demonstrated that his DMO process extends to three-dimensional data sets. Following Forel's approach, the proposed two-dimensional process can be extended to three dimensions.

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APPENDIX A

Appendix A

Impulse Response

The purpose of Dip Move Out is to replace a nonzero offset data set by a zero offset data set. Both data sets are to be equivalent in the sense that both data sets result in the same subsurface picture after migration.

The prestack migration response to a nonzero offset impulse is the locus $Q(x, z)$ for which all traveltimes SQR are equal. Such a locus is called isochrone. In a constant velocity medium an isochrone corresponding to the two-way traveltime t_n is an ellipse with its foci at the location of source and receiver. The major and minor axes (A and B) of the ellipse are

$$A^2 = \frac{V^2 \cdot t_n^2}{4}, \tag{A-1}$$

$$B^2 = A^2 - h^2. \tag{A-2}$$

The ellipse equation is

$$\frac{x^2}{A^2} + \frac{z^2}{B^2} - 1 = 0. \tag{A-3}$$

The poststack migration response to a zero offset impulse of time t_z is the wavefront at the time $t_z/2$ after initiation. In the case of constant velocity medium it is a circle with midpoint at the source receiver position $B(b,0)$ and radius $t_z V/2$:

$$(x^2 - b^2) + z^2 = \frac{t_z^2 V^2}{4}. \tag{A-4}$$

DMO operator

In order to represent the same subsurface picture, we have to map the nonzero offset isochrone to a set of zero offset isochrones. The circles corresponding to the zero offset data set have to be tangential to the nonzero offset ellipse. Superposition of these circles by migrating the zero offset data set will result in the picture of their envelope, which is the prestack migration result of the nonzero offset impulse. The zero offset data set is called the DMO impulse response.

The circle described by (A-4) is tangent to the ellipse of (A-3) when both equations have the same slope at $P(x,z)$. (The slopes can be expressed by implicit partial differential terms. The difference of these slopes is zero if:)

$$b = x(1 - \frac{B^2}{a^2}). \tag{A-5}$$

Substituting for A and B using (A-1) (A-2) yields an expression for x. Eliminating x and z from the circle (A-4), ellipse (A-3) results in:

$$t_0^2 = (t_n^2 - \frac{4h^2}{V^2})(1 - \frac{b^2}{h^2}). \tag{A-6}$$

Because the isochrone ellipse is the impulse response of the nonzero offset data set, replacing this ellipse by impulse responses of a zero offset data set gives us the impulse response of the DMO operator. DMO can be achieved by replacing each nonzero offset trace sample by its DMO impulse response, which is defined by equation (A-6).

Velocity Independence

Forel (1988) shows how the DMO process can be split in a velocity independent shrinking of the input trace and a standard NMO step.

First an array of zero offset locations has to be designed. Each nonzero offset trace contributes to all zero offset traces whose combined source-receiver location lies in between the nonzero offset source and receiver. b is the distance between zero offset location and the midpoint of the nonzero offset experiment.

In the first step the nonzero offset trace time t_n is transformed into t_1:

$$t_n = t_1 \frac{h}{k}, \tag{A-7}$$

where the artificial offset k is defined according to (1)

$$k^2 = h^2 - b^2 \tag{A-8}$$

The new trace is added into an offset panel (k, t_1) at the zero offset location. Substituting (A-7), (A-8) into (A-6) yields the NMO relationship (3) for (t_0, t_1):

$$t_0^2 = t_1^2 - \frac{4k^2}{V^2} \tag{A-9}$$

After processing all nonzero offset traces a standard normal move out of the (k, t_1) panels results into a zero offset data set. The final NMO step permits a traditional velocity analysis. Unlike conventional NMO processing, DMO does not assume any particular reflector dip.

The equation set (A-3), (A-4) and (A-5) can be solved for (x, z) instead of (tn, tz). The points $P(x, z)$ which fulfill the described conditions for varying traveltimes are called the locus of constant replacement point (lcr). Each P is the actual tangential point of an ellipse t_n and a circle centered at a fixed B. The two-way traveltime $t(PB)$ is t_0. The traveltime $t(SPR)$ is t_n. The lcr is a circle:

$$(x - x_0)^2 + z^2 = r^2 x_0 = \frac{h^2 + b^2}{b} r = \frac{h^2 - b^2}{2b} \tag{A-10}$$

Center location and radius are time independent.