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# Multiple attenuation using Abelian group theory

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## ABSTRACT

This paper assumes a flat layered earth model and offset recording geometry. We apply velocity transform on the space-time shot gather into a domain, each of whose attributes forms an Abelian group. An attribute that is *addable* irrespective of ordering forms an Abelian (or commutative) group. Several attributes form Abelian groups. We implement two of them at this stage, the zero-offset two way travel time $T$, and Dix's parameter $T.V_{nmo}^2$. Addability of attributes dictates that in this domain sets of multiples – both primary and multiples – lie at even intervals along parallel straight lines. The next most important property of an Abelian group is closure; in other words, linear combination of different Abelian group attributes also forms an Abelian group. We form linear combination of the $T$ and $T.V_{nmo}^2$ to steer the waterbottom multiples and peglegs straight down the $T$ axis. The periodic character of reverberation events in this domain helps design predictive multiple attenuation operators. We apply predictive deconvolution on this domain to attenuate the waterbottom multiples and peglegs. Then we conjugate velocity transform the deconned result back to the space-time domain. We test the method on a Solid elastic model synthetic dataset and on real data. The results show the efficacy of the method to attenuate waterbottom multiples and peglegs simultaneously. Theoretically, recursive application of the procedure will attenuate all deep earth intrabed multiples. However, problems remain for this method in velocity transform, and we point out them in this paper.

## INTRODUCTION

Near-surface reverberations contaminate our CMP velocity analysis and stack image. Furthermore, the present AVO analysis incurs an even higher demand on these reverberations removal. Thus removing reverberations from reflection seismograms has been a longstanding problem of exploration geophysics. Standard predictive multiple attenuation techniques in marine reflection seismology are based on the periodicity of multiples. Since periodicity of multiples is only satisfied in a one-dimensional impulse response earth model, such techniques find trouble when applied to an offset

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section, for multiples are not periodic in time for a given nonzero offset. Researchers invent new domains to help. Taner (1981) applied predictive deconvolution along radial traces. Alam and Austin (1981) and Treitel et al. (1982) applied predictive deconvolution in the slant-stack domain. Their methods suffer the disadvantage that the period of waterbottom multiples varies from trace to trace in the new domains, and fail to attenuate water layer peglegs originating from subsurface reflections.

This paper discusses Abelian group theory (Muir, 1987) and applies the theory to perform velocity transformation on common shot gathers into Abelian group attributes domain for the separation of primary and multiple reflections (including peglegs). The method applies to layered earth models with finite sets of events. An attribute that is addable independent of ordering (zero-offset travel-time $T$, for example) forms an Abelian group. The results are that sets of multiples and peglegs lie equally spaced along parallel straight lines in this domain. We apply a simple rotation to turn them straight down the $T$ axis, resulting in a new Abelian group attributes domain. Predictive multiple attenuation techniques work well in this domain. Conjugate inverse transforming all the way back to the space-time domain gives the reverberations attenuated section. Because this new Abelian group domain can handle the waterbottom multiples and peglegs efficiently at the same time, it is a prospectively good domain to work in. One main application would be AVO analysis in structure-free but variable lithology areas. Constructing a velocity transform that has a good inverse is the key to the success of this method.

## THE ABELIAN GROUP

### Abelian group attributes and their properties

Attributes that are addable independent of ordering are:

1. $g1 = T$ (two-way, zero-offset travel time)
2. $g2 = T.V_{nmo}^2$ (Dix's parameter)
3. $g3 = T.V_{nmo}^4.m$, which includes the anelliptic factor ($m = 1 + 4.q_w - q_w^2$) in NMO equation
4. log2 Hilbert envelope amplitude (in units of 0.25?)
5. Frequency-dependent attenuation.

At this stage, we implement only the first two attributes, $(g1 = T, g2 = T.V_{nmo}^2)$. The addability of $T$ and $T.V_{nmo}^2$ means that waterbottom multiples and peglegs lie evenly sampled along each of these two axes. Then they lie evenly sampled along parallel lines in the $(g1, g2)$ plane, the slope being the water velocity squared. Figure

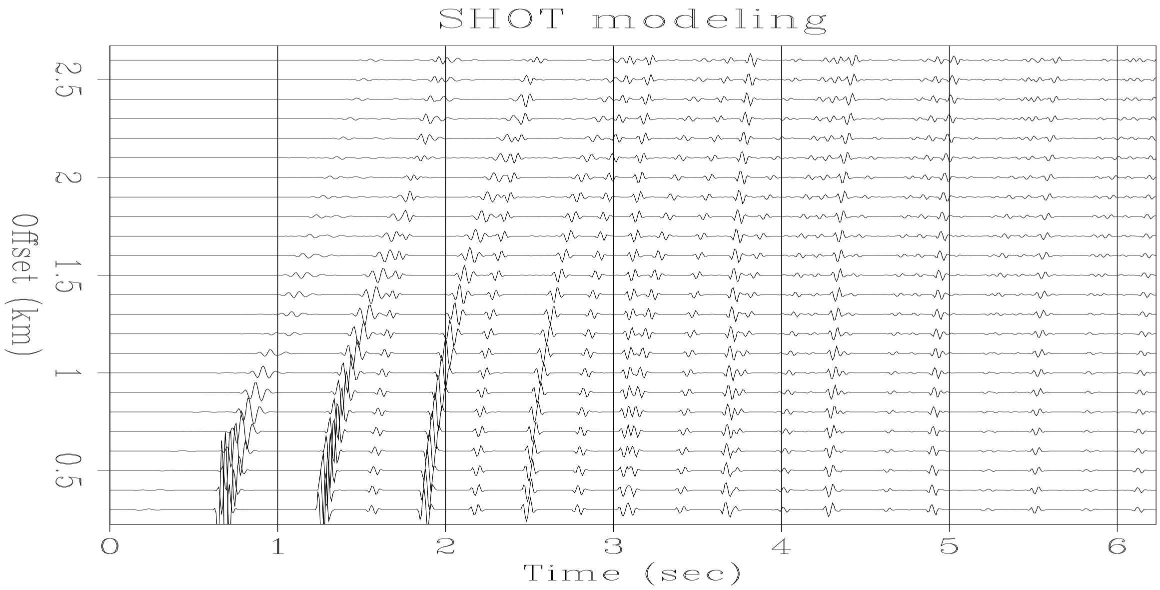
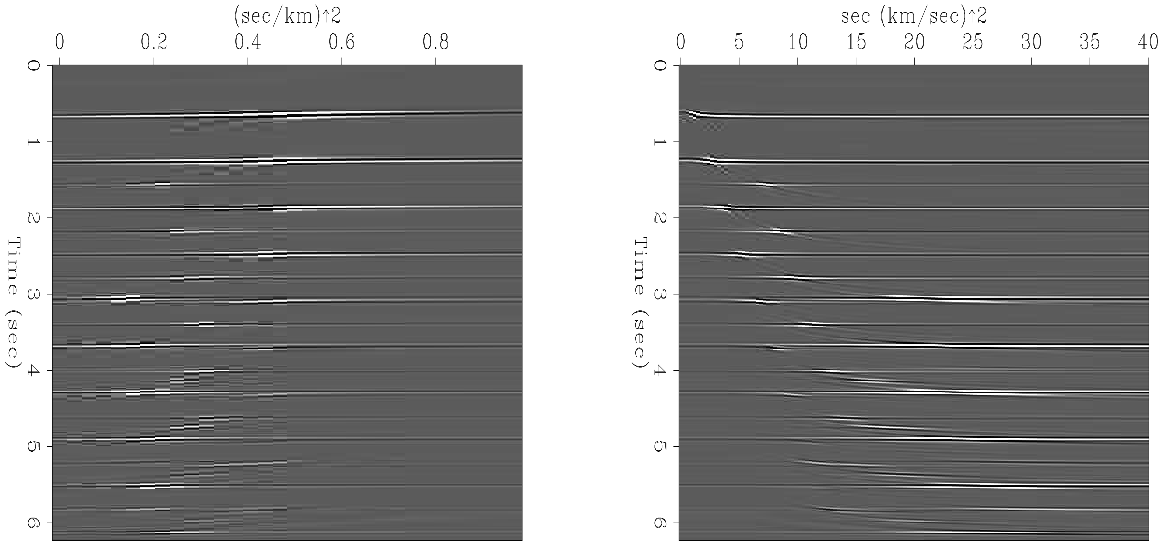



Figure 1: Synthetic data with water-related reverberations. mo2-Syndata [ER]

1 is a Solid elastic modeling dataset that has geology similar to that of the Geco Barents Sea dataset we will study later.

We apply velocity transform by NMO Kirchhoff summation on this dataset into the zero-offset two way travel time and slowness squared domain $(T, 1/V_{nmo}^2)$ (Claerbout, 1992). The zero-offset travel time is already an Abelian group, $g1 = T$. We apply a shear by nearest neighbor interpolation along the slowness squared axis to transform into the Dix's parameter, $g2 = T.V_{nmo}^2$. The results are shown in Figure 2, where the waterbottom multiples and peglegs lie evenly spaced along parallel straight lines in the Abelian group plane. The events in the velocity space span a large area because of the insensitivity of velocity transformation.

The next most important property of the Abelian group is *closure*. Closure means that any linear combination of different Abelian group attributes also forms an Abelian group. If we have a set of vectors $v$, which forms such a group, then for any constant matrix, $M$, the set of vectors $M.v$, also forms such a group. We introduce a shear into the $(g1, g2)$ coordinate frame in Figure 2 by replacing $(T.V_{nmo}^2)$ with the linear combination $(T.V_{nmo}^2 - T.V_{water}^2)$ since we are concerned with the water-bottom multiples and their peglegs (or some other lower-bounding velocity if other intrabed multiples are concerned). The slope of the waterbottom multiples and peglegs – $V_{water}^2$ in $(g1, g2)$ plane can either be known from field water velocity measurement or from looking at the Hilbert envelope of the data, and then finding the shear on the data (i. e., by varying $V_{water}^2$) that maximizes the sum of the envelope energy over axis $T$. Then the waterbottom multiples and peglegs are steered straight down the $T$ axis. Thus in this domain, predictive multiple attenuation operators will work well. This process yields the new Abelian group attributes:




(a) sloth spectrum (b) Abelian Dix's parameter

Figure 2: (a) is the zero-offset travel time versus slowness squared; (b) is the Abelian group attributes zero-offset travel time versus Dix's parameter, events lie along parallel straight lines. mo2-Synsloth [ER]

1. $g1' = g1 = T$
2. $g2' = g2 - g1.V_{water}^2 = T.V_{nmo}^2 - T.V_{water}^2$.

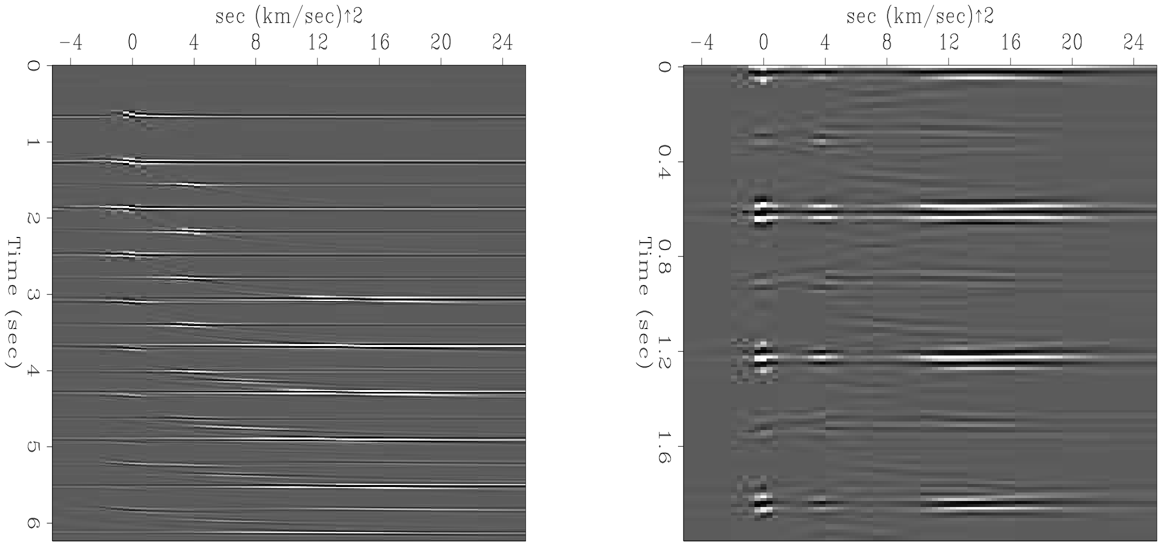
### Going to the $(g1', g2')$ domain

There are two ways to go to $(g1', g2')$ domain, an indirect and a direct one. The indirect way is to map from $(T, 1/V_{nmo}^2)$ to $(g1', g2')$ by including the shear $V_{water}^2$. The result is shown in Figure 3a. Figure 3b is its autocorrelation. This figure shows that the waterbottom multiples and peglegs are periodic in this new Abelian group domain, and the period is constant from trace to trace.

We can apply gapped predictive deconvolution on this $(g1', g2')$ in Figure 3a. Conjugate mapping all the way from the deconned Abelian group back to the space-time domain gives the water-related reverberations attenuated section.

Two problems with this mapping are that it requires two more (forward and conjugate via $(T, 1/V_{nmo}^2)$ ) mappings, while the amplitude in the $(g1', g2')$ domain is less regular than in the second method described below. So we move on to the second mapping method, a direct one.

The other way to go to $(g1', g2')$ is to do velocity transform mapping directly from the space-time domain. The conjugate mapping from $(g1', g2')$ to material parameters consists of:




(a) Abelian group (g1',g2') (b) autocorrelagram

Figure 3: (a) is the Abelian group attributes $(g1', g2')$, (b) is its autocorrelogram. Water-related reverberations are periodic and lie directly under each other on the $g1' = T$ axis. mo2-SynAbelian [ER]

1. $T = g1'$
2. $V_{nmo}^2 = g2'/g1' + V_{water}^2$.

Considering the velocity uncertainty, for a point in $(g1', g2')$ domain, we use also its left and right neighbors along the $g2'$ axis to invert for material parameters, yielding a $T$, and three velocities squared. We apply velocity transform by NMO Kirchhoff summation over the hyperbola strip defined by the three velocities squared. The weight given to each summing element is defined by a triangle weighting function, an approximate to a Gaussian function (Jedlicka, 1989). The apex of the triangle is at the NMO time of the median velocity squared. The area under the triangle is $1/\sqrt{T}$, i. e., we are honoring the 2-D Green's function. The result of summation gives the value at the corresponding point in $(g1', g2')$ domain. The conjugate mapping is the spraying out of the point value in $(g1', g2')$ domain into the NMO hyperbola strip Kirchhoff trajectory in space-time domain (Claerbout, 1991). One caution is that the coordinate $g2' = 0$ must be sampled because it directly samples the water bottom multiples.

The problem with this velocity transform mapping method arises from the different velocity sensitivity in the relation $V_{nmo}^2 = g2'/g1' + V_{water}^2$ between shallow and deep parts. When $g1' = T$ is small, a small change in $g2'$ results in a big change in $V_{nmo}^2$. We discard the negative $V_{nmo}^2$ as computed. As a result, the velocity is coarsely sampled at the shallow part, whereas the velocity is densely sampled at the deep part. We apply a weighting function $1/V_{nmo}^2$ to balance between the number of small and large velocity values used, and to underweight the ridiculously high velocities as computed at the shallow part. And because the 2-D Green's function favors the near-offset strongly, after conjugate inverse velocity transform, the far offset data is attenuated. We include an extra weighting function $1/\cos^2\theta$, where $\cos\theta$ is the Kirchhoff obliquity factor. From our experiences, the results are acceptable when $T > 0.3 second$.

## TESTING ON SYNTHETIC AND REAL DATA

### Test on Solid elastic model data

We apply the above direct velocity transform method to map between the space-time domain and the Abelian group $(g1', g2')$ domain. Figure 4 displays the synthetic data as in Figure 1, and the data after velocity transform to $(g1', g2')$ and back to the space-time domain. The far offset data are attenuated by the velocity transform. For all the mappings here, we use one-pass conjugate mapping. Conjugate gradient iteration gives a better result, but is the last resort. Figure 5 is the data in $(g1', g2')$ domain before and after gapped predictive decon. On the gapped predictive deconned $(g1', g2')$, we apply a *cosine* muting on the left of $g2' = 0$. Figure 6 shows the space-time data after conjugate velocity transforming the deconned $(g1', g2')$ data in Figure 5b and the original water-related reverberations-free data. The reverberations attenuated section shows that at near offset the first waterbottom multiple remains strong; because the hyperbola curvature is large at near offset, it maps to a large area in velocity space. Its amplitude relationship in velocity space thus deteriorates. The gapped predictive deconvolution does not work well here. The remedy to this problem lies in further improved velocity transform.

### Field data

When being applied to field data, one problem is the sensitivity of the operator to noise spikes. We test our forward/conjugate velocity transform operator on noise spikes. When a noise spike is at far offset, it is attenuated in the result. When a noise spike is at near offset, it spreads into a few traces in the result.

Field data comes from a Barents Sea common shot dataset donated to SEP by Geco. The water bottom is deep, flat, and hard. The near offset is 294m, the hydrophone interval 50m. There are 48 traces per shot. The hard waterbottom multiple reflections are so strong that they dominate the whole section. The water bottom primary reflection begins at 600msec. There are two strong primaries: one at 1.5 seconds; the other at 4 seconds. Both have pegleg multiple trains associated with them. We use the wavelet deconned result of another of our papers in this report (Mo, 1992) as input. The input and result are shown in Figure 7. The multiples and peglegs are attenuated in this processing. In all the processings for this dataset, we

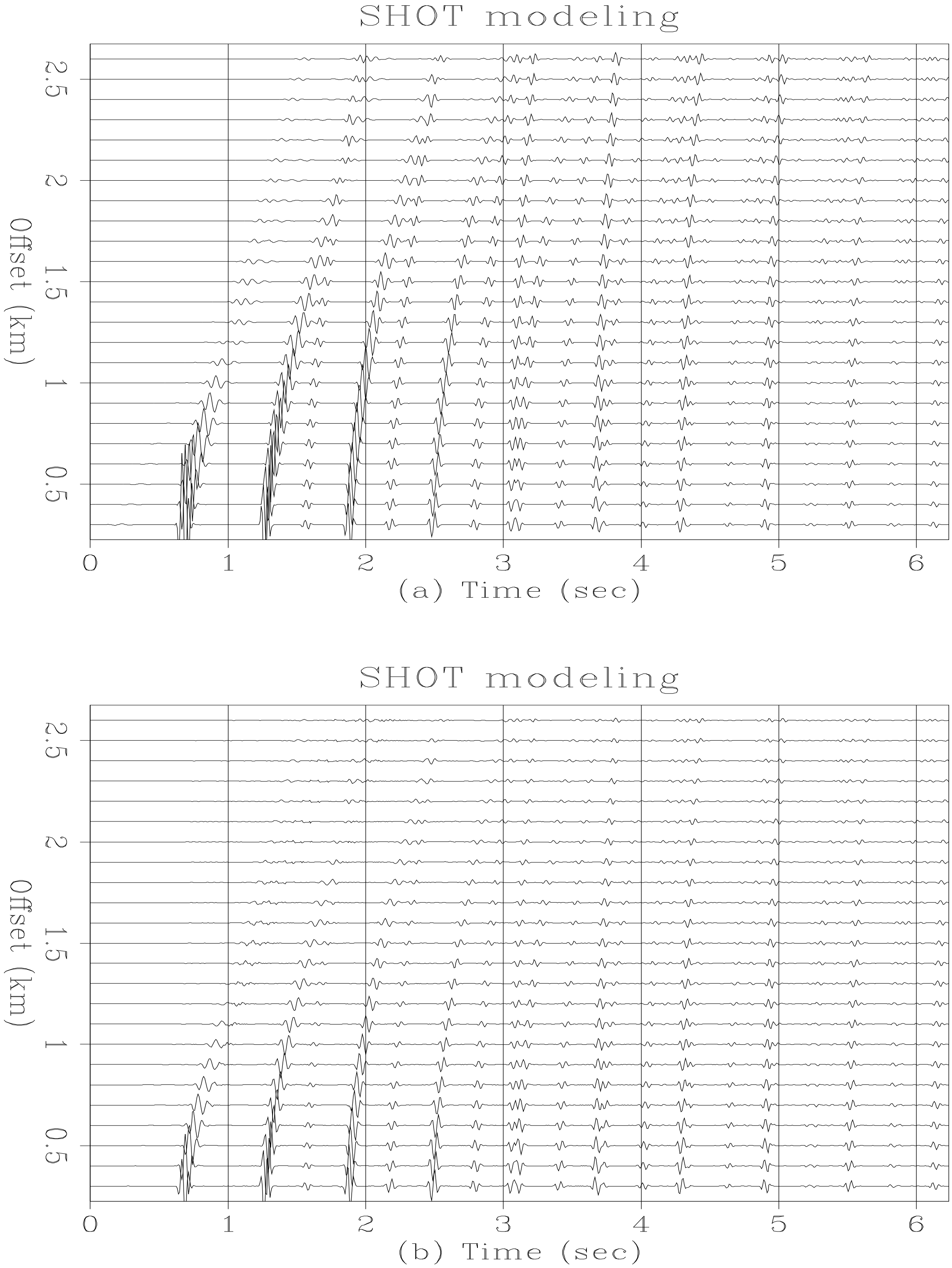



Figure 4: (a) is the input synthetic data as in Figure 1; (b) is the data after conjugate velocity transforming back from $(g1', g2')$ in Figure 5a. The far offset data are attenuated by the velocity transform. $\boxed{\text{mo2-SynMM}}$ [ER]

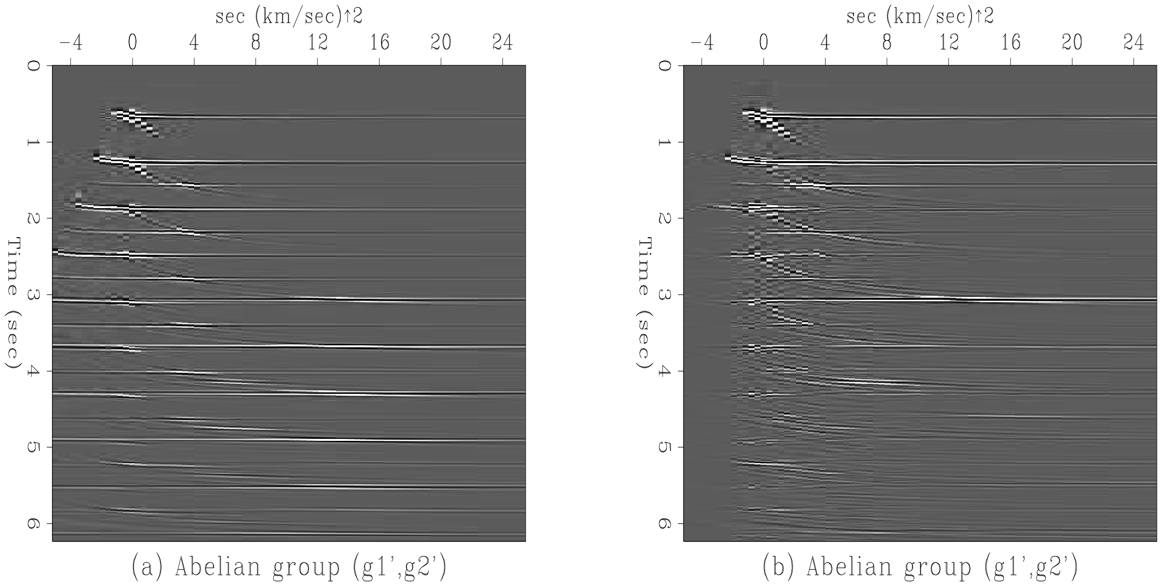


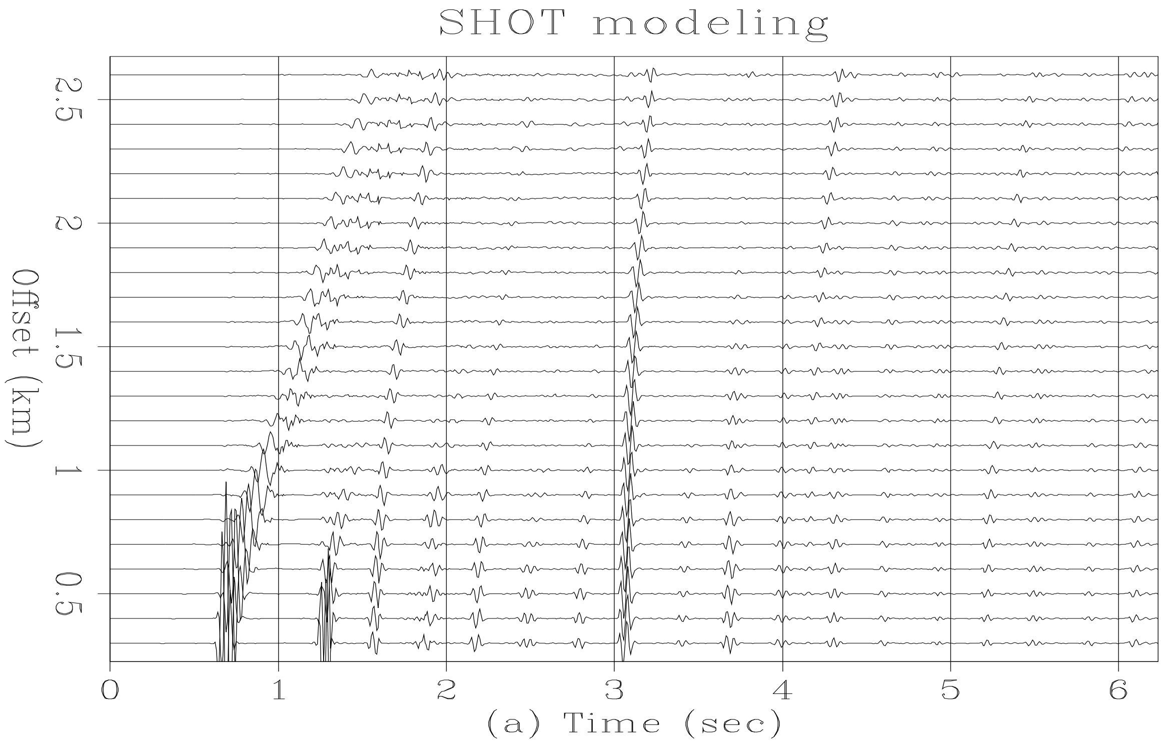

Figure 5: (a) is the Abelian group data $(g1', g2')$; (b) is the $(g1', g2')$ data after gapped predictive decon, $gap = 550ms, filterlength = 550ms$. mo2-SynMMAbelian [ER]

use the same processing parameters as in processing the above synthetic data. This dataset was processed before by Morley (1982).

## DISCUSSION AND CONCLUSIONS

We have showed that the Abelian group attributes domain $(g1' = T, g2' = T.V_{nmo}^2 - T.V_{water}^2)$ is a good domain for waterbottom multiples and peglegs attenuation. We have achieved the correct timing in the mapping between the space-time domain and the Abelian group attributes domain. However, the amplitude remains a problem owing to the different velocity sensitivity between shallow and deep parts in the velocity transform. The geometrical progression of amplitude between primary and multiples in the Abelian group domain is not strictly satisfied. One remedy to the second problem might be to mute the far offset data along a slanted straight line, so that the data of near/far offset have the same recording angular aperture with respect to the source.

Beside the periodicity as discussed above, the other idea is sparsity. By implementation of more Abelian group attributes, events will be increasingly sparse, and primaries and multiples will be more easily differentiated. This helps us to avoid identifying multiples and peglegs as primaries. The next Abelian group attribute candidate to be implemented is anelliptic factor. This would better model non-hyperbolic effects that might be either extrinsic and due to layering, or intrinsic, if the layered material was anisotropic. As $m = g3/(T.V_{nmo}^4)$, there is still the problem of different sensitivity between small and large $T$. It seems to me that this different sensitivity



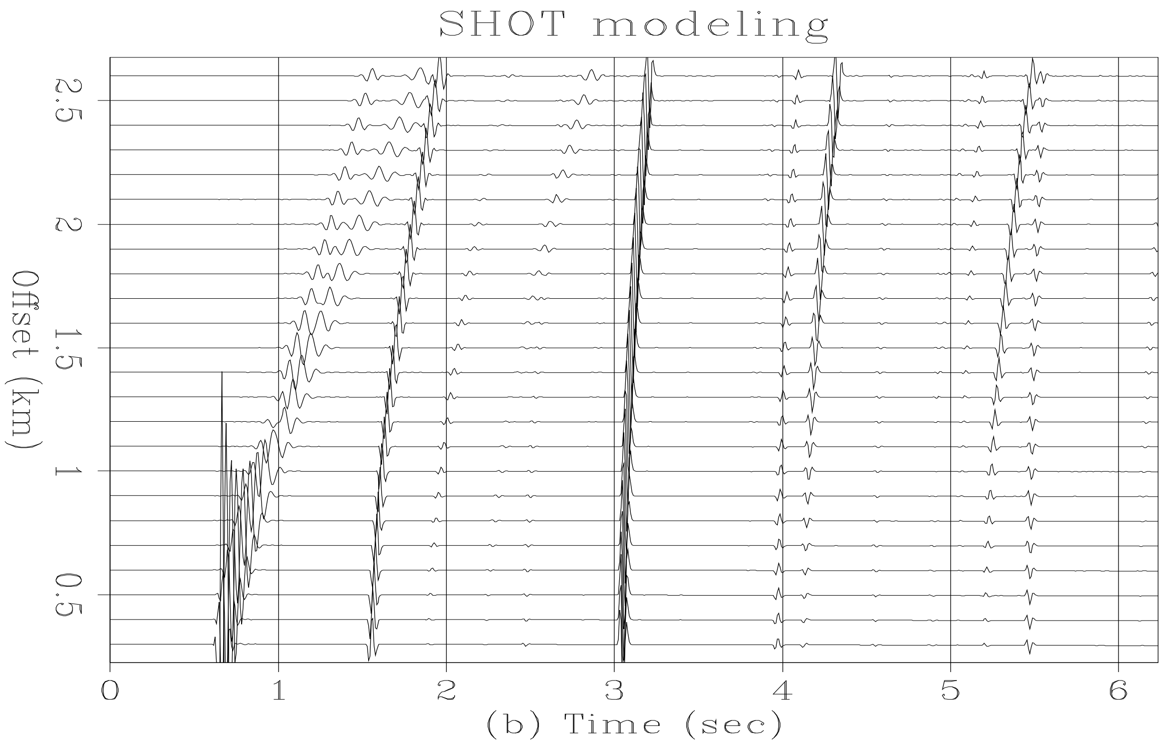
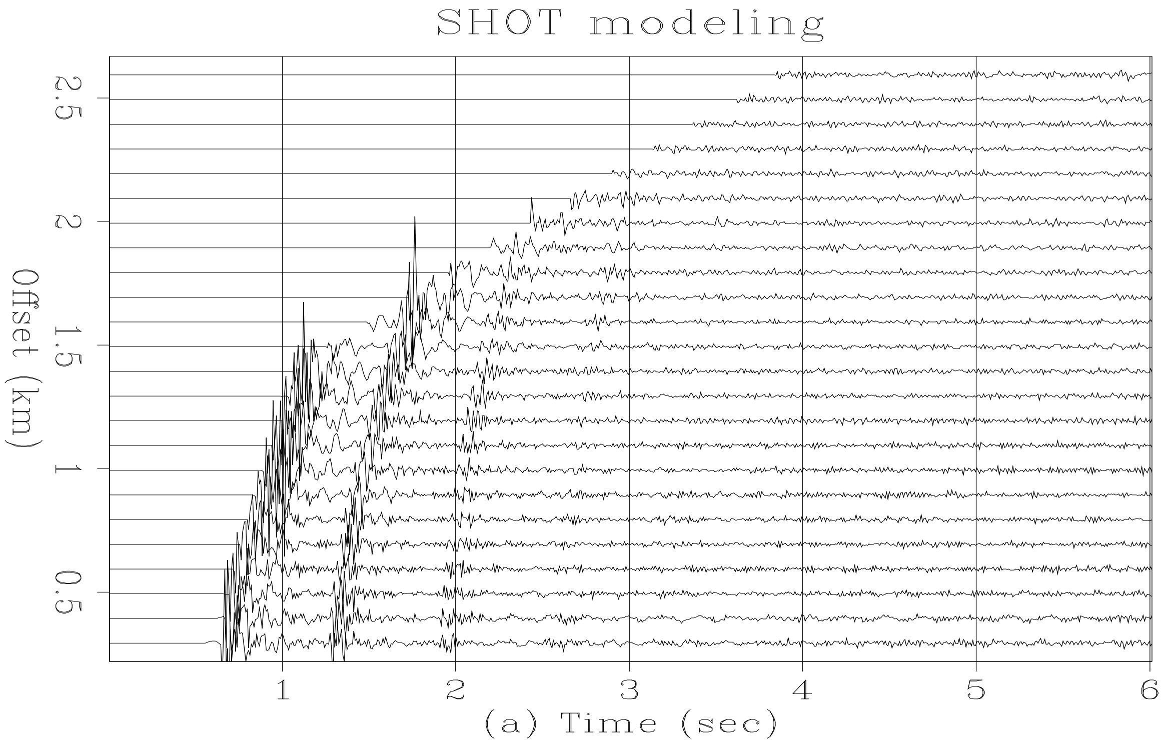




Figure 6: (a) is the water related reverberations attenuated section; (b) is the Solid elastic modeling section free of water related reverberations. mo2-SynMultAtten [ER]



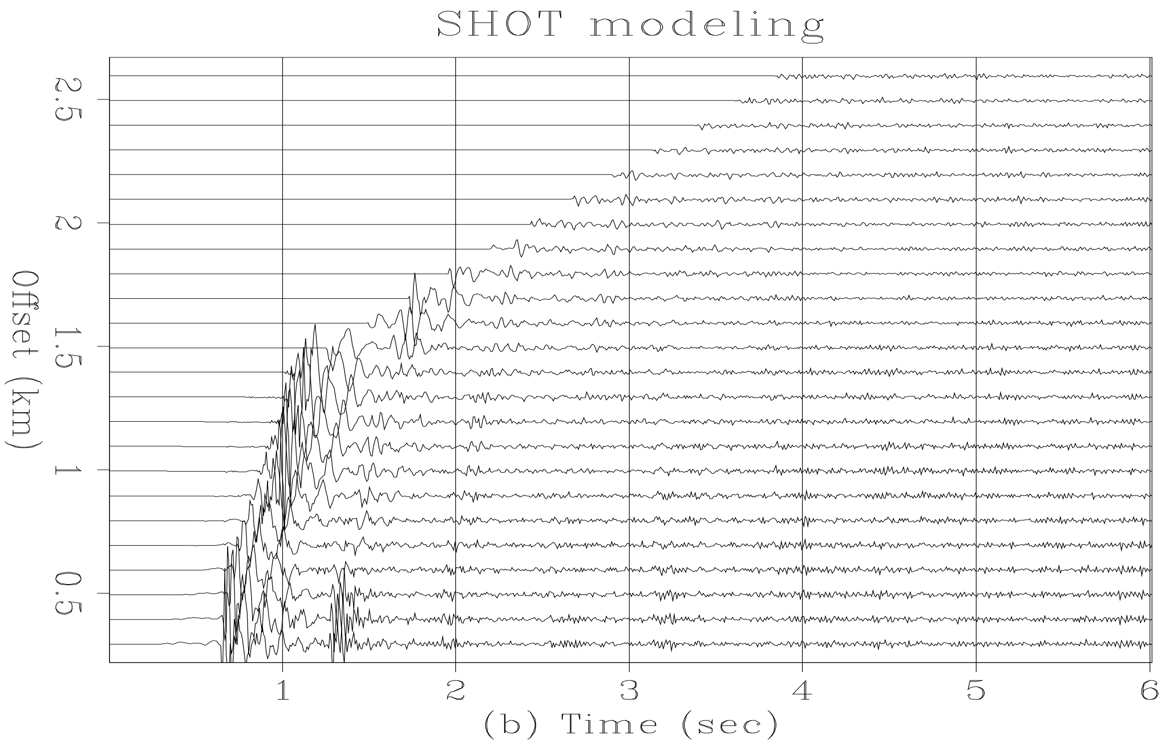




Figure 7: (a) is the Geco common shot gather after wavelet compression and muting to remove direct and head waves; (b) is the water-related reverberations attenuated section. $\boxed{\text{mo2-GMMultAtten}}$ [ER]

does not offset the first one. Since the use of the two weighting functions in the paper is not satisfactory enough, we need a better strategy to handle the different velocity sensitivity between shallow and deep parts in the velocity transform.

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## REFERENCES

Claerbout, J. F., 1991, Earth Soundings Analysis: Processing versus Inversion: Blackwell Scientific.

Claerbout, J. F., 1992, Anti aliasing: SEP–**73**, **??**–**??**.

Jedlicka, J., 1989, Stochastic normal moveout: SEP–**60**, 165–178.

Mo, L., 1992, Robust wavelet deconvolution: SEP–**73**, 37–42.

Morley, L., 1982, Predictive techniques for marine multiple suppression: Ph. D. thesis, Stanford University.

Muir, F., and Dellinger, Joe, 1985, A practical anisotropic system: SEP–**44**, 55-58.

Muir, F., 1987, Layered models, equivalence, and Abelian groups: SEP–**56**, 59–70.

Sherwood, J., Solid elastic modeling program: Geophysical Development Corporation.

Taner, M. F., 1980, Long-period sea-floor multiples and their suppression: Geophys. Prosp., **28**, 30–48.

Treitel, S., Gutowski, P. R., and Wagner, D. E., 1982, Plane-wave decomposition of seismograms: Geophysics, **47**, 1375–1401.

Woodward, M., and Muir, F., 1985, Anisotropic NMO Removal: SEP–**44**, 67–73.