Stanford Exploration Project, Report 73, November 18, 1997, pages 249–257

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Finding the amplitudes of migration hyperbolas from the kinematics

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keywords: *deconvolution, amplitudes, Fourier transform*

ABSTRACT

In migration and DMO, the kinematic problem is easier to solve than the amplitude problem. Where a purely analytical approach is normally used to obtain the amplitude of a migration operator, we attempt to obtain amplitude information by combining an assumption of a flat spectrum with the kinematical formulation. This hyperbola whitening is done with a 2-D prediction-error filter. This technique is applied to three hyperbolas: a kinematic-only hyperbola, a amplitude-corrected hyperbola, and an amplitude-corrected hyperbola with a half-derivative filter. The results show that the assumption of a flat spectrum conflicts with the existence of an evanescent zone, creating unresolved difficulties.

INTRODUCTION

The conventional approach to migration is to derive analytically the positions and the amplitudes of the operator coefficients before the application. Much effort has gone into calculating the correct amplitude response of migration operators, and a considerable cost is involved in applying them. DMO and prestack migration amplitude problems also appear to be difficult problems if amplitude characteristics are required, but the kinematics are fairly straightforward.

In this paper, we try to ignore much of the physics of the problem by attempting to start with a simple geometrical description of the positions of an operator, that is, the kinematic part of the problem, and then deriving the amplitude and phase information by applying a 2-D whitening operator to this kinematic-only part of the operator. This 2-D whitening operator may be described as a unitary operator that maps data from x-t space to x-z space along with the geometrical transformation. The 2-D whitening operator we use is based on the forms for 2-D prediction-error filters discussed by Claerbout (1992), page 198.

This approach might allow simpler and more efficient application of migration

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and DMO operators. If a short operator can be used, and if that operator allows the amplitude and phase information of the migration operator to be specified, migration and DMO processes might be implemented by a two-step process of applying the 2-D prediction-error filter operator to the input data, then applying a simple kinematic operation to the data. The 2-D prediction-error filter operator may be applied before or after the kinematic operations.

In this paper, we show results of applying this technique to three cases. The first case is a purely kinematic hyperbola, one with uniform amplitude. The second case is a hyperbola corrected with simple spherical divergence and cosine amplitude corrections. The third case is a hyperbola with the amplitude corrections and a half-derivative filter.

Although the prediction-error filter whitens the spectrum of the kinematic-only and the amplitude-corrected hyperbola, the calculated operator shows noise being built up in the evanescent zone. The amplitude-corrected hyperbola with the half-derivative filter is significantly damaged by the filter, while we would expect little change if the process had been successful.

MIGRATION OPERATOR ASSUMPTIONS

We assume that a migration operator maps events from x-t space outside the evanescent zone to events in x-z space, and that this mapping is unitary, or white. To do this we need a unitary migration operator. The migration operator should look white outside the evanescent zone, but be zero within the evanescent zone. If we take an operator with position information only and calculate an operator that will whiten it as above, we should get a reasonable approximation to the correct migration operator. For this we can used a 2-D prediction-error filter. While we expect some problems in the evanescent zone, we hope all the energy on the input will be moved to the output with just the proper phase changes due to dips.

This approach to migration is different from other approaches in that we are ignoring much of the physics of the problem, or at least we are taking a different view of the physics. The unitary operator being developed may be considered to be somewhere between the conjugate operator and the inverse. (See Claerbout (1992), pages 99-145.) It might be possible to extend this approach to DMO and prestack migration.

TWO-DIMENSIONAL PREDICTION ERROR FILTERING

If we attempt to calculate an operator that will annihilate a hyperbola, this operator should eliminate any predictable part of this hyperbola. The result of filtering the hyperbola with this operator should be our desired output, a whitened hyperbola.

The operator we calculate was expected to have a form like:

$$
\begin{array}{ccccccc}
\cdot & \cdot & \cdot & 1 & a & a & a \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a
\end{array}
\tag{1}
$$

A dot indicates a zero and an a indicates the individual filter coefficients. This prediction-error filtering is done with a spatial prediction-error filter similar to that of `pe2` from Claerbout(1992), page 193, that calls the conjugate-gradient routine in Claerbout(1992), page 137.

The first attempt with the filter above produced hyperbolas that were asymmetric, a result that was unexpected. To make the operator symmetric, we zeroed out the last half of the first row.

$$
\begin{array}{ccccccc}
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a
\end{array}
\tag{2}
$$

By calculating a two-dimensional filter that removes the predictable part of the hyperbola, the filter will be whitened as desired. The evanescent zone is likely to create a problem if we cannot suppress the whitening by providing a small amount of white light.

Since the evanescent zone produces problems shown later in this paper, an operator of the following form could be used.

$$
\begin{array}{ccccccc}
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & a & a & a & \cdot & \cdot \\
\cdot & a & a & a & a & a & \cdot \\
a & a & a & a & a & a & a \\
a & a & a & a & a & a & a
\end{array}
\tag{3}
$$

The challenge is zeroing the proper coefficients to suppress the noise created in the evanescent zone. This idea needs more development before it is implemented.

WHITENING HYPERBOLAS WITH 2-D PREDICTION-ERROR FILTERS

For comparison, we tried whitening three types of hyperbolas: a purely kinematic hyperbola, a hyperbola with some amplitude corrections, and a hyperbola with the amplitude corrections and a half-derivative filter. A hyperbola with only the kinematics was built by placing points on a grid along a hyperbolic path with a little interpolation. The next best approximation to a correct hyperbola was to add cosine and spherical divergence terms. Finally, a half-derivative filter was added to the amplitude corrected hyperbola. Hyperbola whitening was applied to these three cases with a 2-D prediction-error filter. If this version of hyperbola whitening works, we expect similar results from all three cases, and we expect the operator applied to the hyperbola with amplitude and half-derivative corrections to do almost nothing.

Each hyperbola was passed through the 2-D prediction-error filter, and the input and the output spectrums were displayed. The prediction-error filters were 3 elements in the trace direction and 25 elements in the time direction. The results are shown in Figures **??** to **??**.

Figure **??** shows some whitening of the spectrum of the hyperbola without amplitude corrections, but the evanescent zone shows some noise buildup. The spectrum shows a spike at the origin, indicating that some undesired D.C. is being passed, but the results are encouraging. The ridges in the spectrum may be caused by a filter that was too short in the time direction.

Figure **??**, with the amplitude corrected hyperbola, shows that the amplitudes of the limbs of the hyperbola are significantly lowered after the prediction-error filter. This is likely to be the result of the buildup of the operator in the evanescent zone. While the spectrums in Figure **??** and Figure **??** appear similar, and the amplitudes of the filtered hyperbolas are close, there are some differences, especially on the limbs.

The spectrum of the hyperbola with amplitude and half-derivative corrections has noise inside the evanescent zone after the application of the prediction-error filter, as seen in Figure **??**. It is disappointing to see the spectrum is less flat after filtering than before. The prediction-error filter has done significant damage to this hyperbola, where little change was expected. Once again, the damage may be attributed to the buildup of the zone. It is interesting to note that the filtered hyperbolas and their spectrums look similar in each of the figures.

CONCLUSIONS

The initial results of hyperbola whitening are inconclusive, but there are some encouraging results. The spectrum of a purely kinematic hyperbola is whitened considerably after the 2-D prediction-error filter, but noise has been generated in the evanescent zone. When a hyperbola with amplitude and half-derivative filter corrections is put into the prediction-error filter, it appears that the D.C. component of the spectrum is enhanced. The noise in the evanescent zone and the high D.C. component of the spectrum may be attributed to the prediction-error filter's action on the evanescent zone. As noted above, the prediction-error filter might be modified to avoid noise in this zone.

After prediction-error filtering, each hyperbola is followed by a series of ripples. Ripples may be seen in the whitened hyperbola and in the spectrum of the whitened

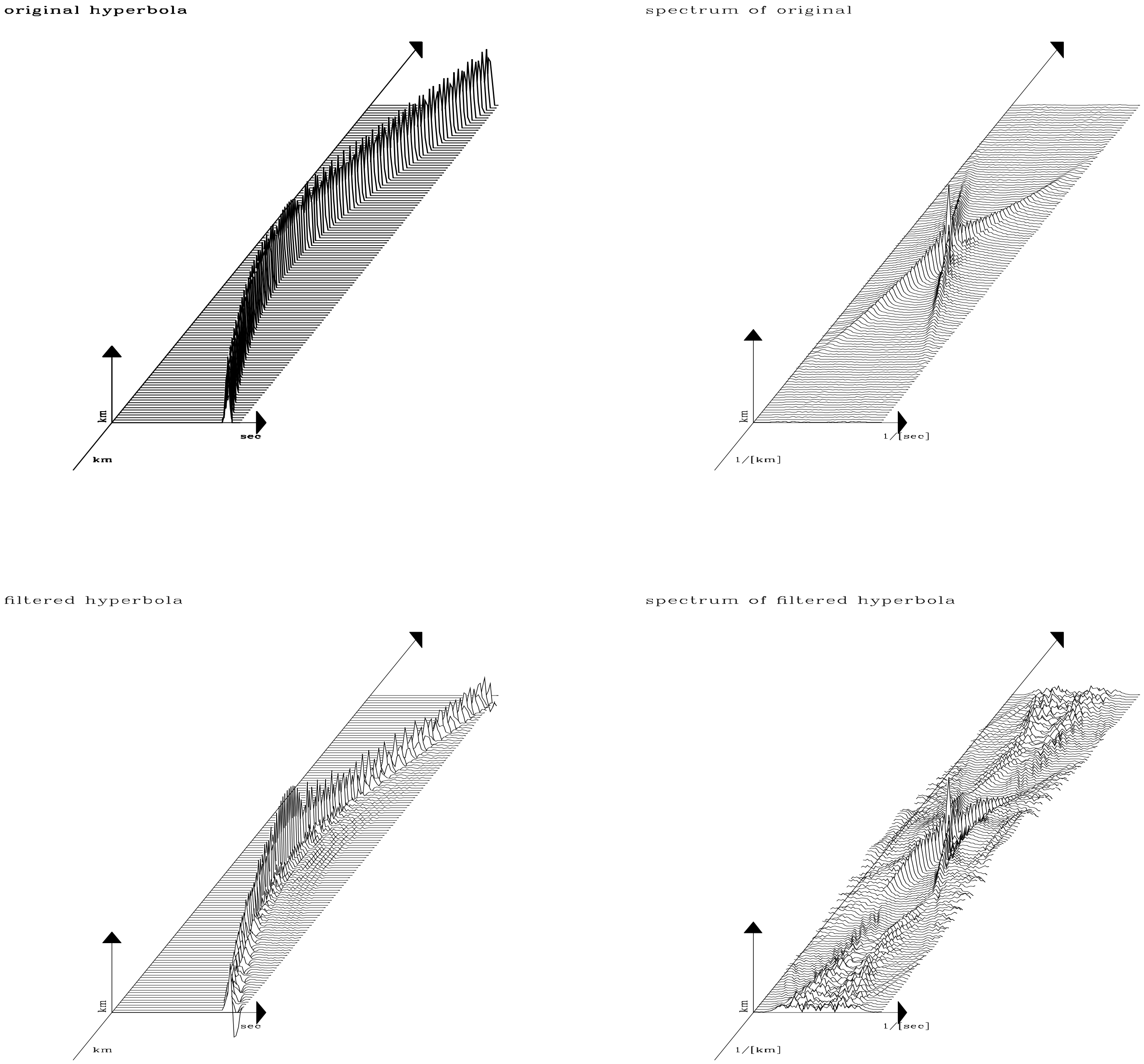

Figure 1: A hyperbola without amplitude corrections. $\boxed{\text{ray3-all1}}$ [ER]

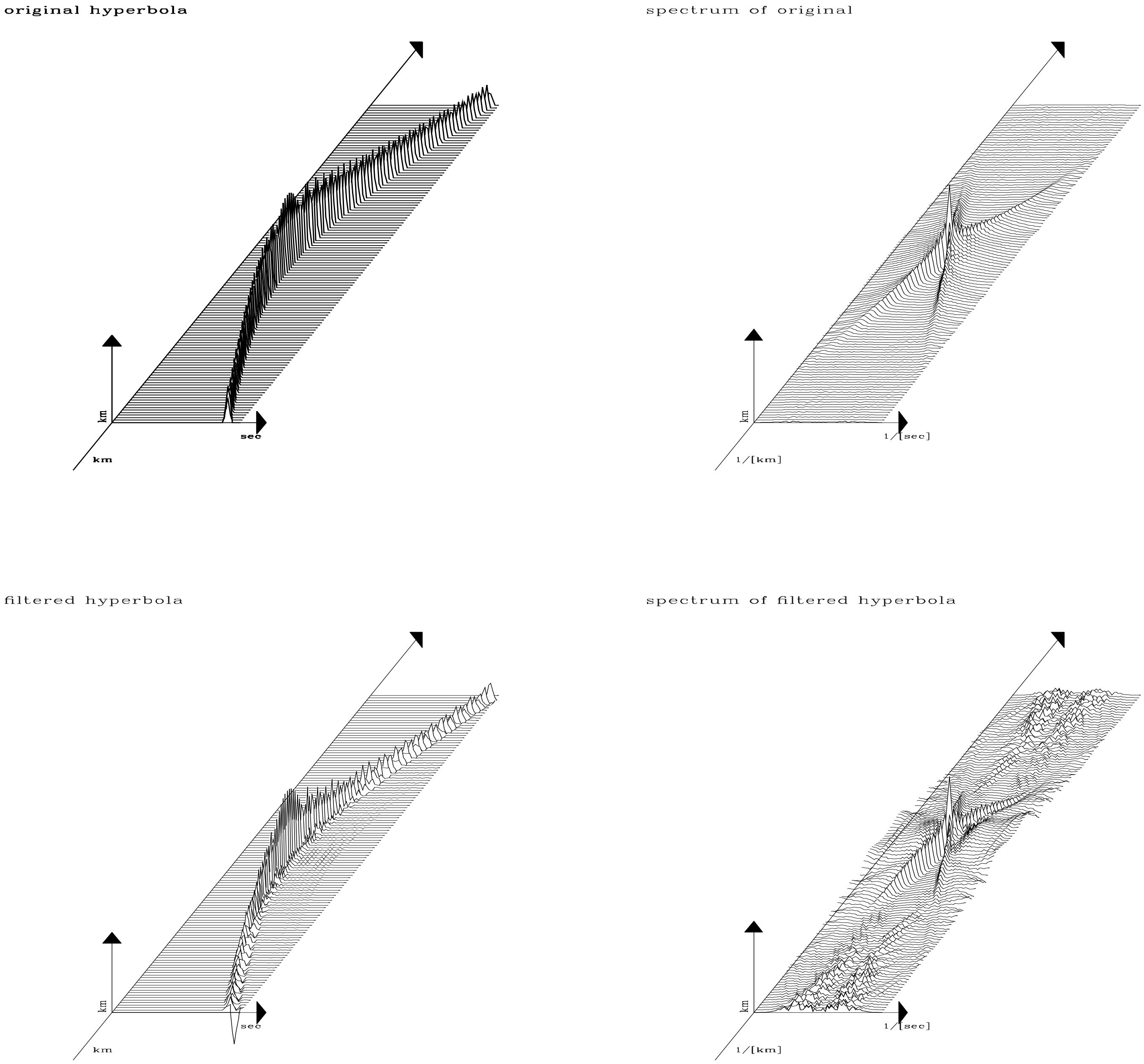

Figure 2: A hyperbola with amplitude corrections. $\boxed{\text{ray3-all2}}$ [ER]

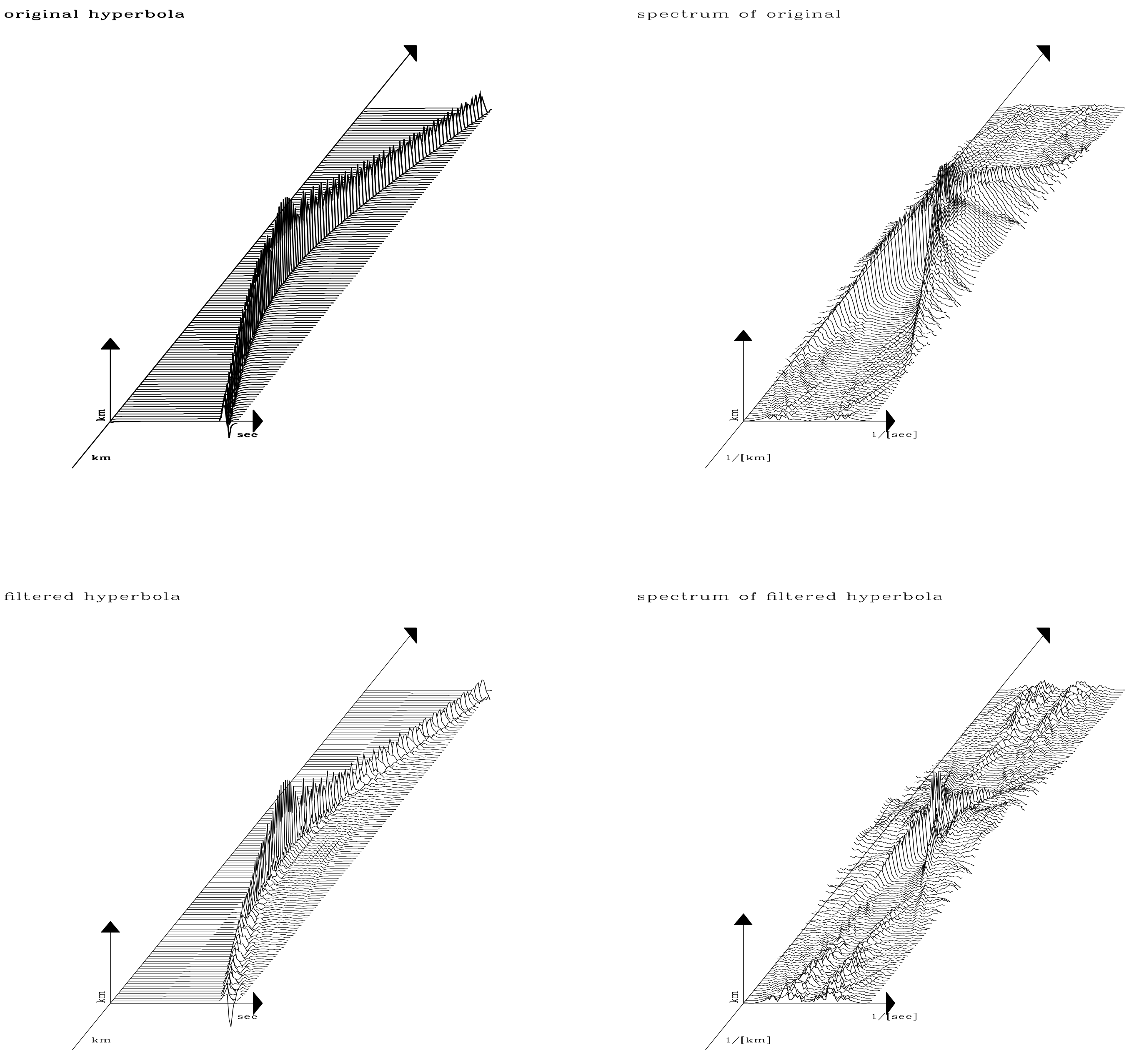

Figure 3: A hyperbola with amplitude and half-derivative corrections. ray3-all3 [ER]

hyperbola. This effect may be caused by the number of filter coefficients in time being too small. Unfortunately, increasing the size of the filter may increase the computation and application costs of the process.

The noise being generated in the evanescent zone may be eliminated by constraining some elements of the prediction-error filter to be zero. Accurate control over the filter constraints may be a problem, since we are using only three coefficients in the trace direction. Controlling the evanescent-zone noise buildup is the next step in developing this process.

REFERENCES

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