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Traveltime tomography in azimuthally anisotropic media

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ABSTRACT

I propose a method of seismic traveltime tomography in azimuthally anisotropic media consisting of dipping transversely isotropic layers. The model used for velocities is elliptical with variable inclination of the axis of symmetry. When used with the double elliptic approximation, the method can be used to fit general transversely isotropic media. This method is a generalization of conventional tomographic inversion techniques where the interfaces of the model are allowed to change their positions and to be eliminated from the inversion as iterations proceed, which allows a better estimation of the remaining unknowns. Ray tracing is performed in anisotropic models. The technique is tested with cross-well synthetic and field data where both strong anisotropy and strong velocity contrasts are present.

INTRODUCTION

The model space needed to perform seismic traveltime tomography consists of two separate models: one for the velocities and one for the heterogeneities. The selection of each of these models should be made depending on any prior knowledge we may have about the true medium. Even if we don't know anything about the medium, the selection should be guided by the type of information we want the extract from the data.

The most common choice for the model of the velocities is also the simplest one: isotropic. By contrast, the model of heterogeneities is often the most complex: a fine 2-D grid (McMechan, 1983). Although the selection of the model for the velocities is usually guided by a simple form of prior knowledge (i.e, "isotropic velocities usually work"), the selection for the model of heterogeneities implicitly assumes that we don't know anything about the spatial variations expected in the medium. When available, this information should be introduced in the model as proposed by Van Trier (1988), Harlan (1989) and Michelena (1990), among others.

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From the inversion point of view, velocity and heterogeneities are closely related. When the model for the velocities is isotropic but the real medium is not, the image of heterogeneities obtained is distorted. On the contrary, if we want to estimate velocity anisotropy in the medium but the heterogeneities are not properly described, we might get wrong estimates for the velocities. These problems can be solved by describing properly both velocities and heterogeneities and then estimating both *simultaneously*, as I propose in this paper. However, this might be difficult to do for a general anisotropic and heterogeneous medium.

Velocity anisotropy, heterogeneity, or both at the same time, can affect the traveltimes nonlinearly depending on how strong they are and how complex the model that describes them is. When one of these nonlinearities is neglected, the computations related to the other one can be simplified. For example, assuming weak anisotropy, Pratt and Chapman (1992) and Chapman and Pratt (1992) proposed to do the ray tracing in isotropic media and to use those rays to invert in the anisotropic model. Michelena et al. (1992) simplify the problem even more and present examples where the velocity contrasts are small enough such that the rays traced on the elliptically anisotropic models are straight and the only nonlinearity that remains is in the estimation of the anisotropy. Another way to simplify the simultaneous estimation of velocities and heterogeneities is by assuming simple models for both. This is the approach I follow in this paper.

The simplest anisotropic velocity model we can assume is elliptical. Elliptical anisotropy is a useful model when the aperture of the experiment is such that the slowness surface is not properly sampled and therefore, it is difficult to estimate more than two velocities: one based on the arrivals along a certain axis (direct velocities) and other based on the curvature around that axis (NMO velocities).

Elliptical anisotropy is not a good approximation for velocities at all angles in a transversely isotropic media. However, it is a good *paraxial* approximation that can be used in two steps (once around the horizontal and once around the vertical) to approximate globally the slowness surface and impulse response of any wave type in a transversely isotropic medium (Muir (1990) and Dellinger and Muir (1991)). Because the approximation uses two perpendicular ellipses, it is called the double elliptic approximation. What is needed to use this approximation is an inversion procedure that fits traveltimes with ellipses once a model for heterogeneities has been chosen.

The simplest model to describe the heterogeneities of the medium is 1-D. If dips are present, we can allow the interfaces of the 1-D model to have variable slope. However, if the spatial variations in the medium are more complex, we may need to use models that specifically account for them, as proposed by Van Trier (1988) and Harlan (1989). It may seem that with a fine 2-D parametrization we should have, in one hand, enough degrees of freedom to explain all kinds of "unexpected" variations in the medium and in the other hand, we should be able to explain the data better. This is only partially true. Michelena at al. (1992) show that for certain media ($\approx$ 1-D, weakly anisotropic) field data can be fitted equally well either with 1-D anisotropic models or with 2-D isotropic models, the latter having 2 orders of magnitude more of degrees of freedom. In the same paper, Michelena et al. also show that the problem of estimating velocity anisotropy in layered media is more stable and the results are more accurate when the discretization is 1-D than when the discretization is 2-D.

The model for velocities and heterogeneities I will use in this paper is a compromise between the simplicity of 1-D isotropic and the complexity of 2-D anisotropic models (Michelena at al., 1992). This compromise addresses three issues: how general the medium can be, how stable the inversion procedure is, and how difficult the ray tracing is. Such an "intermediate" model consists of consists of homogeneous elliptically anisotropic blocks separated by straight interfaces of variable dip and intersect. Interfaces may change position during the inversion process but are never allowed to cross in the area of interest. Byun (1982) proposes a procedure to trace rays in this type of model. In Michelena (1992), I review Byun's procedure and show the details of the actual implementation when the axes of symmetry are tilted.

Certain types of azimuthally anisotropic media can be also approximated with this model of velocity and heterogeneity, in particular those formed by dipping transversely isotropic layers. This is done by considering also the inclination of the axis of symmetry as a variable. The estimation of this variable may help the interpreter to characterize the amount of structural deformation that have undergone the different formations in the reservoir and may also help in the identification of structural unconformities.

A theoretical discussion of the technique is followed by examples with cross-well synthetic data and field data from BP's Devine test site.

FORWARD MODELING

The traveltime for a ray that travels a distance d in an homogeneous medium with elliptical anisotropy and axis of symmetry forming an angle γ with respect to the vertical (Figure 1) is

$$t = \sqrt{\frac{(\Delta x \cos\gamma + \Delta z \sin\gamma)^2}{V_\perp^2} + \frac{(-\Delta z \cos\gamma + \Delta x \sin\gamma)^2}{V_\parallel^2}}, \tag{1}$$

where $\sqrt{\Delta x^2 + \Delta z^2} = d$ and $V_\parallel$ and $V_\perp$ are the velocities in the directions parallel and perpendicular respectively to the axis of symmetry. In Appendix A, I explain how to derive this expression.

If the model is described as a superposition of N homogeneous orthogonal regions, the traveltime for the i^{th} ray traveling across the j^{th} region is

$$t_{i,j} = \sqrt{\frac{(\Delta x_{i,j} \cos\gamma_j + \Delta z_{i,j} \sin\gamma_j)^2}{V_{\perp_j}^2} + \frac{(-\Delta z_{i,j} \cos\gamma_j + \Delta x_{i,j} \sin\gamma_j)^2}{V_{\parallel_j}^2}}. \tag{2}$$

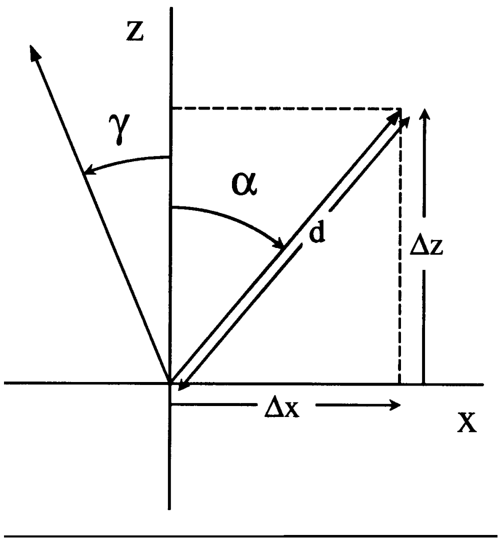

Figure 1: Ray traveling a distance d in a medium with tilted axis of symmetry. α and γ are the angles of the ray and the axis of symmetry respectively with respect to the vertical. reinaldo1-ray-and-axis [NR]

Note that each homogeneous region is characterized by three parameters: two velocities and the angle of the axis of symmetry with respect to the vertical. From now on I will refer to this parameters as interval parameters. In the previous equation, $\sqrt{\Delta x_{i,j}^2 + \Delta z_{i,j}^2}$ is the distance traveled by the i^{th} ray in the j^{th} cell. The sum of expressions like equation (2) can be used to compute the traveltime from source to receiver for a ray that travels in an heterogeneous media, assuming the ray path is known. Byun (1982) and Michelena (1992) explain how to do the ray tracing.

Figure 2 shows the type of model that I will consider in this paper. It consists of homogeneous elliptically anisotropic blocks separated by straight interfaces of variable dip (a_j) and intercept (b_j). I assume that all the axes of symmetry for the different layers lie in the same plane of the survey geometry. If $\Delta x_{i,j}$ and $\Delta z_{i,j}$ are defined as

$$\Delta x_{i,j} = x_{i,j+1} - x_{i,j} \tag{3a}$$
$$\Delta z_{i,j} = z_{i,j+1} - z_{i,j} \tag{3b}$$

the expression (2) for the traveltime $t_{i,j}$ of the i^{th} ray in the j^{th} cell becomes

$$t_{i,j} = \sqrt{\Delta X_{i,j}{}^2 S_{\perp_j}^2 + \Delta Z_{i,j}{}^2 S_{\|_j}}, \tag{4}$$

where $S_{\perp_j}$, $S_{\|_j}$, $\Delta X_{i,j}$ and $\Delta Z_{i,j}$ are equal to

$$S_{\perp_j} = \frac{1}{V_{\perp_j}},$$
$$S_{\|_j} = \frac{1}{V_{\|_j}},$$

$$\Delta X_{i,j} = \Delta x_{i,j} \cos\gamma_j + (a_{j+1}x_{i,j+1} + b_{j+1} - a_j x_{i,j} - b_j)\sin\gamma_j,$$
$$\Delta Z_{i,j} = -(a_{j+1}x_{i,j+1} + b_{j+1} - a_j x_{i,j} - b_j)\cos\gamma_j + \Delta x_{i,j}\sin\gamma_j.$$

$(x_{i,j}, z_{i,j})$ is the point of intersection between the i^{th} ray and the j^{th} interface. If the axis of symmetry is vertical ($\gamma_j = 0$), it follows that $\Delta X_{i,j} = \Delta x_{i,j}$ and $\Delta Z_{i,j} = \Delta z_{i,j}$.

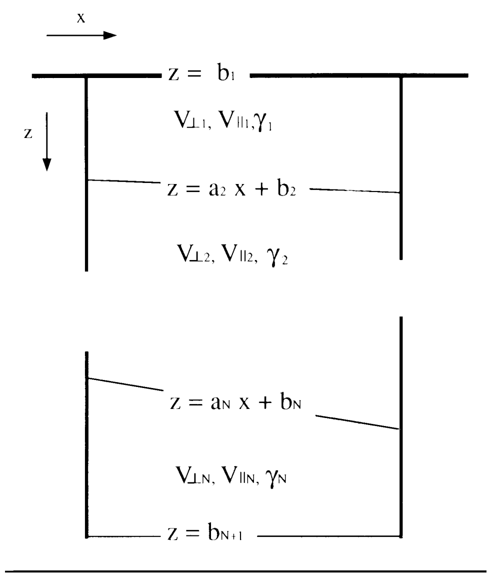

Figure 2: Model of velocities and heterogeneities. The top and bottom interfaces are horizontal ($a_1 = a_{N+1} = 0$) and located at known depths. reinaldo1-inv-model [NR]

Besides the interval parameters previously described, I have added two more parameters to describe how the boundaries that separate different intervals may change their positions. I call these parameters boundary parameters. Figure 2 shows how to count both intervals and boundaries.

The total traveltime for a ray that travels from source to receiver is

$$t_i(\boldsymbol{m}) = \sum_{j=1}^{N} t_{i,j}(\boldsymbol{m}) \qquad i = 1, ..., M, \tag{5}$$

where $\boldsymbol{m}$ is the vector of model parameters of $5N$ elements:

$$\begin{aligned} \boldsymbol{m} &= (m_1, ..., m_N, m_{N+1}, ..., m_{2N}, m_{2N+1}, ..., m_{3N}, m_{3N+1}, ..., m_{4N}, m_{4N+1}, ..., m_{5N}) \\ &= (S_{\perp_1}, ..., S_{\perp_N}, S_{\|_1}, ..., S_{\|_N}, \gamma_1, ..., \gamma_N, b_1, ..., b_N, a_1, ..., a_N), \end{aligned} \tag{6}$$

and M is the total number of traveltimes.

Equation (5) is the system of nonlinear equations that relates the model parameters with the measured traveltimes. A linearized version of this equations will be used in the next section to solve the inverse problem. As explained in Figure 2, b_1 and a_1 are known. This makes the number of variables in $\boldsymbol{m}$ equal to $5N - 2$.

INVERSE MODELING

When solving the inverse problem, the goal is to estimate two different sets of coupled unknowns: the model parameters and the ray paths. The usual way to decouple them is by invoking Fermat's principle, which "justifies" the trick of assuming one to estimate the other in an iterative fashion, as long as the magnitude of the changes from one step to the next are kept small.

Once the ray paths have been estimated, the system of nonlinear equations (5) needs to be solved in order to find a new model where rays are going to be traced again. One way to do this is as a sequence of linearized steps starting from a given initial model $\boldsymbol{m_0}$. The first step is to approximate (5) by its first order Taylor series expansion centered about a given model $\boldsymbol{m_0}$:

$$\begin{aligned} t_i(\boldsymbol{m}) &\approx t_i(\boldsymbol{m_0}) + \nabla t_i(\boldsymbol{m_0}) \cdot (\boldsymbol{m} - \boldsymbol{m_0}) \\ &= t_i(\boldsymbol{m_0}) + \sum_{n=1}^{5N} \underset{\sim}{\mathbf{J}}_{in} (m_n - m_{0n}), \end{aligned} \tag{7}$$

where the elements of the Jacobian $\underset{\sim}{\mathbf{J}}_{in}$ are

$$\underset{\sim}{\mathbf{J}}_{in} = \left. \frac{\partial t_i}{\partial m_n} \right|_{\boldsymbol{m}=\boldsymbol{m_0}} .$$

The explicit form of these derivatives is given in Appendix B.

If we assume that $t_i(\boldsymbol{m})$ represents one component of the vector $\boldsymbol{t}$ of measured traveltimes, we can compute the perturbations $\Delta \boldsymbol{m}_n = (m_n - m_{0n})$ once the traveltimes in the reference model $\boldsymbol{m_0}$ has been calculated. The perturbation $\Delta \boldsymbol{m} = (\boldsymbol{m} - \boldsymbol{m_0})$ is the solution of the following system of equations

$$\underset{\sim}{\mathbf{J}} \Delta \boldsymbol{m} = \Delta \boldsymbol{t} \tag{8}$$

where $\Delta \boldsymbol{t}_i = t_i(\boldsymbol{m}) - t_i(\boldsymbol{m_0})$.

In practice, only a fraction r of the correction $\Delta \boldsymbol{m}$ is added to the given model

$$\boldsymbol{m_1} = (\boldsymbol{m_0} + r \Delta \boldsymbol{m})$$

where r (the step length) is usually small to avoid large changes in the ray paths from one iteration to the next.

The system of linear equations (8) will be solved using the LSQR variant of the conjugate gradients algorithm (Nolet, 1987)

Constraints

Prior information about the real medium can be introduced in the inversion in two ways: with the starting model and by imposing bounds on the variations of the parameters being estimated. In this particular algorithm, these two forms of prior information are *essential* for a reliable estimation of the vector $\boldsymbol{m}$.

If the starting model is “far” from the “true” answer the algorithm may not converge at all, or if it does, it may produce a wrong answer. This may happen with any nonlinear inversion. However, it can be shown that if the starting model is “close” to the true answer, a linearized solution of the nonlinear system of equations (5) converges quadratically (Gill et al., 1981). Therefore, any information introduced in the initial model (especially about dips) will help to algorithm to converge to a reliable estimate of the real medium.

Since the interfaces are allowed to change position from one iteration to the next, the algorithm has to make sure that each new model does not contain crossing interfaces in the area of interest. If two interfaces cross after adding $r\Delta\boldsymbol{m}$ to the given model, the step length r is reduced until those interfaces do not cross after the correction. Once the model has been updated and there are no crossing interfaces, the algorithm checks whether thin layers have been created or not and if so, those layers are eliminated from the inversion, reducing by a multiple of 5 the number of model parameters (five parameters per each eliminated layer). This is done by comparing the thickness of all layers with a predetermined minimum layer thickness, as long as the corresponding boundaries are parallel (also within a predetermined range of angles). If the predetermined minimum thickness is too small, the solution may contain layers with artificially high (or low) velocities and if the minimum is too big, spatial resolution in the model may be lost.

Whenever information is available about the position and dip of certain layers of the medium (from well logs for example) it should also be used to constrain the corresponding interfaces in the model.

Which is the axis of symmetry?

Every ellipse has two axes of symmetry. The inversion proposed in this paper can estimate the inclination with respect to the vertical of either one of them depending on which axis is closer to the initial model. In either case, the estimated parameters will always describe the same *elliptical velocity function*.

Another way to understand why the inversion can estimate the inclination of either axis of symmetry is by examining the traveltime equation (1). An identical equation can be obtained if we switch in (1) $V_{\parallel}$ and $V_{\perp}$ changing at the same time γ by $\gamma \pm \pi/2$. This means that the traveltimes are affected only by the elliptical function of velocities regardless of how such a function is described. Figure 3 shows how a given ellipse can be described with two different sets of parameters.

If the inversion procedure is forced to estimate the same axis of the ellipse (either the major or the minor) for every layer, it can be more difficult (or impossible) to get a reliable estimate of the real elliptical velocity function of the medium, especially in cases when the axis of symmetry in the initial model is far from the true answer.

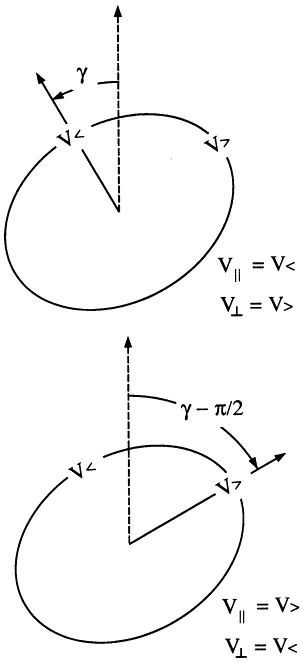

Figure 3: The same ellipse represented in two different ways by interchanging $V_\perp$ and $V_\parallel$ and rotating the axis of symmetry $-\pi/2$. Traveltimes are affected by the velocity function (elliptical in this case) regardless of how that function is described. reinaldo1-two-sets [NR]

SYNTHETIC EXAMPLE

In this section, I will show the application of the algorithm previously explained to the inversion of synthetic traveltimes from a cross-well geometry. 4992 traveltimes were generated through the model shown in Figure 4 for a geometry where sources were located within a range of ± 45 degrees with respect to the horizontal at each receiver position. The ray tracing algorithm described in Michelena (1992) was used to compute the synthetic traveltimes and to trace the rays needed in the nonlinear inversion.

The starting model used for the inversion was homogeneous isotropic described by 17 horizontal layers of equal thickness. The inclination used for the axis of symmetry was $\gamma = 0$ in all layers. Figure 5 shows the initial positions of the boundaries in the starting model. By starting the iterations with this model I wanted to test how the interfaces arrange themselves to create a dipping layer not present in the initial model.

The inversion was constrained by not allowing parallel layers (within ± 5 degrees) to be thinner than 15 ft. When this condition was met, the corresponding layer was eliminated, reducing the number of model parameters. No smoothing was applied to the model after each iteration.

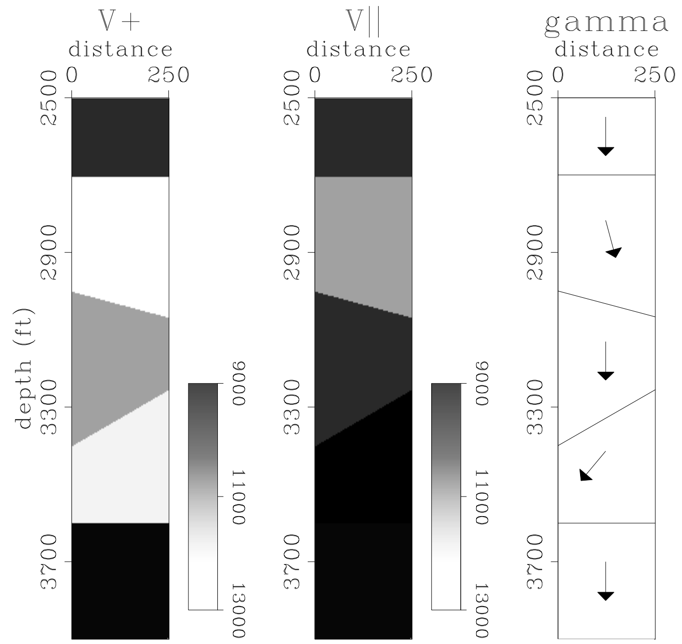

Figure 4: Synthetic model used to test the inversion. The third interface is dipping 15 degrees and the fourth one -30. The inclination of the axis of symmetry in the second layer is -15 degrees and 40 degrees in the fourth layer. The ratio $\frac{V_\perp}{V_\parallel}$ at the fourth layer is 1.26. The gray scale shows variations in velocity. "$V+$" stands for $V_\perp$, "$V||$" for $V_\parallel$ and "gamma" for γ. reinaldo1-syn-model [NR]

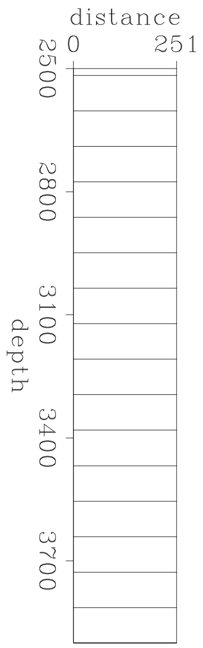

Figure 5: Initial position of the interfaces in the starting model (homogeneous isotropic). The velocity is equal to 12000 ft/sec and the inclination of the axes of symmetry with respect to the vertical is zero for all layers. reinaldo1-initial-model-syn [NR]

Figure 6 shows the result of the inversion after tracing rays 35 times, with few conjugate gradient iterations between each ray tracing. Notice how boundaries have changed positions with respect to their initial values. Notice also that the initial 17 layers were reduced to 10 to allow the positioning of the dipping ones at the correct depths with the correct dips. The inclination of the axes of symmetry estimated by the algorithm is also correct. Figure 7 compares a profile at $x = 0$ of the real model (Figure 4) and the estimated one (Figure 6). The agreement is almost perfect.

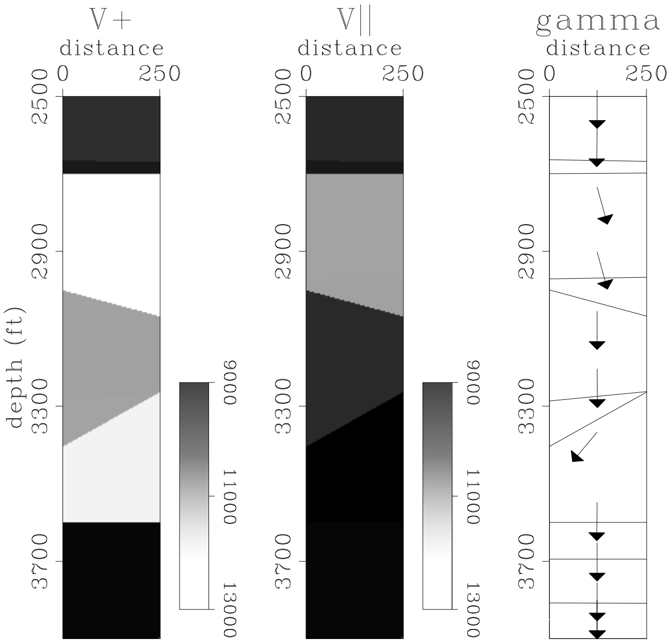

Figure 6: Result of the inversion. Notice how the interfaces have changed their initial positions. The position of each arrow's head shows the interval that correspond to each γ. The gray scale shows variations in velocity. "$V+$" stands for $V_{\perp}$, "$V||$" for $V_{||}$ and "gamma" for γ. reinaldo1-final-model-syn [NR]

In this example, it was possible to estimate correctly both small and moderate dips (between 0 and 30 degrees), but the question of the maximum and minimum dips that can be retrieved from the data will depend in general upon the aperture of the recording geometry, the interval between sources and receivers and the frequency.

FIELD DATA EXAMPLE

In this section, I show the application of the technique previously explained to a cross-well data set recorded at BP's Devine test site, located southwest of San Antonio, Texas. A sketch of the geology at this site is shown in Figure 8. This test site has been cited in recent publications to illustrate the application of different techniques. For example, Harris (1988) reports cross-well seismic measurements using a cylindrical piezoelectric bender transducer both as a source and as a receiver. Miller and Chapman (1991) and Onishi and Harris (1991) concentrate on the problem of estimating velocity anisotropy from cross-well data (tomographically and by head-wave analysis, respectively). Lazaratos et al. (1991) present reflection images also from the cross-well data and Raikes (1991) studies the propagation of S-waves from a multicomponent VSP.

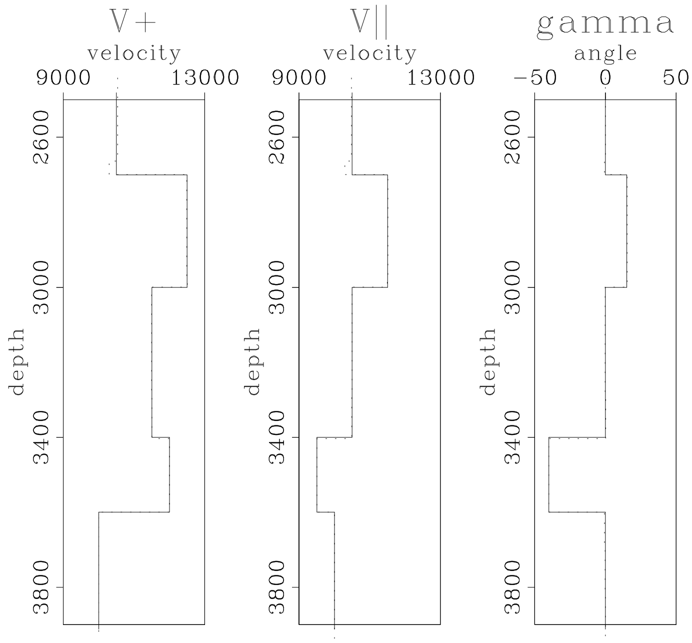

Figure 7: Profile at $x = 0$ of the real model and the estimated one (Figures 4 and 7 respectively). The continuos line represents the real model and the dashed one the result of the inversion. The agreement between real and estimated parameters is almost perfect. "V+" stands for $V_\perp$, "$V||$" for $V_{||}$ and "gamma" for γ. reinaldo1-final-model-profile-syn [NR]

The cross-well data I will use in this paper are P-waves with frequencies between 200 Hz and 4000 Hz. They were recorded between two cased boreholes (Wilson 2 and Wilson 4) whose separation at the surface was 330 ft. Receivers were separated 10 ft and sources 20 ft. Figure 8 shows the corresponding sonic logs at each well. Although the variations are mainly 1-D, it is possible to identify from the logs small dips ($\approx$ 1 degree) from one well to the other as well as small lateral variations within thin layers (for example, the top of the Del Rio clay).

1660 traveltimes were picked from a small data set of only 26 gathers. From these traveltimes only those corresponding to angles less than 45 degrees between source and receiver were kept for the inversion.

Figure 9 shows positions of sources and receivers for 10% of the data set. Notice how irregular the ray coverage is specially in terms of horizontal rays, not present in large portions of the medium. The horizontal velocities estimated in those areas, unlike the rest of the medium, will be the horizontal NMO velocities, that is, horizontal velocities of the best fitting ellipses.

The starting model (not shown) consisted of 140 horizontal layers 5 ft thick. The velocity for all layers was 12000 ft/sec. Layers thinner than 1 ft were not allowed in the model. The inversion was not constrained to match the vertical velocities or depth of certain layers using information derived from the sonic logs, although the information of dips is already present in the initial model.

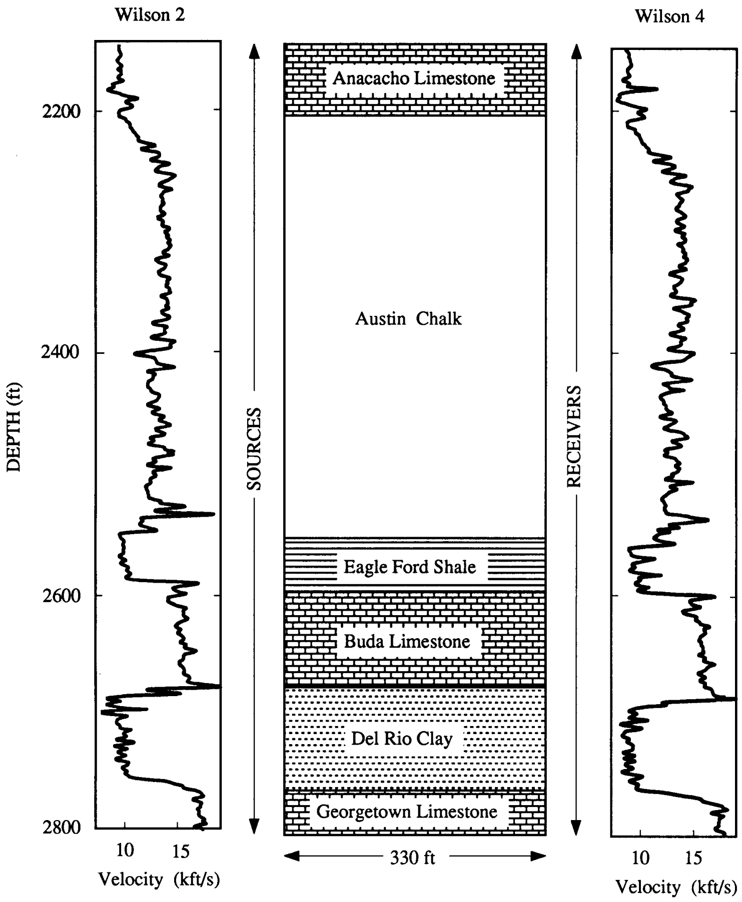

Figure 8: Sketch of the geology at the Devine test site with the sonic logs at each well. Although mostly flat layered, small 2-D variations can be seen (modified from Lazaratos et al., (1991). reinaldo1-devine-site [NR]

The inversion produced a model with horizontal layers (all dips less than 0.1 degree) and vertical axes of symmetry. Therefore, $V_\perp = V_x$ and $V_\| = V_z$. Figure 10 shows the horizontal and vertical velocities as well as an average sonic log from the two wells, blocked every 7 ft. The first thing we notice from Figure 10 is that $V_x \geq V_z$ for almost all depths. We also notice that V_z is much closer to the sonic log than V_x, which is more than 30% larger than the log velocity in the shale and clay intervals. The vertical velocity contrast between limestone and shale is greater than 70%. Still,

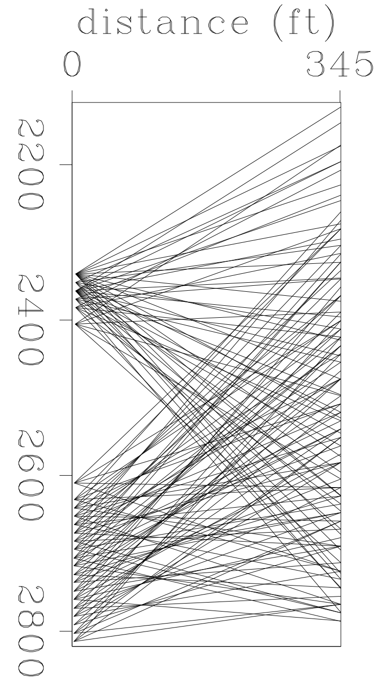

Figure 9: Ray coverage for the experiment at the Devine test site for a fraction (10%) of the total data set. Notice how irregular the coverage is. reinaldo1-devine-rays [NR]

the inversion does a good job in estimating V_z. For this model, the average absolute value of the residuals is 0.2 ms (twice the sampling interval). 12 layers were eliminated during the inversion procedure.

These results agree with the ones presented by Miller and Chapman (1991) and Onishi and Harris (1991).

Figure 10 also shows that the elliptical velocity model explains most of the P-wave anisotropy at this site. However, note that the estimated vertical velocity (NMO velocity) is in general smaller than the log velocity, which indicates that the elliptical model is not fully adequate to describe the possible transverse isotropic nature of this medium. In a few places (top of the clay, for example) the estimated vertical velocities are larger than the log velocity. This is probably because of lateral variations in the real medium that are not correctly described by the model of heterogeneities.

In order to better characterize the nature of the anisotropy, additional wave types and measurement directions (from a VSP survey for example) are needed.

CONCLUSIONS

A tomographic inversion technique that fits seismic traveltimes with elliptical velocity functions has been presented. The elliptical parameters estimated with this inversion can be inserted into the double elliptic approximation to estimate more general transversely isotropic models, when data from different geometries is used.

Since the inclination of the axis of symmetry is also a variable in the inversion procedure, certain types of azimuthally anisotropic media can be approximated, in particular those formed by dipping transversely isotropic layers.

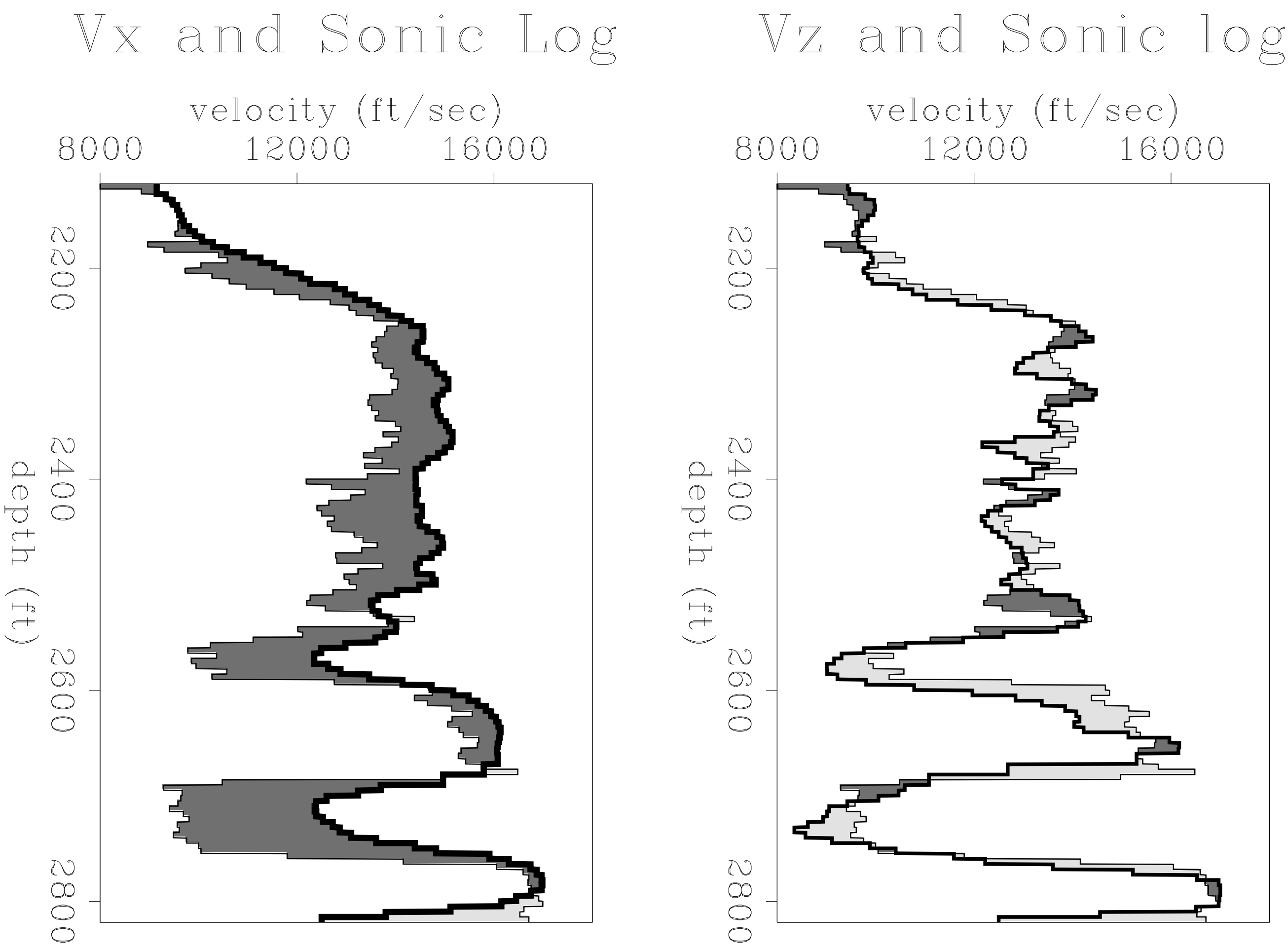

Figure 10: Result of the inversion. An average sonic log (thinner curve) blocked every 7 ft is compared with the two estimated velocities. The vertical velocity is closer and better correlated to the log. Notice how the amount of anisotropy changes through the model, reaching a peak at the shale and clay intervals. The model is described by 128 layers. reinaldo1-devine-log-vel [NR]

The model for heterogeneities is described as a superposition of homogeneous orthogonal regions whose boundaries may change their positions as iterations proceed but are not allowed to cross in the area of interest. When two parallel interfaces move too close to each other, one of them (along with its upper interval) is eliminated from the inversion, reducing the number of unknowns. The technique was successfully applied to synthetic and field data where both strong anisotropy and strong velocity contrasts were present.

ACKNOWLEDGMENTS

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REFERENCES

Byun, B. S., 1982, Seismic parameters for media with elliptical velocity dependencies: Geophysics, **47**, 1621–1626.

Chapman, C. H. and Pratt, R. G., 1992, Traveltime tomography in anisotropic media: Theory: Geophys. J. Intl., **109**, 1–19.

Dellinger, J. and Muir, F., 1991, The double elliptic approximation in the group and phase domains: SEP–**70**, 361–366.

Gill, P. H., Murray, W., and Wright, M. H., 1981, Practical Optimization: Academic Press.

Harlan, W. S., 1989, Tomographic estimation of seismic velocities from reflected raypaths: presented at the 59th Ann. Internat. Mtg., Soc. Expl. Geophys.

Harris, J., M., 1988, Cross-well seismic measurements in sedimentary rocks: presented at the 58th Ann. Internat, Mtg., Soc. Expl. Geophys.

Lazaratos, S. K., Rector, J. W., Harris, J. M. and Van Schaack, M., 1991, High resolution imaging with cross-well reflection data: presented at the 61st Ann. Internat. Mtg., Soc. Expl. Geophys.

Lee, M.,W.,1990, Traveltime inversion using transmitted waves of offset vertical seismic profiling data: Geophysics, **55**, 1089–1097.

McMechan, G.A., 1983, Seismic tomography in boreholes: Geophys. J. Roy. Astr. Soc., **74**, 601–612.

Michelena, R. J., 1990, Traveltime inversion with constraints: STP–**1**, paper H.

Michelena, R. J., 1992, Kinematic ray tracing in anisotropic layered media: SEP–**73**.

Michelena, R. J., Muir, F. and Harris, J. M., 1992, Anisotropic traveltime tomography: submitted for publication in Geophysical Prospecting.

Miller, D. E., and Chapman, C. H., 1991, Incontrovertible evidence of anisotropy in cross-well data: presented at the 61st Ann. Internat. Mtg., Soc. Expl. Geophys.

Muir, F., 1990, A modified anisotropic system: SEP–**67**, 41–42.

Nolet, G., 1987, Seismic wave propagation and seismic tomography, *in* Nolet, G., Ed., Seismic tomography: D. Reidel Publ. Co., 1–23.

Onishi, M., and Harris, J., M., 1991, Anisotropy from head waves in cross-well data: presented at the 61st Ann. Internat. Mtg., Soc. Expl. Geophys.

Pratt, R. G., Chapman, C. H., 1992, Traveltime tomography in anisotropic media: application: Geophys. J. Intl.,**109**, 20–37.

Raikes, S., 1991, Shear-wave characterization of the BP Devine test site, Texas: presented at the 61st Ann. Internat. Mtg., Soc. Expl. Geophys.

Van Trier, J., 1988, Migration velocity analysis using geological constraints: presented at the 58th Ann. Internat. Mtg., Soc. Expl. Geophys.

APPENDIX A

The expression for the ray velocity in a medium with elliptical velocity dependency is given by the expression

$$\frac{1}{V^2(\alpha)} = \frac{\cos^2\alpha}{V_{\|}^2} + \frac{\sin^2\alpha}{V_{\perp}^2}, \tag{A-1}$$

where α is the ray angle measured from the axis of symmetry (positive counterclockwise) and $V_{\|}$ and $V_{\perp}$ are the velocities in the directions parallel and perpendicular to the axis of symmetry (Figure A-1-a). When the axis of symmetry is vertical, the angle that measures the direction of propagation of the ray with respect to the vertical is the same as the group velocity angle.

When the axis of symmetry is rotated an angle γ (Figure A-1-b), the expression for the ray velocity becomes (for the same ray direction):

$$\frac{1}{V^2(\alpha)} = \frac{\cos^2(\alpha-\gamma)}{V_{\|}^2} + \frac{\sin^2(\alpha-\gamma)}{V_{\perp}^2} \tag{A-2}$$

where $\alpha-\gamma$ is the angle from the axis of symmetry to the ray (group velocity angle).

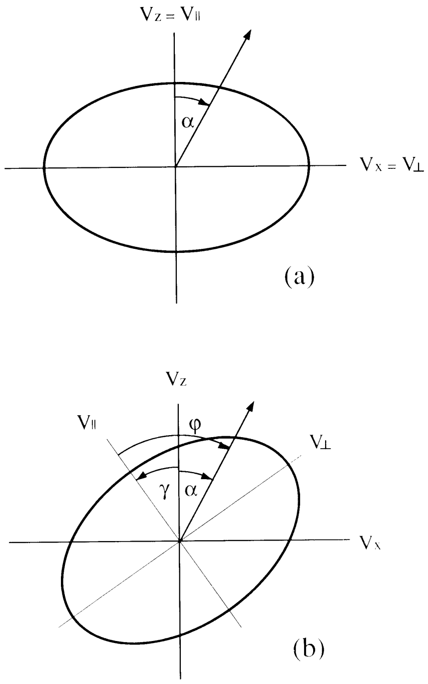

Figure A-1: Ray velocity as a function of direction in an elliptically anisotropic medium: (a) Axis of symmetry vertical (ray angle = group velocity angle). (b) Axis of symmetry tilted (ray angle = α; group velocity angle = ϕ). reinaldo1-tilted-ellipses [NR]

If the ray travels a distance d between two points (Figure 1),

$$d = \sqrt{\Delta x^2 + \Delta z^2} \tag{A-3}$$

the corresponding traveltime t is

$$\begin{aligned} t^2 &= \frac{(d\cos(\alpha-\gamma))^2}{V_{\|}^2} + \frac{(d\sin(\alpha-\gamma))^2}{V_{\perp}^2} \\ &= \frac{(d\cos\alpha\cos\gamma + d\sin\alpha\sin\gamma)^2}{V_{\|}^2} + \frac{(d\sin\alpha\cos\gamma - d\cos\alpha\sin\gamma)^2}{V_{\perp}^2} \end{aligned} \tag{A-4}$$

To further simplify this equation we need to know the values of $d\cos\alpha$ and $d\sin\alpha$. In order to do this, we need to be careful about the sign of α (clockwise or counter-clockwise) for the given ray direction. We also need to be careful about the sign of Δx and Δz. It turns out that regardless of how the signs of these quantities are defined (as long as they are consistent) the final expression for t^2 is always, as expected, the same. The result is

$$t^2 = \frac{(-\Delta z\cos\gamma + \Delta x\sin\gamma)^2}{V_{\|}^2} + \frac{(\Delta x\cos\gamma + \Delta z\sin\gamma)^2}{V_{\perp}^2}. \tag{A-5}$$

This is the expression for the traveltime of a ray that travels between two points separated by a distance d in an homogeneous elliptically anisotropic medium with axis of symmetry forming an angle γ with the vertical. This equation is the heart of the inversion procedure proposed in this paper.

APPENDIX B

In this appendix, I show the expressions for the partial derivatives of the traveltime t_i (equation (5)) with respect to the model parameters m_k, where m_k is a component of the vector $\boldsymbol{m}$:

$$\begin{aligned} \boldsymbol{m} &= (m_1, ..., m_N, m_{N+1}, ..., m_{2N}, m_{2N+1}, ..., m_{3N}, m_{3N+1}, ..., m_{4N}, m_{4N+1}, ..., m_{5N}) \\ &= (S_{\perp_1}, ..., S_{\perp_N}, S_{\|_1}, ..., S_{\|_N}, \gamma_1, ..., \gamma_N, b_1, ..., b_N, a_1, ..., a_N). \end{aligned}$$

First, the derivatives with respect to the interval parameters $S_{\perp_j}$, $S_{\|_j}$ and γ_j $(1 \le j \le N)$:

$$\frac{\partial t_i}{\partial m_k} = \begin{cases} \frac{\Delta X_{ik}^2 S_k}{t_{ik}} & \text{if } 1 \le k \le N \\ \frac{\Delta Z_{ik}^2 S_k}{t_{ik}} & \text{if } N+1 \le k \le 2N \\ \frac{\Delta X_{i,k} S_k^2(-\Delta x_{i,k}\sin\gamma_k + \Delta z_{i,k}\cos\gamma_k)}{t_{i,k}} + & \\ \frac{\Delta Z_{i,k} S_{k+N}^2(\Delta z_{i,k}\sin\gamma_k + \Delta x_{i,k}\cos\gamma_k)}{t_{i,k}} & \text{if } 2N+1 \le k \le 3N \end{cases}$$

Note that for the interval parameters the derivatives with respect to the j^{th} variable depend only upon the properties of the j^{th} layer.

The derivatives of the traveltime with respect to the boundary parameters (a_j and b_j), depend on the properties of the medium above and below that boundary, as follows:

$$\frac{\partial t_i}{\partial m_k} = \begin{cases} \frac{-\sin\gamma_k \Delta X_{i,k} S_k^2 + \cos\gamma_k \Delta Z_{i,k} S_{k+N}^2}{t_{i,k}} + \\ \frac{\sin\gamma_{k-1} \Delta X_{i,k-1} S_{k-1}^2 - \cos\gamma_{k-1} \Delta Z_{i,k-1} S_{k-1+N}^2}{t_{i,k-1}} & \text{if } 3N+2 \leq k \leq 4N \\ \frac{\partial t_i}{\partial m_{k-N}} x_{i,k} & \text{if } 4N+2 \leq k \leq 5N \end{cases}$$

Since the traveltime t_i is not affected by the position of the top boundary, $\frac{\partial t_i}{\partial a_1} = \frac{\partial t_i}{\partial b_1} = 0$.

When a ray travels horizontally, only the first N components of the vector of partial derivatives are non zero. For the forward modeling this is not problem and it simply means that the horizontal component of the velocity is the only parameter that affects the traveltime of a horizontally traveling ray. Problems arise, however, when we try to invert these traveltimes because there are infinite combinations of the other parameters that satisfy the data equally well (null space). This translates into instability of the inversion procedure. For these reason, this inversion does not use rays that travel exactly along the horizontal.

After making the appropriate simplifications in the above equations for the case of isotropic media, it results a set of equations similar to the ones obtained by Lee (1990). Note two misprints in Lee's equations.