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Modeling, adjoint modeling, and migration in V(z) media, using mono-frequency Green's functions

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ABSTRACT

Modeling and migration are very simple to implement if pre-tabulated Green's functions are available. I use Green's functions, calculated at a few frequencies to estimate, the Green's functions for all frequencies. In a V(z) medium the Green's function is independent of surface location therefore only one set needs to be calculated. I calculate this set by one way extrapolation in polar coordinates. They are then used to perform prestack modeling and migration of data. Because they are calculated in polar coordinates they can be used to model and image overturned waves.

INTRODUCTION

Modeling and migration can be expressed in terms of frequency-dependent Green's functions of the wave equation. However, practical implementations do not use the Green's functions explicitly. The reason for this is the prohibitive cost of calculating and storing all the required functions.

One common way to overcome this problem is to use an asymptotic form of the Green's functions. These are usually of the form $Ae^{-i\omega\tau}$ where τ can be found by solving the eikonal equation and A can be found by solving the transport equation. When these approximate functions are substituted into the migration equations they reduce the calculation to a line or surface integral form. This is computationally very efficient but there may be problems when the asymptotic form is not a good approximation to the Green's function.

In a previous paper (Nichols, 1993) I discussed calculating mono-frequency Green's functions using one way wave equations in polar coordinates. The complete Green's function is the superposition of all the mono-frequency Green's functions. Using currently available computers, it may be too expensive to routinely calculate these

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functions for all frequencies. In this paper I examine the possibility of calculating the Green's functions at selected frequencies and estimating the other frequencies.

Once the Green's functions at all frequencies have been estimated they can be used for pre-stack and post-stack modeling and migration. If the medium has velocities that are independent of the horizontal coordinate, only one set of Green's functions needs to be calculated. Because the Green's functions are calculated in polar coordinates they are not limited to waves traveling at less that 90 degrees to the vertical. They can be used to image reflections due to overturned waves. I use a single set of Green's functions for modeling and migration in a V(z) medium with overturned waves.

Since the Green's functions are calculated using the wave equation they contain all the correct amplitude and phase information for the waves, this is difficult to do with Green's functions calculated from the eikonal equation, since the eikonal equation does not carry any phase information.

The true amplitude nature of the wave equation Green's functions reveals a problem with using the adjoint of the modeling operator as an imaging operator. In the noise free case, an inverse operator would be an ideal imaging operator that would invert both the amplitude and phase information. The adjoint operator correctly inverts the phase but is squares the amplitude. An alternative operator is one that removes the phase and attempts to approximately recover the original amplitudes. This process is sometimes referred to as migration/inversion (Lumley and Beydoun, 1991).

CALCULATING GREEN'S FUNCTIONS

Mono-frequency Green's functions

In an earlier paper I described a method for calculating mono-frequency Green's functions by using one-way equations in a polar coordinate frame. This method has the advantage that high dips can be correctly modeled using a paraxial operator. I use a 45° extrapolation operator which will accurately model waves propagating at up to 45° to the *polar* coordinate grid. These waves may be traveling at much higher angles to the the *rectangular* coordinate frame and may even be propagating at angles greater than 90° (overturned waves).

The 2-D polar coordinate wave equation,

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2} = \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2}$$

can be approximated by a paraxial equation in the frequency domain,

$$\frac{\partial P}{\partial r} = \left(\frac{-1}{r^2} + i\sqrt{\alpha}\right) P - \left[\frac{\frac{i\sqrt{\alpha}}{2r^2}\frac{\partial^2}{\partial \theta^2}}{1 - \frac{\alpha}{4r^2}\frac{\partial^2}{\partial \theta^2}}\right] P$$

where,

$$\alpha = \sqrt{\frac{\omega^2}{v^2} - \frac{1}{4r^2}}.$$

This equation can be used for outward extrapolation of a single frequency from some initial radius r_0, to give a solution on the whole (r, θ) plane. The initial solution at $r = r_0$ is the solution for a homegenous medium.I choose to express this solution in terms of amplitude and wrapped phase, $A(r, \theta, \omega)$ and $\phi_w(r, \theta, \omega)$. Starting from the known phase at $r = r_0$ I perform a phase unwrapping to obtain the unwrapped phase $\phi(r, \theta, \omega)$. The amplitude and phase are then mapped back to the rectangular coordinate frame using a simple bilinear interpolation.

Figure 1 shows the amplitude and unwrapped phase for a polar coordinate Green's function in a medium with velocity that is a linear function of depth. Figure 2 shows the same Green's function mapped back to rectangular coordinates.

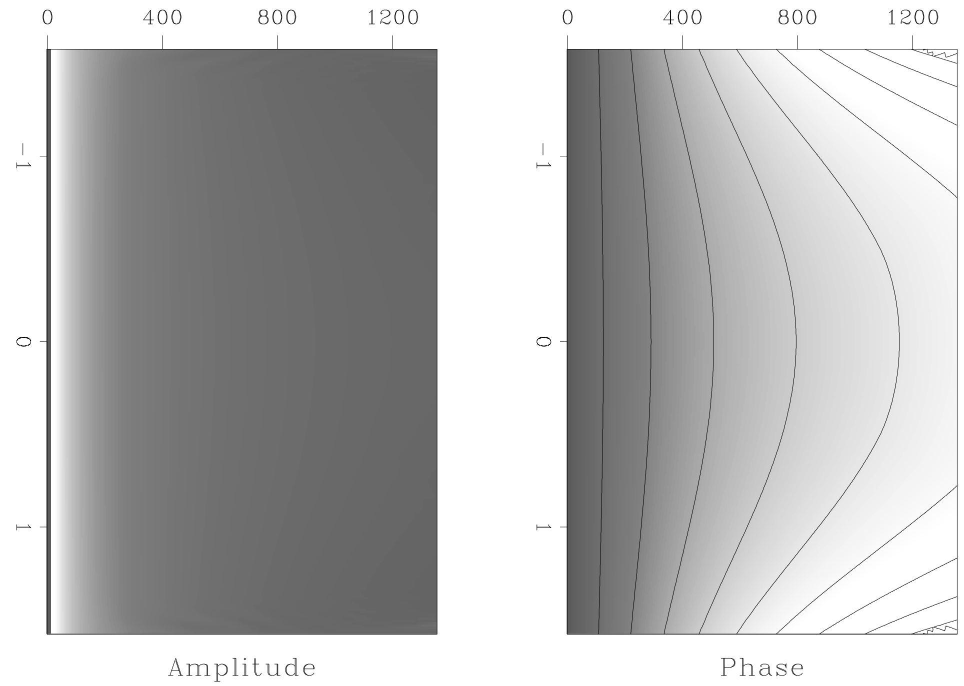

Figure 1: Mono-frequency Green's function in polar coordinates. The medium has a velocity gradient as a function of depth, v0=6000m/s dv/dz = 15 (m/s)/m. Frequency is 50Hz. Left frame is amplitude, right frame is unwrapped phase. dave1-polar-greens [CR]

All of these calculations are done in parallel for a number of frequencies. If all the frequencies are calculated we then have a complete Green's function. However, calculating all the frequencies can be very expensive so I only calculate a limited number. The missing frequencies must then be estimated from the ones calculated.

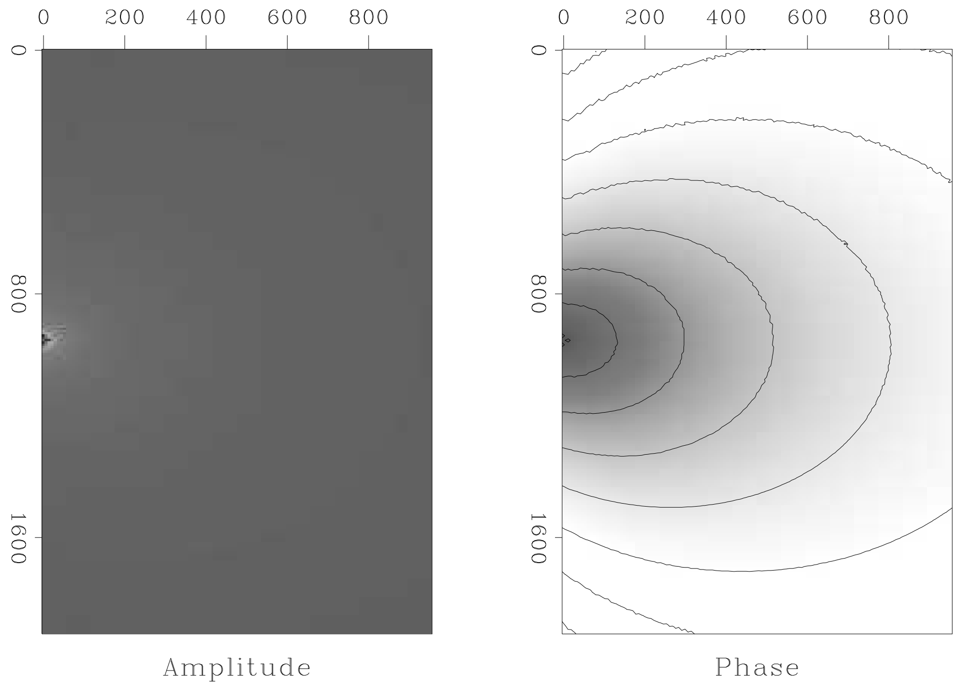

Figure 2: Mono-frequency Green's function in rectangular coordinates. The medium has a velocity gradient as a function of depth, v0=6000m/s dv/dz = 15 (m/s)/m. Frequency is 50Hz. Left frame is amplitude, right frame is unwrapped phase. $\boxed{\text{dave1-rect-greens}}$ [CR]

Interpolation of Green's functions to all frequencies

The simplest possible interpolation scheme would be to assume that the Green's function could be modeled as a single, non-dispersive, zero phase event. It would then be characterized by an amplitude, $A(x,z)$ and frequency independent traveltime $\tau(x,z)$. To estimate these two parameters we only need the Green's function calculated at one frequency,$G(x,z,\omega_0) = A(x,z,\omega_0)e^{i\phi(x,z,\omega_0)}$. The amplitude is assumed to be the same for all frequencies and the traveltime is simply given by the unwrapped phase divided by the frequency, $\tau(x,z) = \phi(x,z,\omega_0)/\omega_0$. The Green's function for all frequencies is then given by, $G(x,z,\omega) = A(x,z)e^{i\omega\tau(x,z)}$. This simple scheme has little advantage over eikonal equation based methods. It uses the same simple model for the Green's function, it does have the advantage of using a calculation in the seismic data bandwidth (rather than at infinite frequency) but it has the disadvantage that it breaks down rapidly in the presence of multiple events.

The next simplest method will also fail in the presence of multiple arrivals but it provides a more general model of the event. This model assumes that both the amplitude and phase are linear functions of frequency over some bandwidth $\omega_0 \leq \omega \leq \omega_1$. Within this band the Green's function can be modeled by an initial amplitude and phase, and an amplitude and phase gradient. Two Green's functions at frequencies ω_0 and ω_1 are required to fit this model,

$$A_0(x,z) = A(x,z,\omega_0) \tag{1}$$
$$\delta_\omega A(x,z) = (A(x,z,\omega_1) - A(x,z,\omega_0))/(\omega_1 - \omega_0) \tag{2}$$
$$\phi_0(x,z) = \phi(x,z,\omega_0) \tag{3}$$
$$\delta_\omega \phi(x,z) = (\phi(x,z,\omega_1) - \phi(x,z,\omega_0))/(\omega_1 - \omega_0) \tag{4}$$

The Greens function at any frequency within this band can then be calculated by,

$$G(x,z,\omega) = A(x,z,\omega)e^{i\phi(x,z,\omega)} \tag{5}$$
$$A(x,z,\omega) = A_0(x,z) + \delta_\omega A(x,z)(\omega - \omega_0) \tag{6}$$
$$\phi(x,z,\omega) = \phi_0(x,z) + \delta_\omega \phi(x,z)(\omega - \omega_0) \tag{7}$$

This model allows the event to have a constant phase shift (as expected for overturned waves) and for amplitude to vary as a function of frequency. If this model is used piecewise within the seismic bandwidth it can also be used to model a weakly dispersive event.

Figure 3 shows a time slice through the Greens' function for the same velocity gradient medium used earlier. Three mono-frequency Green's functions were calculated and used to estimate all the frequencies. The time slices were generated by applying an inverse FFT. The expanding wavefront has overturned waves near the top of the section. Figure 4 shows a depth slice through the Green's function. The phase changes on the limbs of the hyperbola correspond to the point at which the waves overturn. It is encouraging that all the major features expected in this Green's function have been recovered from an extrapolation of only three frequencies.

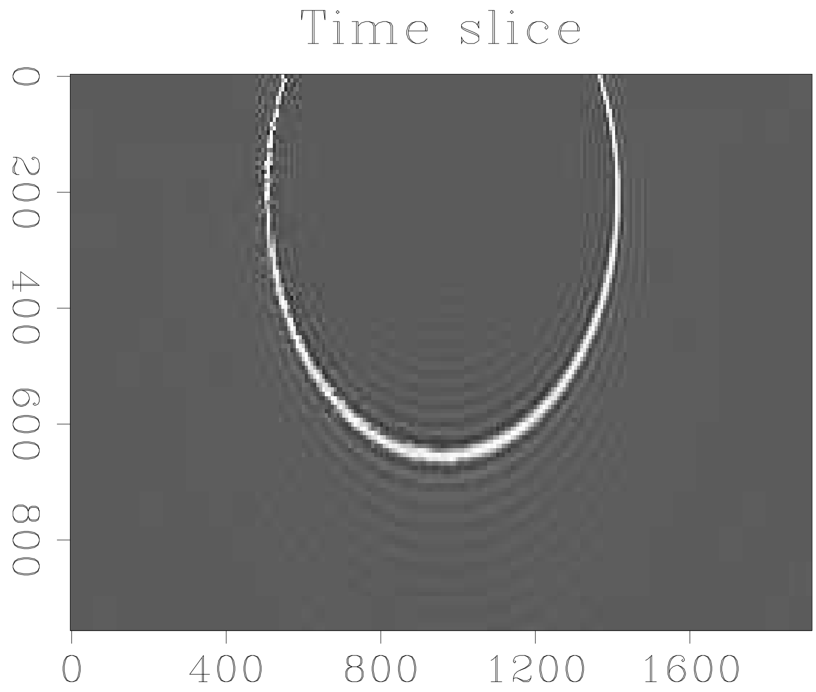

Figure 3: Time slice through the full Green's function in a V(z) medium. Note the overturned waves near the top of the panel. dave1-tsl-greens [CR]

When there are multiple arrivals at a particular location both of these simple interpolation methods will fail. In this situation an interpolation scheme based on a multiple event model must be used. This is a subject for future research, I will not discuss it further in this paper.

MODELING IN A V(Z) MEDIUM

In a general medium, the scattered wavefield received at position g, on the surface from a shot at position s, is given by,

$$P(g, s, \omega) = \int\int G(x, z; s, \omega) G(x, z; g, \omega) S(x, z) \; dx \; dz$$

where $S(x, z)$ is the scattering function in the subsurface. This formula suggests that we need to calculate a Green's function for every surface position. However if the velocity field is only a function of depth and not of x, we only need one Green's function. The integral can then be expressed as a convolution of the Green's function over x and an integral over depth.

$$P(g, s, \omega) = \int\int G(x - s, z; \omega) G(x - g, z; \omega) S(x, z) \; dx \; dz$$

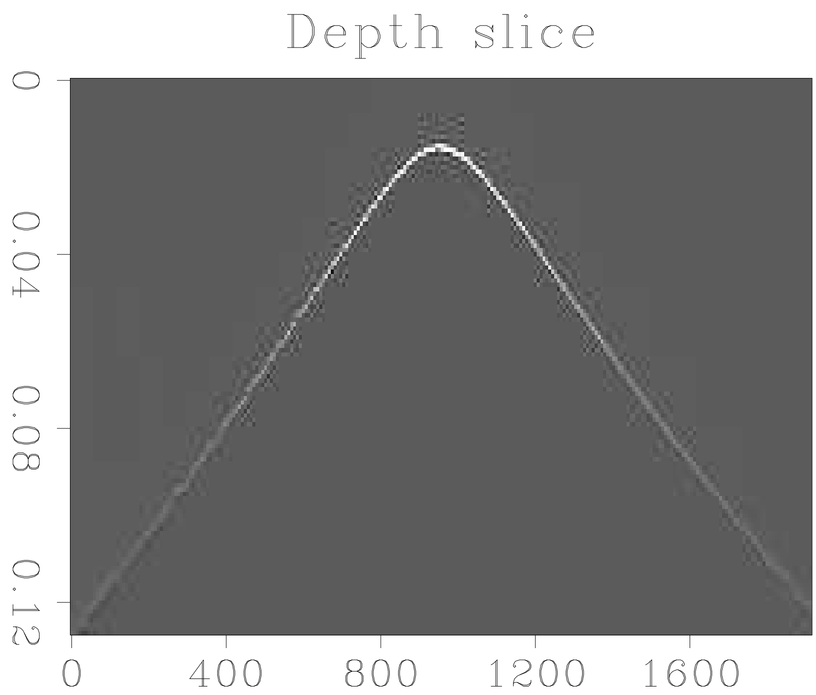

Figure 4: Depth slice through the full Green's function in a V(z) medium. Note the phase change on the limbs of the hyperbola. dave1-zsl-greens [CR]

For a constant offset panel in CMP coordinates (y, h) this can be further simplified to a convolution over a constant offset Green's function, and an integral over depth.

$$P(y, \omega; h) = \int \int G(x - y, z; \omega, h) S(x, z) \; dx \; dz$$

Where $G(x-y, z; \omega, h) = G(x-y+h/2, z; \omega) G(x-y-h/2, z; \omega)$. The constant offset Green's function is merely the one-way Green's function multiplied by a shifted copy of itself.

Figure 5 is the result of constant offset modeling for three different offsets. Again only three frequencies were calculated to generate the Green's function. The first panel is the scattering function, two spikes in the center of the panel. The second panel is the zero offset modeling result, the third panel is for an offset of 300m and the fourth panel is at an offset of 750m. In the zero offset panel the upward curvature of the limbs of the hyperbola and the phase change due to overturned waves can clearly be seen. In the far offset panel an interesting effect is visible, the top of the table-top is concave. The traveltime to a point under the midpoint is greater than the traveltime to a point to one side.

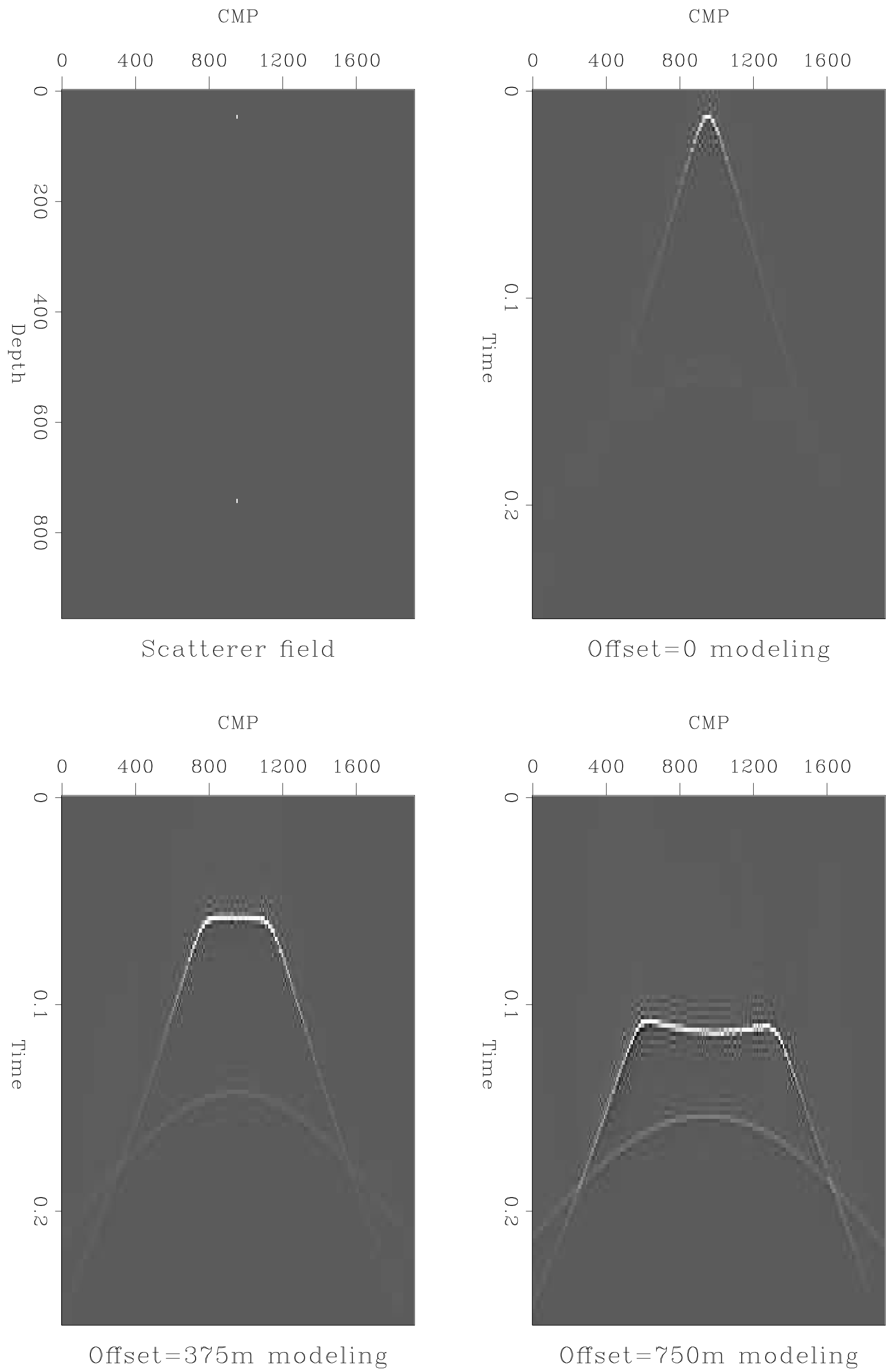

Figure 5: Constant offset modeling in a V(z) medium dave1-const-mod [CR]

ADJOINT MODELING AND MIGRATION

Migration is often considered to be the adjoint of modeling, this would suggest the following adjoint operation as a reasonable constant offset imaging operation.

$$\hat{S}(x,z;h) = \int\int G^*(x-y,z,\omega;h)P(y,z,\omega;h)\ dy\ d\omega$$

When I apply this process to the result of constant offset modeling the result is very disappointing. Figure 6 shows the result of adjoint modeling on the three constant offset panels shown in figure 5. The deeper event has been imaged with such a small amplitude that it isn't even visible. This is because the Green's function I am using are not merely kinematically correct they also have the correct amplitudes. The adjoint operation correctly inverts the phase but is squares the amplitude. This would not be a problem if the integration had been applied over infinite space and time. The truncation of the modeling and adjoint modeling operators results in a loss of energy from the image. The loss of energy is greatest at the bottom of the section where only a small piece of the hyperbola will be imaged.

One way to overcome this problems is to apply a time dependent gain before the migration. This will amplify events that are later in time so that the amplitudes of the images of deeper events will be increased. However it will also increase the amplitude of the later parts of hyperbolas belonging to shallow events, this is not desirable. What should really be applied is a depth dependent gain after migration. Many authors, e.g. (Lumley and Beydoun, 1991), attempt to account for the effects of truncation by dividing by a "diagonal Hessian" operator, this is an approximation to the full inversion of the modeling operator. This process is often called "migration inversion." In a V(z) medium this becomes a depth and offset dependent scaling after migration.

Migration/inversion & preconditioning

Wave equation modeling can be regarded as a linear operator, the domain of the operator is a the scattering field, and the range is a seismic dataset. When the scattering field and seismic data are discretized they can be represented as finite dimensional vectors. The modeling operator, A, maps from a scatterer vector, s to a data vector d.

$$As = d$$

When imaging we wish to obtain an estimate of s from the measured data d. One possible operation is to apply the adjoint operator,

$$\hat{s} = A'd$$

If the operator is nearly unitary this will be a good estimate of the model. When the operator is not unitary, as when our modeling operator is truncated, this will not be

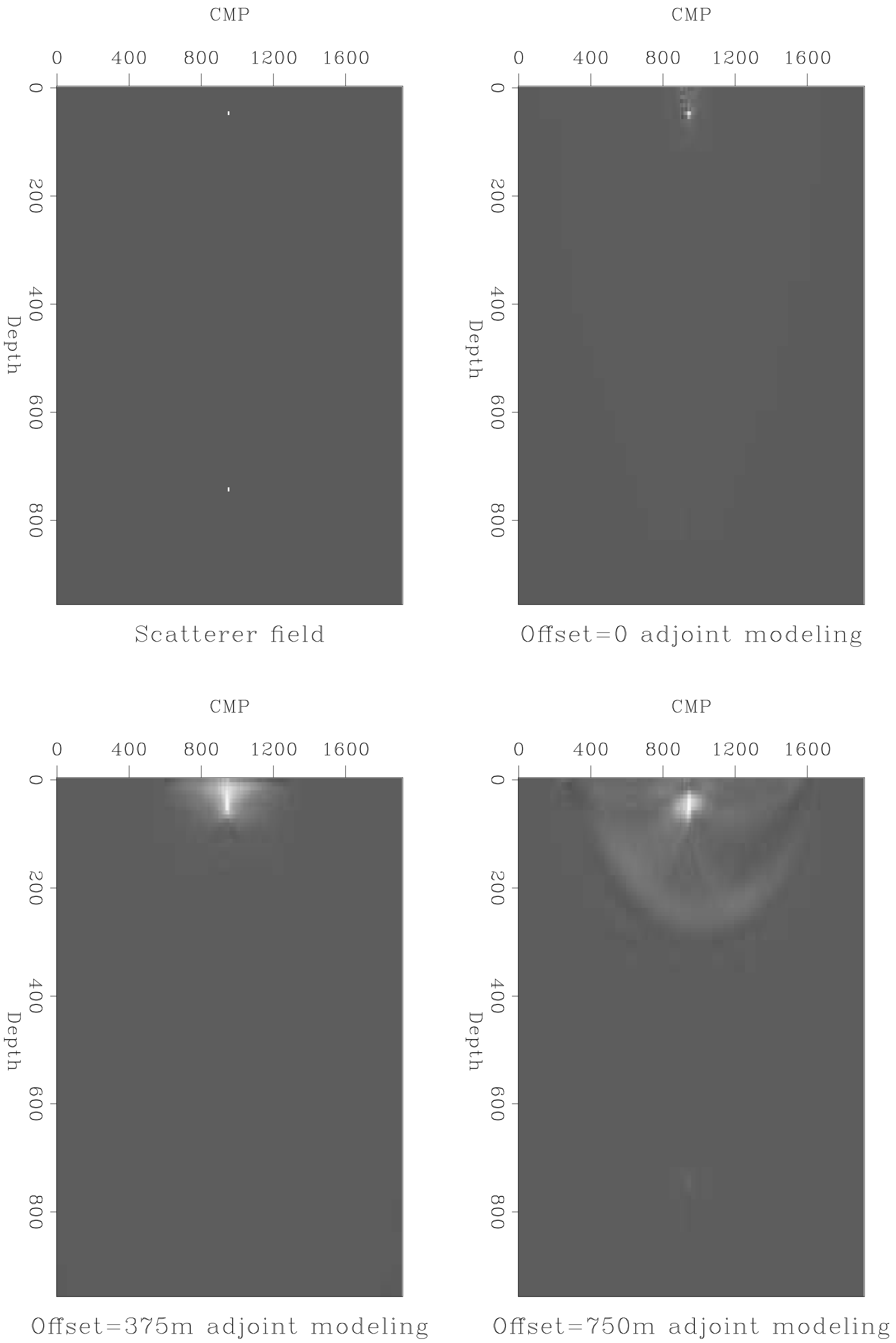

Figure 6: Adjoint modeling applied to the constant offset panels of the previous figure. The deeper events have not been imaged at the correct amplitude. dave1-adj-mig [CR]

a good estimate. The estimate can be improved by solving the least squares inverse problem

$$\min_{\hat{s}} ||A\hat{s} - d||^2$$

If a conjugate gradient algorithm is used the application of the adjoint can be regarded as the first iteration of that algorithm.

Another method for obtaining an estimate is to note that the solution of the least squares problem is given by

$$\hat{s} = (A'A)^{-1}A'd$$

Calculation of $(A'A)^{-1}$ is very expensive but we might obtain an acceptable result by approximating this inverse. If we assume $A'A$ is diagonally dominant we can approximate the inverse by $(Diag(A'A))^{-1}$, where $Diag$ extracts the diagonal of $A'A$ and sets all other elements to zero. So the estimate of the solution is given by,

$$\hat{s} = (Diag(A'A))^{-1}A'd. \tag{8}$$

This procedure can also be interpreted as the first step of an iterative algorithm. In this case the problem being solved is a preconditioned version of the original problem. $M = (Diag(A'A))^{-1/2}$ is a diagonal preconditioner that equalizes the norm of the columns of the operator A. The preconditioned least squares problem is,

$$\min_{\hat{t}} ||AM\hat{t} - d||^2 \tag{9}$$

$$\hat{s} = Mt. \tag{10}$$

One step of the preconditioned iterative algorithm followed by application of the preconditioner to solve for $\hat{s}$ is exactly the same as equation (8).

In many applications it is relatively simple to calculate $(Diag(A'A))$. For the modeling algorithm used here it is,

$$(A'A)(x, z; h) = \int\int G^*(y, z, \omega; h)G(x - y, z, \omega; h)\ dy\ d\omega.$$

If the modeling is not limited in spatial aperture, the weight is just a function of depth and offset panel. It is a depth and offset dependent scale applied after adjoint modeling rather than the time dependent scale usually applied before migration in seismic processing.

Figure 7 shows the constant offset migrations calculated using this weight function. The amplitudes of the two scatterers have been correctly recovered. The diagonal weight only corrects the relative amplitudes of the result, it does not correct the spectrum of the result, the full inverse operator $(A'A)^{-1}A'$ would correct the spectrum as well.

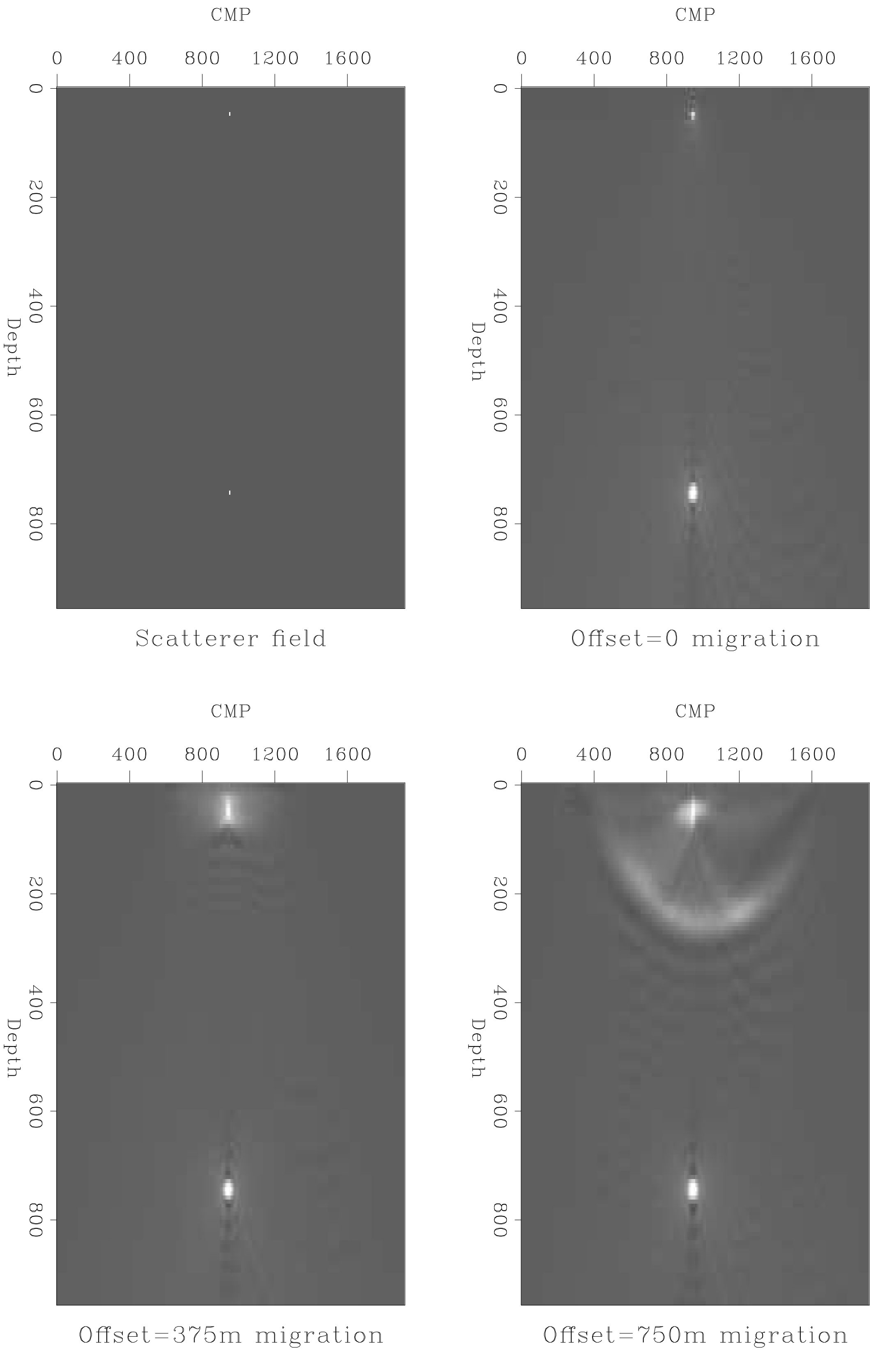

Figure 7: Constant offset migration/inversion in a V(z) medium. dave1-ata-mig [CR]

CONCLUSIONS

Green's functions generated at a few frequencies can be interpolated to all frequencies of interest and used for modeling and migration. I have shown that Green's function calculated in a polar coordinate frame can be used to model and migrate overturned waves in a V(z) medium.

If the Green's functions contain the true amplitude information, i.e. they are not not just kinematic operators, the migration algorithm must be modified to account for the amplitude effects. The simplest scheme is to use a diagonal approximation to the true inverse operator. This scheme produced satisfactory results for the geometry used in my examples.

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