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Short Note

Interpolation in Stolt migration

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INTRODUCTION

Downward continuation and imaging in the isotropic case (Gazdag, 1978) can be written as

$$M(z, k_x) = \int d\omega e^{ik_z z} P(\omega, k_x), \tag{1}$$

where

$$k_z = -\text{sign}(\omega)\sqrt{\frac{\omega^2}{v^2} - k_x^2}. \tag{2}$$

In equation (1), $P(\omega, k_x)$ represents the Fourier transform of the seismic field $p(t, x)$ recorded at the surface, following Claerbout's (1985) sign convention

$$P(\omega, k_x) = \int dt\, e^{i\omega t} \int dx e^{-ik_x x} p(t, x),$$

and v represents half the velocity, as used in the exploding reflectors model.

We can rewrite equation (1) as time migration replacing the depth steps by equivalent time steps τ:

$$M(\tau, k_x) = \int d\omega e^{ik_\tau \tau} P(\omega, k_x). \tag{3}$$

where we define

$$\begin{cases} \tau &= \frac{z}{v} \\ k_\tau &= v k_z = -\text{sign}(\omega)\sqrt{\omega^2 - v^2 k_x^2}. \end{cases}$$

For a constant velocity medium, Stolt (1978) transforms the integral in ω using a Fourier transform, which can be computed rapidly via a Fast Fourier Transform (FFT) algorithm

$$M(\tau, k_x) = \int dk_\tau e^{ik_\tau \tau} J(k_\tau, k_x) P(\omega(k_\tau, k_x), k_x), \tag{4}$$

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where $J(k_\tau, k_x)$ represents the Jacobian of the transformation from ω to k_τ

$$J(k_\tau, k_x) = \frac{d\omega}{dk_\tau} = \frac{k_\tau}{\sqrt{k_\tau^2 + v^2 k_x^2}}$$

and $P(\omega(k_\tau, k_x), k_x)$ represents the initial data as function of the new variable k_τ.

Interpolation in Stolt migration

Depending on the method used to transform the field $P(\omega, k_x)$ into the interpolated field $P(\omega(k_\tau, k_x), k_x)$, different artifacts can be introduced in the migrated image (Harlan, 1982; Ronen, 1982). One way to eliminate the interpolation and avoid artifacts in the migrated image is to perform a slow Fourier transform in time with irregular values in ω, but regular in k_τ:

$$M(\tau, k_x) = \int dk_\tau e^{ik_\tau \tau} J(k_\tau, k_x) \int dt e^{i\mathrm{sign}(k_\tau)\sqrt{k_\tau^2 + v^2 k_x^2}\, t} P(t, k_x), \tag{5}$$

where ω was replaced by

$$\omega = \mathrm{sign}(k_\tau)\sqrt{k_\tau^2 + v^2 k_x^2}. \tag{6}$$

Actually such an algorithm can be very appealing when implemented on a parallel computer (Blondel, 1993) due to the high degree of parallelism, and can run faster than the sequence of FFT followed by interpolation. The mapping function from (ω, k_x) space to (k_τ, k_x) can be any anisotropic dispersion relation. Dellinger et al. (1990) and Ecker and Muir (1993) show examples of anisotropic migration operators by modifying the dispersion relation (2).

By comparing the standard Stolt migration with the slow Fourier transform Stolt migration, we can deduce an interpolation scheme that, when used in connection with the standard algorithm, will be equivalent to the slower but more correct one. In matrix format, we can write the slow Fourier FFT for a single wavenumber k_x as:

$$M(k_\tau; k_x) = \begin{bmatrix} P(\omega_1; k_x) \\ P(\omega_2; k_x) \\ \vdots \\ P(\omega_n; k_x) \end{bmatrix} = \begin{bmatrix} 1 & e^{i\omega_1 t_2} & \cdots & e^{i\omega_1 t_n} \\ 1 & e^{i\omega_2 t_2} & \cdots & e^{i\omega_2 t_n} \\ \vdots & & & \\ 1 & e^{i\omega_n t_2} & \cdots & e^{i\omega_n t_n} \end{bmatrix} \begin{bmatrix} P(t_1; k_x) \\ P(t_2; k_x) \\ \vdots \\ P(t_n; k_x) \end{bmatrix}. \tag{7}$$

The variable ω is not evenly sampled as required by the FFT. Each ω_j corresponds to an evenly sampled value of k_τ according to equation (6)

$$\omega_j = \mathrm{sign}(k_{\tau j})\sqrt{k_{\tau j}^2 + v^2 k_x^2}.$$

The ones in the slow FT matrix represent zero values of time in the first column, i.e. $t_1 = 0$.

We can multiply the data by a unit matrix composed from a forward FFT and an inverse FFT before performing the slow FT

$$[P(\omega_i;k_x)] = [SFT]\left[FFT^{-1}\right][FFT]\,[P(t_i;k_x)]\,. \tag{8}$$

We observe that by combining the operations

$$[SFT]\left[FFT^{-1}\right] \tag{9}$$

and calling it interpolation, we obtain the classic Stolt migration algorithm. What is left now is to identify exactly what our interpolation is doing.

The new interpolator

The slow Fourier transform of an evenly sampled signal in time can be written as

$$P_r = \sum_{k=0}^{N-1} p_k e^{i2\pi\frac{rk}{N}}, \tag{10}$$

where the index k is an integer corresponding to the evenly sampled time values, while the variable r is a real number:

$$\begin{cases} \omega_r &= \frac{2\pi r}{Ndt} \\ t_k &= kdt. \end{cases}$$

In equation (9), we take the slow Fourier transform of the inverse fast Fourier transform. Introducing the inverse FFT inside equation (10) we have

$$\begin{aligned} P_r &= \sum_{k=0}^{N-1}\left[\frac{1}{N}\sum_{m=0}^{N-1} P_m e^{-i2\pi\frac{km}{N}}\right] e^{i2\pi\frac{rk}{N}} \\ &= \frac{1}{N}\sum_{m=0}^{N-1} P_m \sum_{k=0}^{N-1} e^{2\pi i\frac{k}{N}(r-m)} \\ &= \frac{1}{N}\sum_{m=0}^{N-1} P_m \frac{e^{2\pi i(r-m)}-1}{e^{2\pi i\frac{(r-m)}{N}}-1}. \end{aligned} \tag{11}$$

The expression inside the sum provides a formula for a new interpolation method

$$P_r = \frac{1}{N}\sum_{m=0}^{N-1} P_m e^{\pi i(r-m)\left[1-\frac{1}{N}\right]} \frac{\sin\pi(r-m)}{\sin\pi\frac{(r-m)}{N}}\ . \tag{12}$$

Observe that for $N\to\infty$ the interpolator becomes identical to the one presented by Rosenbaum and Boudreaux (1981):

$$P_{n+\delta n} = \sum_{m=0}^{\infty} P_m e^{\pi i(n+\delta n-m)} \text{sinc}\ \pi(n+\delta n-m), \tag{13}$$

where we replaced r by $n + \delta n$. A truncated version of the Rosenbaum interpolation formula is used by Harlan (1982), showing better artifact reduction than any other classic interpolation method. The interpolation formula in equation (12) is shown to produce even better results then those found by Harlan (Lin et al., 1993).

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