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Short Note

Comparison of different interpolation methods for Stolt migration

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INTRODUCTION

In Stolt migration, it's very important to select a good algorithm for interpolation in the frequency domain. Incorrectly migrated events caused by a bad interpolator can badly distort the correctly migrated events. Harlan (1982) showed some of the artifacts produced by different interpolators.

In this paper we compare the following five different interpolation methods: nearest-neighbor, linear, linear plus cork-screw, Harlan's 10-point scheme, and Muir's scheme.

DESCRIPTION OF THE FIVE INTERPOLATION METHODS

The formula for Stolt migration is

$$model(x, z) = 2DFT^{-1}\{data[k_x, \omega(k_x, k_z)]\frac{d\omega}{dk_z}\} \tag{1}$$

We first Fourier transform the data set in the (t, x) domain into the (ω, k_x) domain, then map ω into k_z by the dispersion relation as follows:

$$k_z = \sqrt{\frac{\omega^2}{v^2} - {k_x}^2} \tag{2}$$

In this mapping step, we need to use an interpolation operator because of the discrete Fourier transform. Finally, we inverse Fourier transform the data in the (k_z, k_x) domain back to the (z, x) domain.

To compare the five interpolation methods, we implement them in a Stolt migration of three impulses at different depths in anelliptic anisotropy media (Dellinger et

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al., 1993), which causes triplication at the corners of each impulse response. We use the same synthetic data and parameters for each interpolation method in order to compare the results of different interpolators.

Nearest-neighbor interpolation

The nearest-neighbor method implements the interpolation by choosing the value of the nearest sample in the k_z domain as the interpolated value at a certain k_z. Figure 1 shows the impulse response of three spikes in an anelliptic anisotropic media using Stolt migration and nearest-neighbor interpolation in the k_z domain. Many artifacts appear at all depths. In the lower part of the section, the artifacts are even stronger than the real events. These artifacts are caused by wraparound of transformation between the time-space domain and the frequency-wavenumber domain.

Linear interpolation

This method is a two-point interpolation. We input the values of the nearest two points in the k_z domain, then use the following equation to get the value of a point between them:

$$C_{n+\delta n} = (1 - \delta n)C_n + (\delta n)C_{n+1} \tag{3}$$

where $C_{n+\delta n}$ is the value of the point to be interpolated, C_n is the value of the point on its left side, and C_{n+1} is the value of the point on its right side. δn is the value of the distance between the interpolated point and the sample point on its left side.

The linear interpolation in the frequency domain is equivalent to a convolution with a triangle function in the frequency domain (Harlan, 1982). In time-space domain, its counterpart is multiplication by a $sinc^2 \frac{t}{T}$ function. The multiplication by the $sinc^2 \frac{t}{T}$ causes the original values between $T/2$ and T to be weaker than the wraparound between $-T/2$ and 0. So in Figure 2, for which linear interpolation was used, the lower part of the section shows that the artifacts almost replace the original correct information.

Linear plus cork-screw

An improvement to the linear interpolation technique can be made by multiplying the linear interpolator by $\exp(-i\pi\delta n)$, which, in the time-space domain, corresponds to moving the center of the $sinc^2$ function to $T/2$, so that the original values between $T/2$ and T are stronger than the wraparound between $-T/2$ and 0. The linear plus cork-screw operator is

$$C_{n+\delta n} = \exp(-i\pi\delta n)[(1 - \delta n)C_n + (\delta n)C_{n+1}] \tag{4}$$

In Figure 3, we show the implement of this operator. This result is better than Figure 1 and Figure 2, i.e. the artifacts are still obvious for the top and bottom impulse responses, because the amplitude of the $sinc^2$ function varies with time and space. Only the original value at the center of the section is not affected by the artifact because for $\frac{n*T+T}{2}$, the wraparound is multiplied by a zero value of the $sinc^2$ function.

Harlan's 10-point interpolation

Harlan (1982) designed a 10-point *sinc* interpolator as follows:

$$C_{n+\delta n} = \frac{1}{6\pi} \exp\left(-i\pi\delta n\right) \sin(\pi\delta n) \sum_{m=-4}^{5} C_{n+m}(6 - |\delta n - m|)/(\delta n - m) \qquad (5)$$

In the time-space domain this interpolation corresponds to multiplication by a rounded rectangle function centered at $T/2$. This operator almost avoids the wraparound effect. Figure 4 shows the impulse response of Harlan's 10-point interpolator. We can hardly find any artifacts in this display.

Muir's interpolation

To do more precise interpolation in the frequency-domain, Francis Muir derived the following equation:

$$C_{n+\delta n} = \frac{1}{N} \sum_{m=0}^{N-1} C_m \frac{(1 - \exp^{2\pi i(n+\delta n-m)})}{(1 - \exp^{2\pi i(n+\delta n-m)/N})} \qquad (6)$$

This formula is equivalent to the slow Fourier Transform (SFT) but is implemented in the frequency domain. The method provides a fast model of SFT with exact interpolation.

Muir's interpolator shows a perfect impulse response without any artifacts in Figure 5.

CONCLUSION

In order to choose a good interpolator for Stolt migration, we compared the five different interpolation methods. The following table shows the relative differences in accuracy and speed of the five interpolators:

Interpolator	Speed	Accuracy
Nearest-neighbor	++++	+
Linear	+++	++
Linear plus cork-screw	+++	+++
Harlan's 10-point	++	++++
Muir's	+	+++++

Nearest-neighbor and linear interpolators produce too many artifacts. The linear plus cork-screw method reduces some artifacts. Harlan's interpolator provides an accurate interpolation with an acceptable speed. Muir's method is the most accurate but is also the slowest.

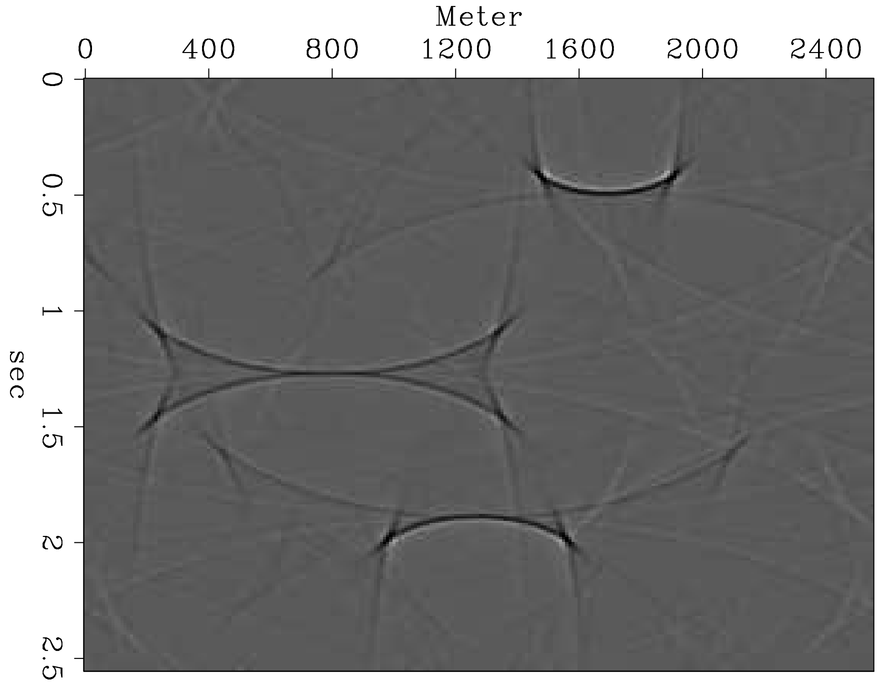

Figure 1: Impulse response by nearest neighbor interpolation $\boxed{\text{intime5-nearnb}}$ [ER]

REFERENCES

Dellinger, J., Muir, F., and Karrenbach, M., 1993, Anelliptic approximations for TI media: Journal of Seismic Exploration, **2**, 23–40.

Harlan, W. S., 1982, Avoiding interpolation artifacts in Stolt migration: SEP–**30**, 103–110.

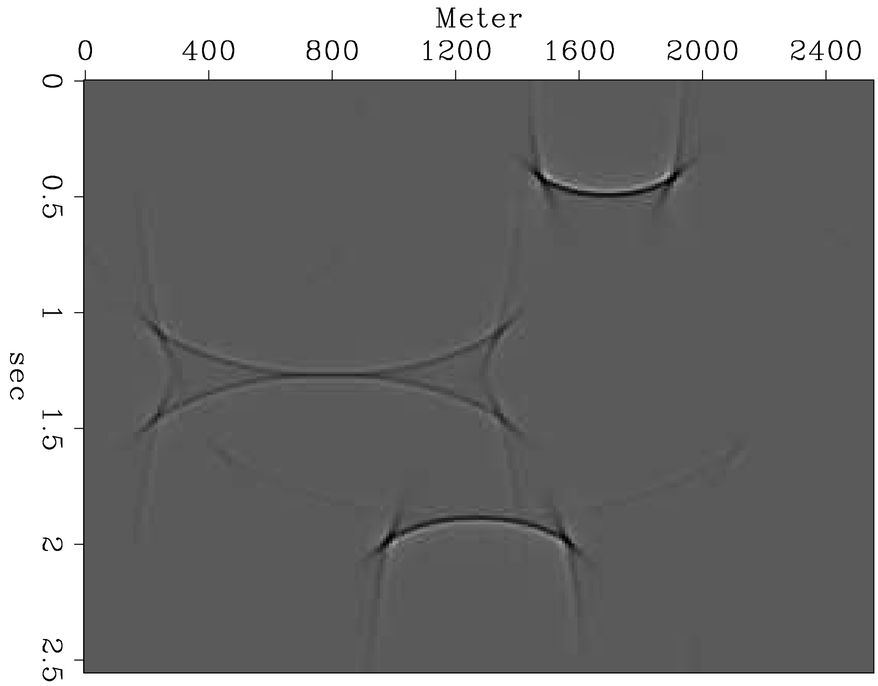

Figure 2: Impulse response by linear interpolation intime5-lin [ER]

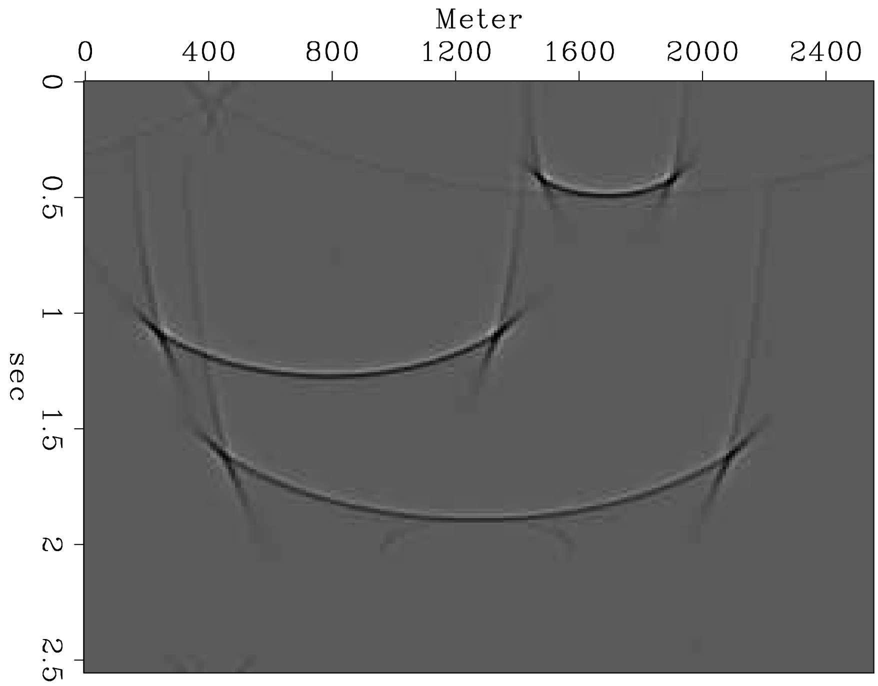

Figure 3: Impulse response by linear plus cork-screw interpolation intime5-linfaze [ER]

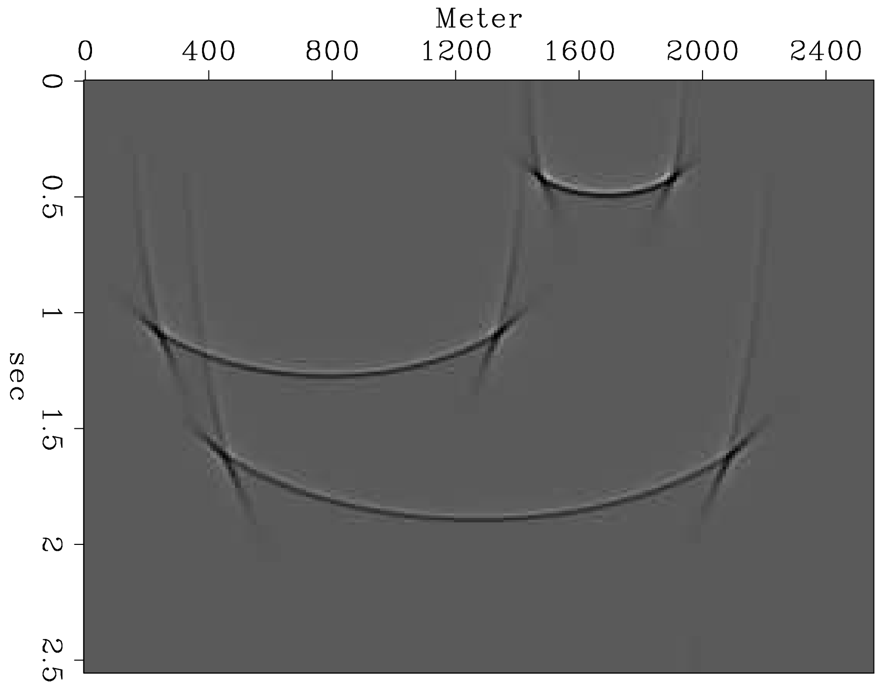

Figure 4: Impulse response by Harlan's interpolation intime5-harlanz [ER]

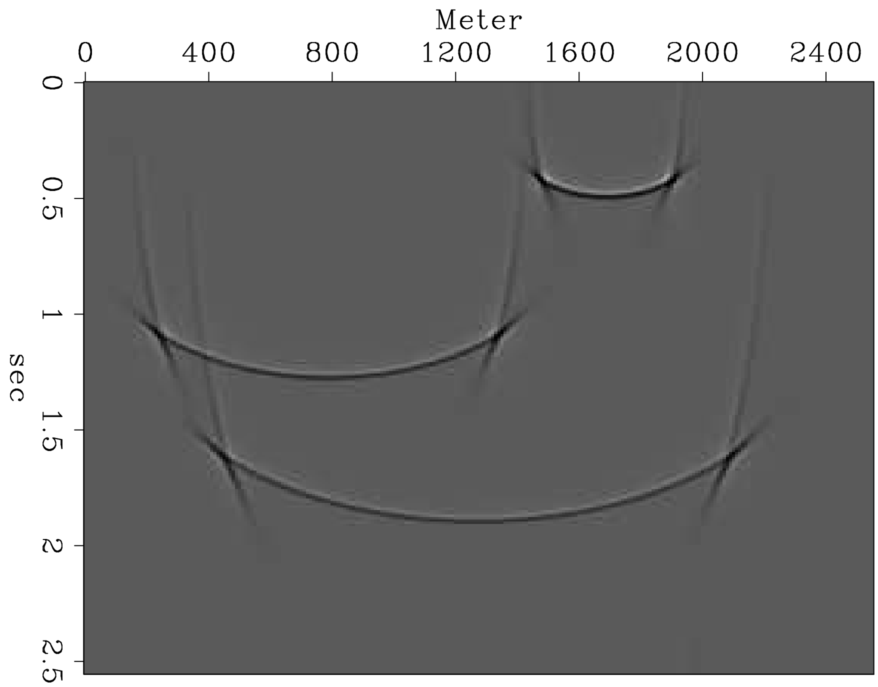

Figure 5: Impulse response by Muir's interpolation intime5-muirz [ER]