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Modeling nonlinear source-surface interactions

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ABSTRACT

The interaction of a seismic source device with the free surface of the earth changes the radiation characteristic to be different from that of the equivalent body point source. Usual seismic sources are built to give enough energy to penetrate a few kilometers in the earth. It is not guaranteed that when the source is activated the surface material behavior stays within a region that can be appropriately described using infinitesimal stress strain relationships. Finite differences can easily model radiation characteristics and wave propagation when the equations of motion are no longer linear. I compare two nonlinear elastic wave equation to model such source behavior.

INTRODUCTION

When generating a time dependent forcing function, nonlinear behavior can occur in the electronic amplification part of the system. These nonlinearities can be avoided by having feedback mechanisms. Another nonlinear mechanism is the transfer of the desired signal into the ground. Most of the signal distortion occurs at this instance with high momentum devices, be it by impacting or vibrating. The subsurface volume in which such mechanisms dominate is rather small, yet such nonlinear behavior shows up in an effective alteration of the signal spectrum and depends on radiation direction. It is this interaction of the device with the earth's surface that cannot be described by using infinitesimal displacements or stresses. When describing nonlinearities we need to separate two effects: one is the finiteness of the variables, the other is the possibility of entering a nonlinear region in the stress-strain relationship. In general the function describing the stress-strain dependency can be arbitrary, as long as some underlying conservation principles are satisfied. It is always possible to expand this function around the equilibrium point at which we measure. A conventional elastic method would only use the linear term of such a Taylor series expansion. This works well if the magnitude of the variables are small. If the magnitude exceeds a certain limit we have to use higher order terms in the approximation in order to describe the medium properly. It is seldom the case that these higher order stiffness constants are measured.

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For this reason, this paper mainly deals with effects introduced by the finiteness of the variables, but not so much with the nonlinear stress-strain relationship. The finiteness of displacements introduces no new material parameters which must be known or estimated. Allowing the displacements to be non-infinitesimal merely means that the volume is deformed by finite displacements. This is equivalent to allowing higher order terms in the Taylor expansion of the displacement field.

This paper does not deal with receiver properties, although the same framework can be applied to modeling and estimating receiver functions. I assume that receiver properties can be effectively estimated as their transfer function. Most of the external effect is assumed to be caused by ground coupling. Stresses and displacements are generally assumed to be small by many orders of magnitudes in the case of receivers.

ABANDONING LINEARITY

Geometric Nonlinearity

In an elastic region, if the deformation is less than 10^{-3}, nonlinearity is assumed to be due to geometry and not due to a nonlinear stress-strain relation. However the boundary between geometric and material nonlinearity cannot be drawn very clearly. Following Biot (1965), it is possible to compute the material deformation to second order as:

$$\epsilon_{kl} = e_{kl} + 1/2(e_{k\mu}\omega_{\mu l} + e_{l\mu}\omega_{\mu k}) + 1/2(\omega_{k\mu}\omega_{l\mu}) \tag{1}$$

$$e_{kl} = 1/2(\frac{\partial u_k}{\partial l} + \frac{\partial u_l}{\partial k}) \qquad \text{and} \qquad \omega_{kl} = 1/2(\frac{\partial u_k}{\partial l} - \frac{\partial u_l}{\partial k}) \tag{2}$$

Neglecting the rotational tensor components ω in (1) results in the usual definition of the elastic symmetric strain tensor e. (All doubly appearing indices are summed over and the ranges are always $i = 1, 2, 3.$) Following Fung (1965) we have a similar formulation for the Eulerian strain tensor:

$$\epsilon^E_{kl} = 1/2(\frac{\partial u_k}{\partial l} + \frac{\partial u_l}{\partial k}) - \frac{\partial u_m}{\partial k}\frac{\partial u_m}{\partial l} \tag{3}$$

Both equation (1) and (3) describe the deformation of a medium to a higher order than the usual elastic strain tensor e. The deformation is a pure geometrical property and not a material property. Consequently the linear Hooke's law:

$$\sigma_{ij} \;=\; c_{ijkl}\;\; \epsilon_{kl} \tag{4}$$

remains unmodified. The relation between stress and strain components is still linear. It is the computation of ϵ_{kl} that changes in each case. When considering the displacement gradients, we can see that higher order gradient components are related

to stress components nonlinearly. However the parameters which link them, the stiffness coefficients c_{ijkl}, are still linear elastic parameters. All previous equations lead to an elastic wave equation of the form

$$\nabla\ C\ \nabla^t\ u = \rho \frac{\partial^2 u}{\partial t^2}, \tag{5}$$

or in the geometrically nonlinear case to a slight modification of the previous expression:

$$\nabla\ C\ \nabla_g^t\ u = \rho \frac{\partial^2 u}{\partial t^2} \tag{6}$$

where ∇_g is the geometrically nonlinear derivative operator.

Material nonlinearity

The next higher level of complexity involves the use of nonlinear relations between stress and strain components. Even when computing displacement gradients to first order, only the higher order material relationship introduces higher order effects in the differential equation. Material nonlinearity might easily be coupled with geometric nonlinearity and drawing a clear boundary between them might be hard in many cases. This kind of nonlinearity introduces new parameters into the differential equation, namely new material constants (higher order elastic parameters). In many practical cases we might not have knowledge of these parameters or they might be hard to estimate.

Negligence of the second and higher order cross terms ensures that a wave equation is linear. If these terms are neglected in the Taylor expansion of $d\mathbf{u}$, we are still confined to examine only a small neighborhood around a reference point. Assuming that products of the displacement gradients are small, assures that the principle of superposition is valid. The validity of this principle can be proved by applying two deformations consecutively. The order of application of the deformations has no effect on the final observed deformation. The principle of superposition is a fundamental property of the linear theory of elasticity.

We have to use equations (1) and (3) as soon as we consider finite displacements of the medium. It is practical to start with this nonlinear description since it introduces only small modifications to existing modeling programs, while admitting some degree of nonlinearity. Algorithms which are based on linear assumptions fail in this case and other methods have to be used to solve the problem. The wave equation therefore will be nonlinear in the spatial domain, in the time domain the principle of superposition is still valid. In order to investigate the impact of nonlinearity on radiation patterns and signal wave forms, numerical examples can be calculated in which nonlinear wave propagation is modeled using finite differences in time and space. Such a model should serve as a lower limit as to what we can expect from nonlinearities.

MODELING WITH FINITE DIFFERENCES

Many geophysical numerical algorithms assume linearity. Thus they will work only on equations that have a linear behavior. For modeling nonlinear phenomena, the choice of algorithms becomes rather restrictive. In most cases an analytical solution is not possible or would involve some sort of linearization. That in turn would defeat the original purpose of investigating nonlinear effects. For modeling nonlinearities finite difference algorithms have the advantage that no linearity assumptions have to be made in order to model nonlinear wave propagation. One problem however remains with using finite difference methods: It is now much harder to calculate stability criteria properly. Many rigorous stability criteria are based on linearity. The usual methods use linear transforms to come up with an easy to calculate estimate of stability and dispersion of the algorithm. In a recent paper Kosik (1993) makes use of a Crank-Nicholson FD scheme which is implicit and thus guaranteed to exhibit stability for a certain range of parameters.

Applicability

For investigating surface source behavior, the previously described notion of finite stresses and strains in a medium can be readily applied to surface source of different types. Multicomponent impulsive or vibratory sources could generate a region in the subsurface in which such nonlinear behavior is appropriate. However such a region might be relatively small compared to the source wavelength. For data processing a far field description with an effective source is more efficient. It allows a different parameterization of source behavior depending on the experiment type and the data itself. However, for modeling of source behavior in order to test source equalization schemes, such as described by Karrenbach (1992), it is valuable to have numerical algorithms that allow accurate modeling of nonlinear source behavior.

CHAOTIC SOURCES

The previous paragraph described nonlinear source behavior of an elastic medium that obeys the usual elastic anisotropic wave equation. However a recent investigation by Walker (1993) shows that the time dependence of source-surface interactions of vibrators can be effectively described using a nonlinear "chaotic" wave equation. Walker uses the well-known Duffing equation to parameterize vibratory sources and nonlinear propagation in the subsurface. The one dimensional Duffing equation is given as:

$$\frac{\partial^2 u}{\partial t^2} + \delta \frac{\partial u}{\partial t} + \alpha u + \gamma u^2 + \beta u^3 = F \cos(\omega t) \tag{7}$$

where α, β, γ and δ are adjustable parameters, which govern the nonlinear behavior. u is a scalar quantity. F is the strength of the periodic forcing function with ω being the temporal frequency.

The time dependent solution $u(t)$ of the Duffing equation exhibits typical behavior for a wide range of parameters. When the source-surface system is at a high energy level, the amplitudes are large. Harmonics are then generated that alter the high frequency part of spectrum of the original source wavelet. When the amplitudes are low, the nonlinear behavior creates subharmonics, that introduce low frequency peaks. Such a period doubling phenomenon is typical of chaotic equations. The typical chaotic behavior, however, happens in bandlimited intervals of the forcing function's spectrum. Outside that intervals, the equation behaves pseudo regular. The following Figures show some of that behavior. Unless otherwise noted the parameters are set to zero. If you are viewing the paper on the screen you can adjust parameters by yourself and you will see immediately the result. Figure 1 shows a constantly growing signal, due to a system that is not damped or limited in any way. Figure 2 shows a damped linear system. The phase space (displacement plotted versus velocity) is a spiral approaching a constant orbit. The time domain response is well behaved and the spectrum shows a single peak developed around the center frequency of the forcing function. In Figure 3, some harmonic peaks are generated in the spectrum beyond the peak of the forcing function. High amplitudes arise by emphasizing the third order term in the equation. In Figure 4 subharmonic peaks are generated in the spectrum below the peak of the forcing function. Relatively low amplitudes are generated, in contrast to Figure 3. Nonlinear behavior is developing in Figure 5 at this parameter setting, which is apparently erratic in the phase space. The spectrum of the forcing function is changed in a non systematic way. Compare this to the structured modifications in Figure 4 and 3. Figure 6 and 7 show slightly different parameter settings and illustrate the unstable character of this equation. Small changes in parameters produce large effects in the solution behavior. The original Duffing equation describes a scalar chaotic process and has been used in the past to model and analyze the effective time behavior of a source-surface system. I would like to extend the equation and use it for multi-component sources in a 3-D medium. Formally, a 3-D 3-c Duffing equation reads then:

$$\nabla C \nabla^t \mathbf{u} + \nabla D \nabla \frac{\partial \mathbf{u}}{\partial t} + \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} + \gamma ||\mathbf{u}||^2 + \beta \mathbf{u} ||\mathbf{u}||^2 = \mathbf{F}(\mathbf{x}, t) \tag{8}$$

I would like to investigate this equation further using finite difference schemes to see if for a reasonable choice of parameters source-surface effects can be modeled. It will be interesting to compare it to 9 component land data to see if such source effects are indeed present in the data. Finding the equivalent linear multi-component source function would prove very useful as a step towards backing source differences out of the data set.

CONCLUSIONS

Nonlinear wave propagation can be described with a purely geometric parameterization or with a nonlinear material characterization (or both). The finite strain approach lends itself as a natural extension to the usual linearly elastic wave propagation

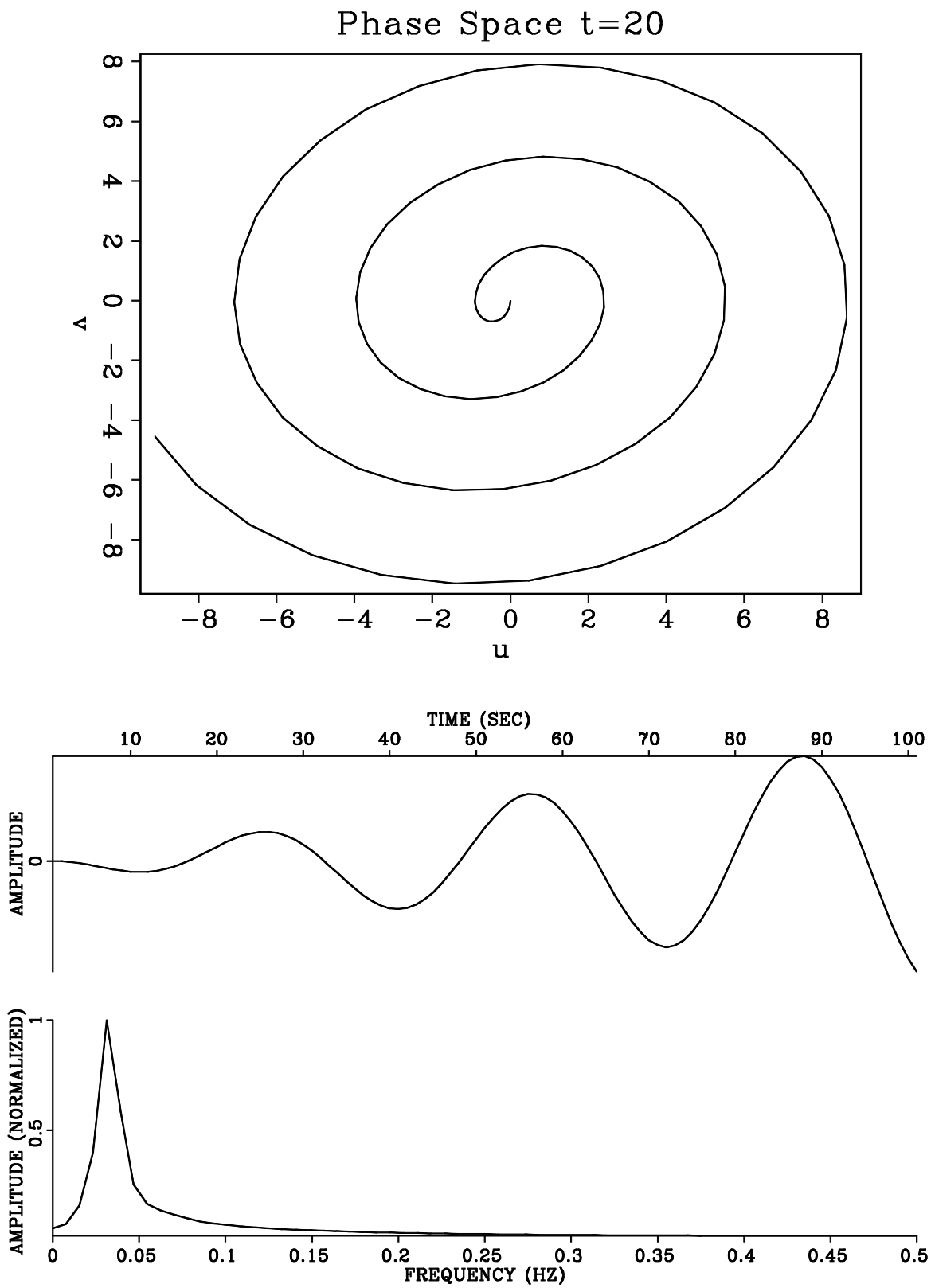

Figure 1: Undamped Duffing equation with α=1, β=0, δ=0, ω=1 and F=1. The movement is linear and growing even after some initial time. martin-undamped [ER]

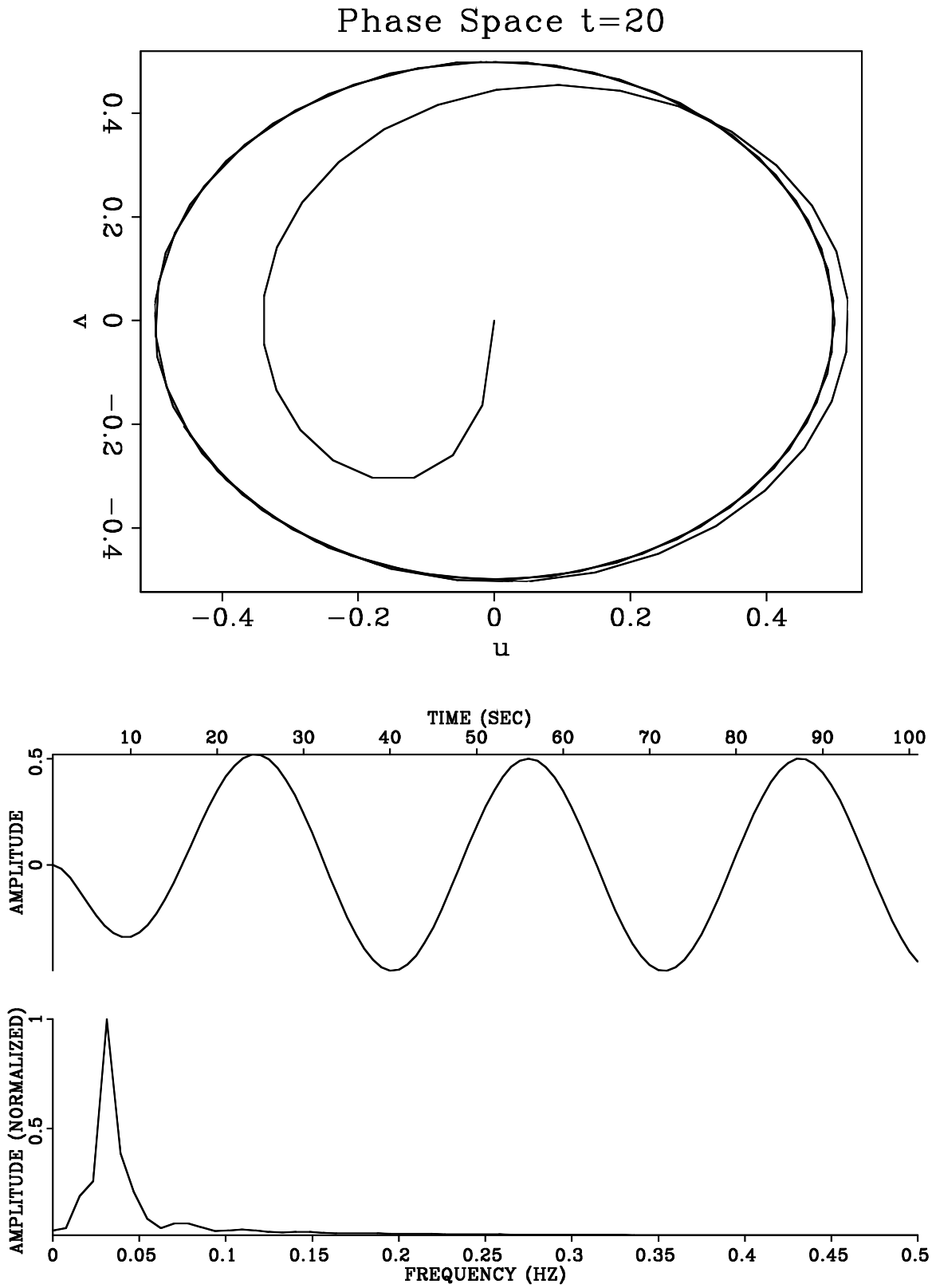

Figure 2: Damped Duffing equation with α=1, β=0, δ=2, ω=1 and F=1. The movement is linear and stable after some initial time. martin-damped [ER]

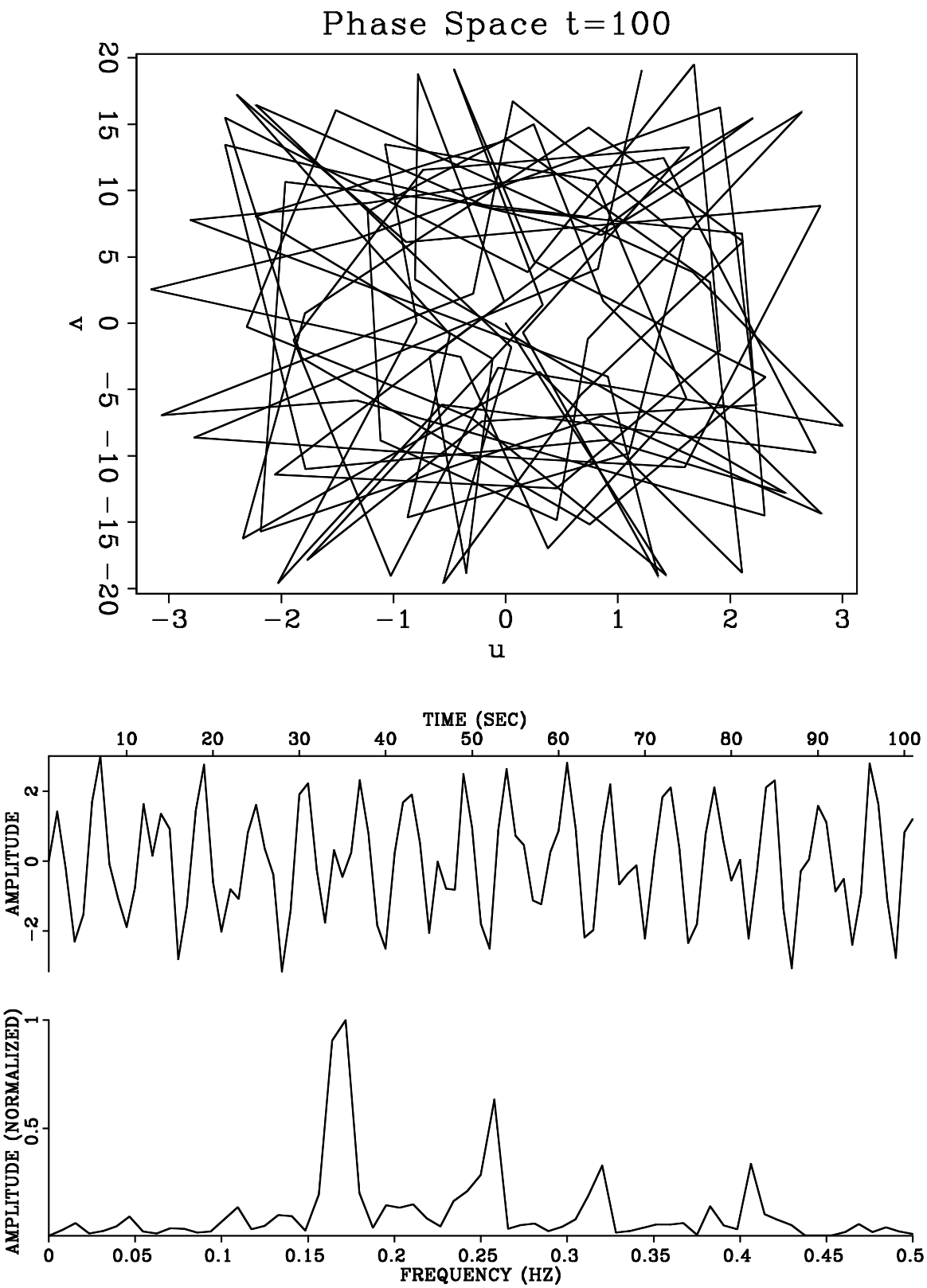

Figure 3: Duffing equation with α=1, β=0, δ=0, γ=10 ω=4.67 and F=1. Harmonic peaks are generated in the spectrum beyond the peak of the forcing function. High amplitudes arise by emphasizing the third order term in the equation. martin-topharm [ER]

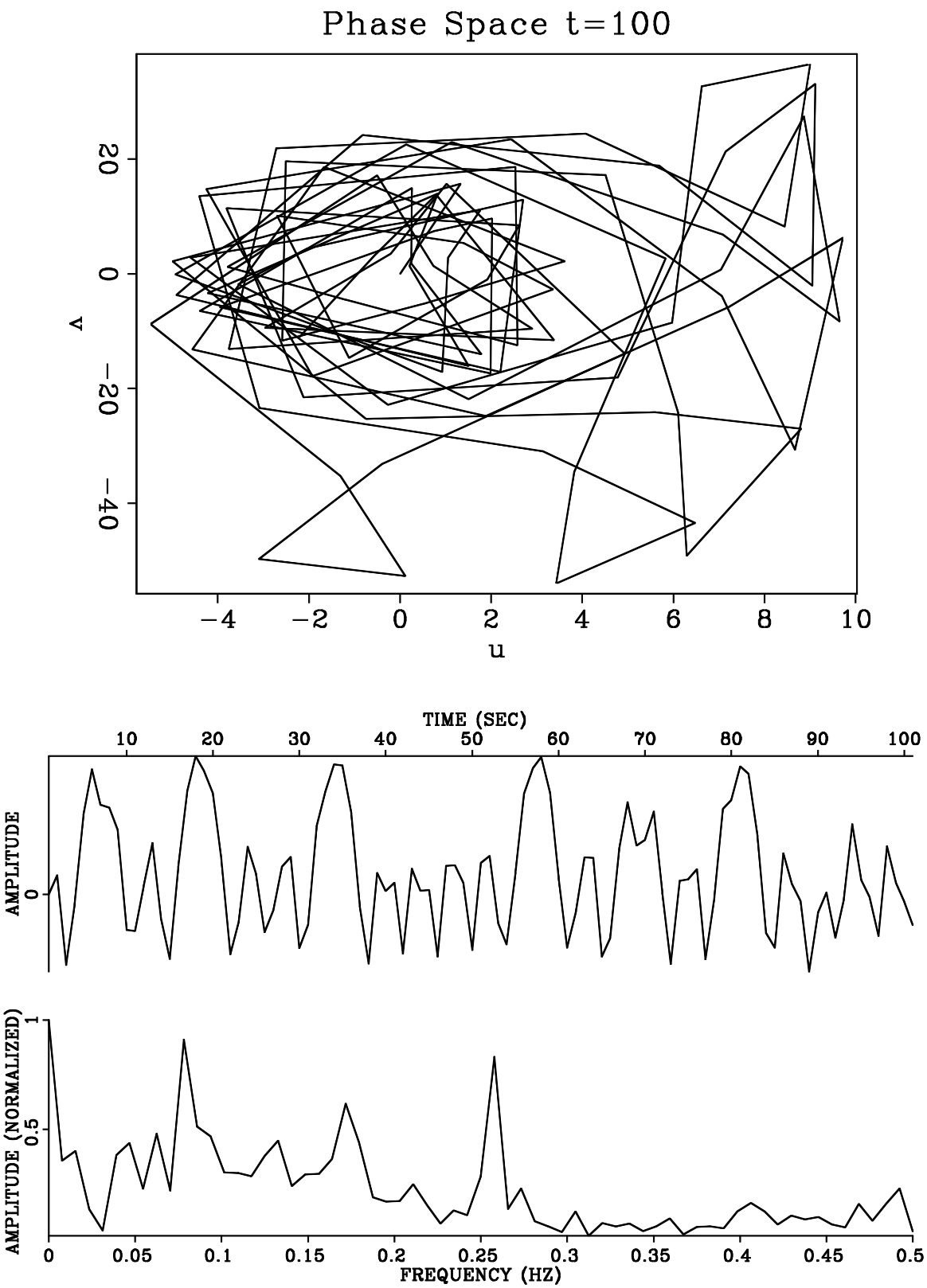

Figure 4: Duffing equation with α=1, β=0, δ=0, γ=0.67 ω=4.67 and F=1. Subharmonic peaks are generated in the spectrum below the peak of the forcing function. Relatively low amplitudes are generated, in contrast to Figure 3. martin-subharm [ER]

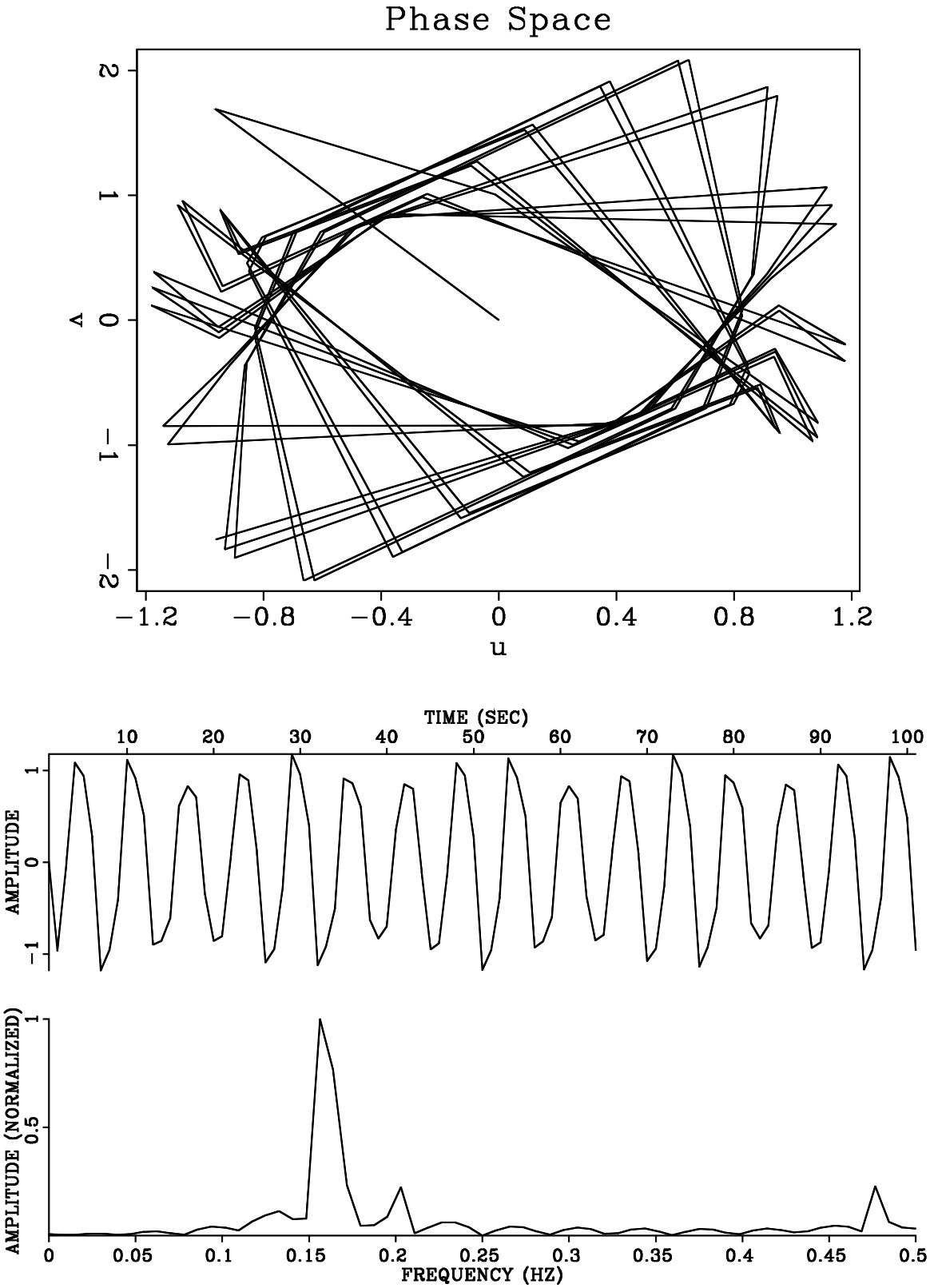

Figure 5: Duffing equation with α=1, β=0, δ=2, γ=10 ω=1.0 and F=10. Nonlinear behavior is developing at this parameter setting, which is apparently erratic in the phase space. The spectrum of the forcing functions changed in a non systematic way. Compare this to the structured modifications in Figure 4 and 3. martin-nonlin [ER]

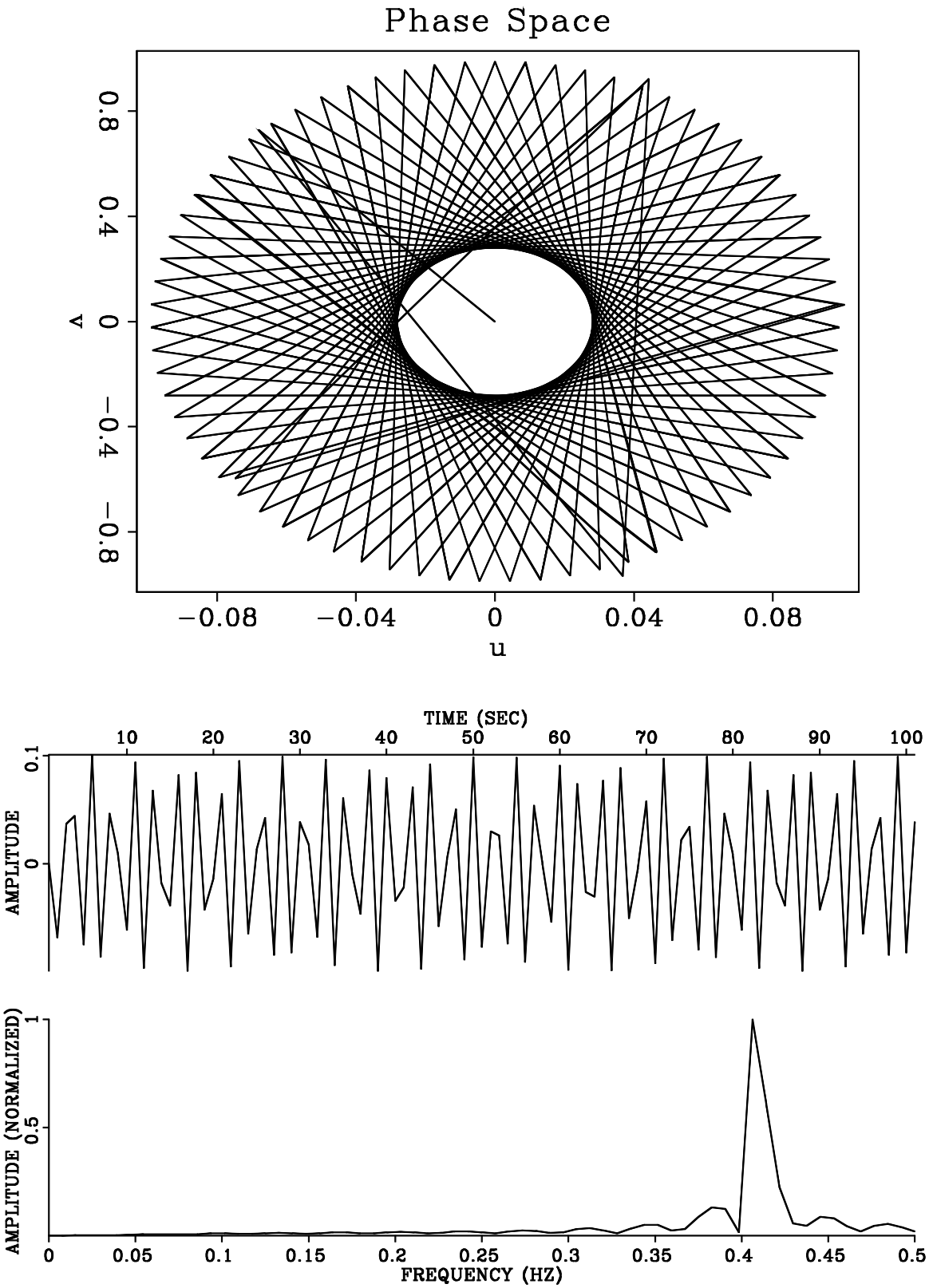

Figure 6: Duffing equation with α=1, β=0, δ=2, γ=10 ω=10.0 and F=10. Nonlinear behavior shows well developed structure in phase space, but the spectrum shows signs of alternate peaks. martin-nonlin2 [ER]

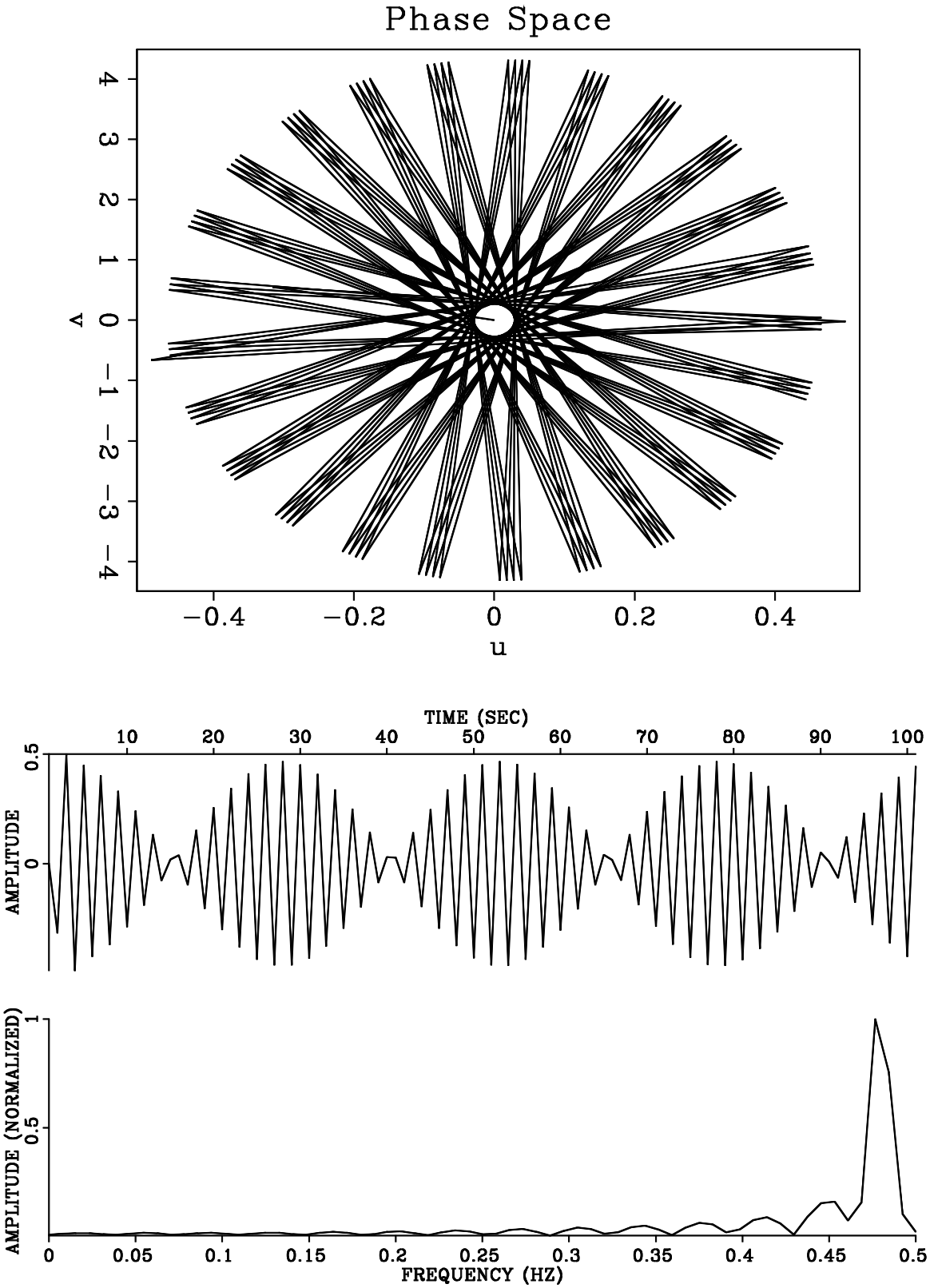

Figure 7: Duffing equation with α=1, β=0, δ=2, γ=10 ω=9.3 and F=40. A slight change in frequency and source strength compared to Figure 6 shows totally different behavior in phase space as well as in physical space. martin-nonlin3 [ER]

description. A more complicated and maybe more realistic treatment incorporates a nonlinear stress-strain relationship. However additional parameters have to be estimated. The one-dimensional Duffing equation is described and an extension to three component 3-D data is proposed, mimicing its 1-D counterpart.

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