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3-D prestack migration of common-azimuth data

Biondo Biondi and Gopal Palacharla[1]

ABSTRACT

In principle, downward continuation of 3-D prestack data should be carried out in a 5-D computational space, even when the dataset to be migrated is only 3-D or 4-D. Unless this efficiency issue is solved, 3-D prestack migration methods based on the solution of the one-way wave equation are uncompetitive with Kirchhoff methods. We present a method for downward continuing common-azimuth data in the frequency-wavenumber domain in the original 4-D space of the common-azimuth data geometry. The method is based on a stationary-phase approximation of the one-way wave equation and can be applied to both phase-shift and Stolt migrations. The proposed migration methods are exact for constant velocity, and approximate for velocity varying with depth. However, results of some numerical experiments on synthetic data show that the approximation is good even in presence of strong vertical velocity gradients.

INTRODUCTION

As computational power of scientific computers keeps growing, 3-D prestack migration is becoming within the reach of the seismic industry. For 3-D prestack migration Kirchhoff methods are often preferred over methods based on the recursive downward continuation of the recorded wavefield because of their flexibility in handling 3-D prestack data geometries (Audebert, 1994). Kirchhoff methods can be employed to efficiently migrate datasets with uneven spatial sampling and datasets that are subsets of the complete prestack data, such as common-offset cubes and common-azimuth cubes. For recursive methods the irregular sampling problem can be addressed with an interpolation preprocessing step, though in practice it can be a challenging task. However, in principle the downward continuation of prestack subsets should be carried out in the full 5-D data space, even when the original subset is 3-D (common offset) or 4-D (common azimuth). As a result of these constraints most of the computations are wasted on propagating components of the wavefield that are either equal to zero or do not contribute to the final image. These potential limitations of recursive methods have led the industry to pursue almost exclusively Kirchhoff methods for 3-D prestack migration (Western and Ball, 1991; Ratcliff et al., 1994), though recursive methods have some intrinsic advantages over Kirchhoff methods. First, they are potentially more accurate and robust because they are based on the full wave-equation and not an asymptotic solution based on ray theory. Second, when they can be used to extrapolate the recorded data without increasing

[1]**email:** not available

their dimensionality (e.g. zero-offset data), they can be implemented more efficiently than the corresponding Kirchhoff methods.

In this paper we develop a method for efficiently imaging common-azimuth data by downward continuing the data in the original 4-D space. An efficient algorithm for migrating common-azimuth subsets has useful practical applications because common-azimuth data are either the result of a collection of actual physical experiments (e.g. single-streamer marine survey with negligible cable feather) or they may be synthesized by preprocessing (Biondi and Chemingui, 1994). Our method is potentially very efficient for migrating common-azimuth data but it is exact (within the limitations of a stationary phase approximation (Bleistein, 1984), as we will discuss in the following section) only for the simple case of constant propagation velocity. For the case of velocity varying with depth we propose a simple and efficient approximation and we explore the range of validity of our approximation by analyzing the results of some simple numerical experiments. But more studies are required to determine its accuracy in the more complex, and interesting, case of velocity being a general function of the spatial coordinates.

An important application of our method that is unaffected by the constant velocity assumption is the efficient computation of migration velocity scans from common-azimuth data by a Stolt migration (Stolt, 1978). A 3-D time-migrated image can be extracted from these multiple-velocity migrations with a similar procedure to the one that is routinely applied to 2-D data. Furthermore, these migration velocity scans can also be effective tools for estimating the velocity function (Shurtleff, 1984; Fowler, 1988).

COMMON-AZIMUTH PHASE-SHIFT MIGRATION

3-D prestack data can be downward continued in the frequency-wavenumber domain by applying the 3-D double square root equation (DSQR). In 3-D the DSQR equation is function of five scalar variables: the temporal frequency (ω), the two components of the midpoint wavenumber vector ($\mathbf{k}_m = k_{mx}\mathbf{x}_m + k_{my}\mathbf{y}_m$), and the two components of the offset wavenumber vector ($\mathbf{k}_h = k_{hx}\mathbf{x}_h + k_{hy}\mathbf{y}_h$). Therefore, in the general case the dimensionality of the computational domain is five. However, the data is not always 5-D but it is often 3-D or 4-D. When the dimensionality of the data is lower than five it would be more efficient to use an algorithm that downward continues the data in less than 5 dimensions, if such an algorithm were available. Common-azimuth data has only four dimensions because the offset vectors between source and receivers are constrained to have the same azimuth. The four axis are: recording time, two midpoints, and the offset along the azimuthal direction. A phase-shift migration of common-azimuth data in a 4-D computational space can be derived by evaluating the stationary-phase approximation of the full 3-D DSQR equation (Appendix A). The resulting expression is

$$\hat{M}(\mathbf{k}_m, z_r) = \int_{-\infty}^{+\infty} dk_{hx} \int_{-\infty}^{+\infty} d\omega\; D_0(\omega, \mathbf{k}_m, k_{hx}) A(\omega, \mathbf{k}_m, k_{hx}, z_r) e^{i\,\Phi_{stat}(\omega, \mathbf{k}_m, k_{hx}, z_r) + i\,\frac{\pi}{4}}. \qquad (1)$$

Where x is the azimuth direction, y is the direction perpendicular to x (in the following we will call y the cross-azimuth axis), D_0 and $\hat{M}$ are respectively the data and the migration results.

The function A is real and thus it is a simple amplitude factor (given also in Appendix A), and Φ_{stat} is the phase function evaluated along the stationary path and it is equal to

$$\Phi_{stat}(\omega, \mathbf{k}_m, k_{hx}, z_r) = \int_{z_r}^{0} dz^{'} DSQR[\omega, \mathbf{k}_m, k_{hx}, k^{'}_{hy}(z_r), z^{'})]; \tag{2}$$

where $DSQR(\omega, \mathbf{k}_m, \mathbf{k}_h, z^{'})$ is the double square root equation

$$DSQR(\omega, \mathbf{k}_m, \mathbf{k}_h, z^{'}) = \frac{\omega}{v(z^{'})} \left\{ \sqrt{1 - \frac{v^2(z^{'})}{4\omega^2}[(k_{mx} + k_{hx})^2 + \left(k_{my} + k_{hy}\right)^2]} + \sqrt{1 - \frac{v^2(z^{'})}{4\omega^2}[(k_{mx} - k_{hx})^2 + \left(k_{my} - k_{hy}\right)^2]} \right\} \tag{3}$$

and $k^{'}_{hy}(z_r)$ is the global stationary path for the phase function.

When the velocity is constant ($v(z^{'}) = V$) the global stationary path is independent of the reflector depth z_r and it can be derived analytically as equal to

$$k^{'}_{hy} = k_{my} \frac{\sqrt{\omega^2 - \frac{V^2}{4}(k_{mx} + k_{hx})^2} - \sqrt{\omega^2 - \frac{V^2}{4}(k_{mx} - k_{hx})^2}}{\sqrt{\omega^2 - \frac{V^2}{4}(k_{mx} + k_{hx})^2} + \sqrt{\omega^2 - \frac{V^2}{4}(k_{mx} - k_{hx})^2}}. \tag{4}$$

However, when the velocity varies with depth the stationary path cannot be easily derived analytically. To overcome this problem we suggest to approximate the global stationary path $k^{'}_{hy}(z_r)$ with the stationary path evaluated locally at each depth level $z^{'}$ of the recursive downward continuation, that is;

$$k^{'}_{hy}(z_r) \approx \hat{k}^{'}_{hy}(z^{'}) = k_{my} \frac{\sqrt{\omega^2 - \frac{v^2(z^{'})}{4}(k_{mx} + k_{hx})^2} - \sqrt{\omega^2 - \frac{v^2(z^{'})}{4}(k_{mx} - k_{hx})^2}}{\sqrt{\omega^2 - \frac{v^2(z^{'})}{4}(k_{mx} + k_{hx})^2} + \sqrt{\omega^2 - \frac{v^2(z^{'})}{4}(k_{mx} - k_{hx})^2}}. \tag{5}$$

These local stationary paths are dependent on velocity at each depth level $v(z^{'})$, but they are independent of the reflector depth z_r. Therefore the integral of equation (2) can be evaluated recursively over depth levels, leading to an efficient migration algorithm. In the following sections we will investigate the validity of this approximation by comparing migration results obtained using the full DSQR equation and results obtained using our approximation in a medium with velocity varying with depth.

In principle the proposed method could be generalized to handle lateral velocity variations by using the Phase Shift Plus Interpolation (Gazdag and Sguazzero, 1984) methodology. However, we think that a better understanding of the implications of the approximation introduced in equation (5) is required before we apply the method to more general cases.

COMMON-AZIMUTH STOLT MIGRATION

In the previous section we have introduced an efficient phase-shift migration algorithm for common-azimuth datasets. Because for constant velocity Stolt migration algorithms are more efficient than phase-shift ones, we will now derive an algorithm for common-azimuth Stolt migration. Probably the most promising application of this new 3-D Stolt migration is the computation of constant migration velocity scans. As it is routinely done for 2-D prestack Stolt, this multiple-velocity migrations can be used for extracting a time-migrated image and for estimating the velocity function in a dip-independent fashion.

The fundamental result of our development is the derivation of a new Stolt change of variables to be applied to common-azimuth data in the frequency-wavenumber domain. As we show in Appendix B, the new change of variables can be recast in a way that shows how in constant velocity, 3-D prestack migration can be seen as the cascade of two processes: 2-D prestack migration along the azimuth direction, and poststack migration along the cross-azimuth direction. Canning and Gardner (1992) showed a similar result, however in their two-pass migration the poststack migration along the cross-azimuth direction comes before the 2-D prestack migration. The migration of a common-azimuth dataset can be expressed as

$$\hat{M}(\tau,\mathbf{k}_m) = \int_{-\infty}^{+\infty} dk_{hx} \int_{-\infty}^{+\infty} dk_\tau \left[\frac{d\omega}{dk_\tau}\right] D_0(\omega(k_\tau,\mathbf{k}_m,k_{hx},V),\mathbf{k}_m,k_{hx})\bar{A}(t,\mathbf{k}_m,k_{hx},\tau)e^{ik_\tau\tau+i\frac{\pi}{4}}. \tag{6}$$

where τ is pseudo-depth, $\bar{A}$ is an amplitude factor and $\omega = \omega(k_\tau,\mathbf{k}_m,k_{hx},V)$ is a dispersion relation derived by using the stationary-phase method. Notice, that to be able to evaluate the integral over τ in equation (6) by Fast Fourier Transforms we need to drop the amplitude factor $\bar{A}$, because it depends on t. Dropping this amplitude term does not affect the usefulness of the method because its main application is for kinematic migrations.

After lengthy algebraic manipulations the dispersion relation $\omega(k_\tau,\mathbf{k}_m,k_{hx},V)$ can be recast as the cascade of two changes of variable: the first is the change of variable for 2-D prestack Stolt migration, i.e.;

$$\omega^2 = \frac{1}{k_{\tau 0}^2}\left(k_{\tau 0}^2 + \frac{V^2}{4}k_{mx}^2\right)\left(k_{\tau 0}^2 + \frac{V^2}{4}k_{hx}^2\right), \tag{7}$$

and the second is the change of variable for zero-offset migration in the cross-azimuth direction, i.e.;

$$k_{\tau 0}^2 = k_\tau^2 + \frac{V^2}{4}k_{my}^2. \tag{8}$$

This result has a simple intuitive explanation in terms of what is known about the kinematics of two-pass 3-D poststack migration (Jakubowicz and Levin, 1983) and 3-D DMO (Hale, 1983). The outline of the reasoning is the following: first we consider that constant velocity 3-D poststack migration can be split along two perpendicular directions (azimuth and cross-azimuth in our case) without loss of accuracy. Second, we consider that for constant velocity

3-D DMO is kinematically exact and reduces to 2-D DMO along the azimuthal direction. Consequently, constant velocity prestack migration of common-azimuth data can be achieved by cascading NMO+DMO with poststack migration along the azimuth direction followed by poststack migration along the cross-azimuth direction. Finally we recognize that the first two steps of this procedure are equivalent to 2-D prestack migration along the azimuthal direction.

Though the dispersion relation $\omega(k_\tau, \mathbf{k}_m, k_{hx}, V)$ is more meaningfully described as the cascade of the two transformations expressed in equations (7) and (8) the regridding of the data can be accomplished in one single step. Consequently, if the additional costs of performing FFT along the cross-azimuth midpoint axis and of handling the large amount of data are neglected, the proposed 3-D prestack Stolt migration of a set of parallel lines with a common-azimuth is comparable to the cost of migrating the same lines with a conventional 2-D prestack Stolt migration algorithm.

NUMERICAL RESULTS

To check the validity of the proposed migration methods and to explore their limits of applicability, we migrated a common-azimuth dataset containing a band limited impulse. As a benchmark for judging the quality of the results obtained using the proposed common-azimuth Stolt and phase-shift migrations, we use the results of applying phase-shift migration to a full 5-D prestack data, thus without any stationary-phase approximation.

First we will analyze the results of both the new migration methods in a medium with constant velocity of 2.5 Km/s. The data was a band-limited impulse (5-40 Hz) delayed .5 seconds and placed at .5 Km offset along the azimuthal direction. Figure **??** shows a vertical slice along the azimuthal direction of the migration results for both the 5-D phase-shift migration (a) and the common-azimuth phase-shift migration (b). Figure **??** shows the horizontal slices taken across the migration results at constant depth of 160 m. There are very small differences between the results; they are mostly caused by numerical artifacts that are larger in the 5-D phase-shift results. We somewhat undersampled the cross-azimuth offset axis of the full 5-D dataset, to accommodate limitations of our test program in handling out-of-core jobs. Figure **??** and Figure **??** compare the results obtained by applying common-azimuth Stolt migration (b) and 5-D phase shift migration (a). The differences between the results are mainly in the numerical artifacts, though the amplitudes behavior is also slightly different. This inaccuracy in amplitudes is probably caused by the neglect of the amplitude term $\bar{A}$ in equation (6), as we discussed in the previous section.

To test the accuracy of phase-shift common-azimuth migration for varying velocity we migrated the same dataset as in the constant velocity test, but with velocity increasing with depth. The velocity function had a constant but strong (5 s^{-1}) vertical gradient. Figure **??** compares the vertical sections along the azimuthal direction of the migrated results obtained by the full 5-D phase-shift method (a) and the 4-D common-azimuth one (b). Figure **??** compares the vertical sections along the cross-azimuth direction. Because of the strong vertical gradient the impulse responses are not semicircle, but they bulge outwardly in the shallow part of the section. Finally Figure **??** compares the horizontal sections taken from the migrated results

../biondo1/./Fig/Comp-ph-v0-y.pdf

Figure 1: In-line vertical sections obtained by using: a) the 5-D phase-shift migration, and b) the common-azimuth phase-shift migration for constant velocity.

../biondo1/./Fig/Comp-ph-v0-z.pdf

Figure 2: Depth slices obtained by using: a) the 5-D phase-shift migration, and b) the common-azimuth phase-shift migration for constant velocity.

../biondo1/./Fig/Comp-st-v0-y.pdf

Figure 3: In-line vertical sections obtained by using: a) the 5-D phase-shift migration, and b) the common-azimuth Stolt migration for constant velocity.

../biondo1/./Fig/Comp-st-v0-z.pdf

Figure 4: Depth slices obtained by using a) the 5-D phase-shift migration, and b) the common-azimuth Stolt migration (b) for constant velocity.

at a depth of 160 m. There are slight differences between the migration results obtained using the two phase-shift methods. The most noticeable are in the amplitudes behavior as a function of the reflector dips (Figures **??** and **??**). The common-azimuth migration shows higher amplitudes for the steeply dipping reflectors. There are also slight differences in the phases of the results. These phase errors are only visible in the depth slices because they vanish along the axis and they are maximum along the direction oriented at an angle of 45 degrees with respect to the axis. However, overall the common-azimuth migration produced a fairly accurate result, notwithstanding the strong velocity gradient.

`../biondo1/./Fig/Comp-ph-vz-y.pdf`

Figure 5: In-line vertical sections obtained by using: a) the 5-D phase-shift migration, and b) the common-azimuth phase-shift migration for velocity increasing with depth.

CONCLUSIONS

3-D common-azimuth datasets can be efficiently migrated in the frequency-wavenumber domain with methods based on the one-way wave equation (Stolt and phase-shift) thanks to a stationary-phase approximation that reduces the dimensionality of the computational space from five to the original four of the common-azimuth geometry. Within the stationary phase assumptions, the proposed methods are exact for constant velocity. For velocity functions varying with depth we have introduced an approximation that leads to an efficient recursive (phase-shift) algorithm. When our approximation is used for migrating synthetic data in presence of strong vertical gradient in velocity the results are very close to the result obtained using a full 5-D phase-shift method.

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Figure 6: Cross-line vertical sections obtained by using: a) 5-D phase-shift migration, and b) the common-azimuth phase-shift migration for velocity increasing with depth.

../biondo1/./Fig/Comp-ph-vz-z.pdf

Figure 7: Depth slices obtained by using: a) the 5-D phase-shift migration, and b) the common-azimuth phase-shift migration for velocity increasing with depth.

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APPENDIX A

DERIVATION OF COMMON-AZIMUTH PHASE-SHIFT MIGRATION

In this appendix we derive the stationary-phase approximation for common-azimuth phase-shift migration. We start by analyzing the adjoint operation; that is, the forward modeling of common-azimuth data. This side-step is not necessary for the derivation of phase-shift common-azimuth migration, but it makes the phase-shift case parallel to the development of Stolt common-azimuth migration (Appendix B) and thus we think that both derivations become clearer. The prestack data in the wavenumber domain can be obtained by summing the contributions of the reflectors at each depth level appropriately delayed (phase shifted in the wavenumber domain) according to the double-square root equation. In mathematical terms the prestack data in the temporal frequency (ω), the midpoint wavenumbers ($\mathbf{k}_m = k_{mx}\mathbf{x}_m + k_{my}\mathbf{y}_m$), and the offset wavenumbers ($\mathbf{k}_h = k_{hx}\mathbf{x}_h + k_{hy}\mathbf{y}_h$) are given by the following integral

$$D(\omega,\mathbf{k}_m,\mathbf{k}_h) = \int_0^{+\infty} dz_r \; M(\mathbf{k}_m,z_r) e^{i\,\Phi(\omega,\mathbf{k}_m,\mathbf{k}_h,z_r)} \tag{B-1}$$

where the phase function is given by

$$\Phi(\omega,\mathbf{k}_m,\mathbf{k}_h,z_r) = \int_{z_r}^{0} dz^{'} \; DSQR(\omega,\mathbf{k}_m,\mathbf{k}_h,z^{'}), \tag{B-2}$$

and the double-square root equation is equal to

$$\begin{aligned} DSQR(\omega,\mathbf{k}_m,\mathbf{k}_h,z^{'}) = \\ \frac{\omega}{v(z')}\left\{\sqrt{1-\frac{v^2(z')}{4\omega^2}[(k_{mx}+k_{hx})^2+\left(k_{my}+k_{hy}\right)^2]} + \right. \\ \left. \sqrt{1-\frac{v^2(z')}{4\omega^2}[(k_{mx}-k_{hx})^2+\left(k_{my}-k_{hy}\right)^2]}\right\} \end{aligned} \tag{B-3}$$

The common-azimuth data corresponding to the azimuth oriented along the in-line direction x can be extracted from the full prestack dataset by integrating the data over all the k_{hy} wavenumbers. The expression for computing a common azimuth dataset is

$$
\begin{aligned}
&D_0(\omega,\mathbf{k}_m,k_{hx}) = \\
&\quad \int_{-\infty}^{+\infty} dk_{hy}\; D(\omega,\mathbf{k}_m,\mathbf{k}_h) = \\
&\quad \int_{-\infty}^{+\infty} dk_{hy} \int_0^{+\infty} dz_r\; M(\mathbf{k}_m,z_r) e^{i\Phi(\omega,\mathbf{k}_m,\mathbf{k}_h,z_r)} = \\
&\quad \int_0^{+\infty} dz_r\; M(\mathbf{k}_m,z_r) \int_{-\infty}^{+\infty} dk_{hy}\; e^{i\Phi(\omega,\mathbf{k}_m,\mathbf{k}_h,z_r)} = \\
&\quad \int_0^{+\infty} dz_r\; M(\mathbf{k}_m,z_r) I(\omega,\mathbf{k}_m,k_{hx},z_r)
\end{aligned}
\qquad \text{(B-4)}
$$

In the stationary-phase regime, the second integral ($I(\omega,\mathbf{k}_m,k_{hx},z_r)$) can be asymptotically approximated by

$$
\begin{aligned}
&I(\omega,\mathbf{k}_m,k_{hx},z_r) \approx \\
&\quad \frac{2\pi}{\sqrt{\Phi''_{stat}(\omega,\mathbf{k}_m,k_{hx},z_r)}}\; e^{i\Phi_{stat}(\omega,\mathbf{k}_m,k_{hx},z_r)+i\frac{\pi}{4}} = \\
&\quad A(\omega,\mathbf{k}_m,k_{hx},z_r)\; e^{i\Phi_{stat}(\omega,\mathbf{k}_m,k_{hx},z_r)+i\frac{\pi}{4}}
\end{aligned}
\qquad \text{(B-5)}
$$

where the phase function Φ_{stat} is equal to the original phase function [equation (B-2)] evaluated along the stationary path k'_{hy}. In constant velocity the stationary path can be derived analytically. Unfortunately, when velocity varies with depth, the stationary path of the integral in equation (B-2) cannot be easily derived analytically and we will introduce an approximation for Φ_{stat}.

Constant velocity

For constant velocity the integral in the expression for the phase function is easily evaluated and the phase function simplifies to

$$
\begin{aligned}
&\Phi_0(\omega,\mathbf{k}_m,\mathbf{k}_h,V,z_r) = \\
&\quad \int_{z_r}^{0} dz'\; DSQR_0(\omega,\mathbf{k}_m,k_{hx},k_{hy},V) = \\
&\quad -DSQR_0(\omega,\mathbf{k}_m,k_{hx},k_{hy},V) z_r
\end{aligned}
\qquad \text{(B-6)}
$$

Consequently, in constant velocity the stationary path for the phase function is equal to the stationary path for the double-square root equation and it can be evaluated analytically. There are two solutions for the stationary path of the double-square root equation, and they are

$$
k'_{hy} = k_{my} \frac{\sqrt{\omega^2 - \frac{V^2}{4}(k_{mx}+k_{hx})^2} \mp \sqrt{\omega^2 - \frac{V^2}{4}(k_{mx}-k_{hx})^2}}{\sqrt{\omega^2 - \frac{V^2}{4}(k_{mx}+k_{hx})^2} \pm \sqrt{\omega^2 - \frac{V^2}{4}(k_{mx}-k_{hx})^2}} \qquad \text{(B-7)}
$$

In choosing between these two solutions, first we notice that one solution is the inverse of the other. Second we consider the limiting case of the in-line offset wavenumber (k_{hx}) equal to zero. In this case one solution diverges while the other (being the one with the minus sign at the numerator) is equal to zero. We accept this second solution because it is consistent with the notion that when both k_{hx} and k_{hy} vanish the double square root equation reduces to the single square root equation that is commonly used for migrating zero-offset data.

Velocity function of depth

When velocity varies with depth the integral in the expression for the phase function [equation (B-2)] cannot be evaluated analytically. Consequently the "global" stationary path cannot be derived analytically, and its numerical computation is expensive. However, from the constant velocity case, we can analytically derive the "local" stationary paths at each depth level,

$$\hat{k}'_{hy}(z') = k_{my} \frac{\sqrt{\omega^2 - \frac{v^2(z')}{4}(k_{mx}+k_{hx})^2} - \sqrt{\omega^2 - \frac{v^2(z')}{4}(k_{mx}-k_{hx})^2}}{\sqrt{\omega^2 - \frac{v^2(z')}{4}(k_{mx}+k_{hx})^2} + \sqrt{\omega^2 - \frac{v^2(z')}{4}(k_{mx}-k_{hx})^2}}. \tag{B-8}$$

An efficient approximation of the phase function evaluated along the global stationary path is the integral along the depth axis of the phase functions evaluated along the local stationary paths. The global stationary function can thus be approximated by the following expression

$$\Phi_{stat}(\omega, \mathbf{k}_m, k_{hx}, z_r) \approx \int_{z_r}^{0} dz' \; DSQR[\omega, \mathbf{k}_m, k_{hx}, \hat{k}'_{hy}(z'), z']. \tag{B-9}$$

3D Migration of constant-azimuth prestack data

From the previous results on common-azimuth modeling we can directly derive an expression for migrating 3D common-azimuth prestack data. In the general case of multi-azimuth data, the reflectivity function is estimated by

$$\hat{M}(\mathbf{k}_m, z_r) = \int_{-\infty}^{+\infty} dk_{hy} \int_{-\infty}^{+\infty} dk_{hx} \int_{-\infty}^{+\infty} d\omega \; D(\omega, \mathbf{k}_m, \mathbf{k}_h) e^{-i\,\Phi(\omega, \mathbf{k}_m, \mathbf{k}_h, z_r)} \tag{B-10}$$

Similarly, the migration of a common-azimuth dataset can be expressed as

$$\hat{M}(\mathbf{k}_m, z_r) = \int_{-\infty}^{+\infty} dk_{hx} \int_{-\infty}^{+\infty} d\omega \; D_0(\omega, \mathbf{k}_m, k_{hx}) I^*(\omega, \mathbf{k}_m, k_{hx}, z_r) \tag{B-11}$$

Substituting equation (A-5) into equation (B-11) gives equation (1) of the main text.

As for modeling, this expression for migration is exact (within the approximation of stationary phase) for constant velocity, but it is approximated for velocity varying with depth.

APPENDIX B

DERIVATION OF COMMON-AZIMUTH STOLT MIGRATION

In this appendix we derive the stationary-phase approximation for Stolt common-azimuth migration. As for the phase-shift case, we start from the stationary-phase approximation of Stolt modeling. The 3-D prestack data in time (t), the midpoint wavenumber ($\mathbf{k}_m = k_{mx}\mathbf{x}_m + k_{my}\mathbf{y}_m$), and the offset wavenumber ($\mathbf{k}_h = k_{hx}\mathbf{x}_h + k_{hy}\mathbf{y}_h$) can be computed by Stolt modeling using the following integral over the pseudo-depth τ,

$$D(t,\mathbf{k}_m,\mathbf{k}_h) = \int_{-\infty}^{+\infty} dk_\tau \; M(k_\tau,\mathbf{k}_m) e^{i\Psi(t,k_\tau,\mathbf{k}_m,\mathbf{k}_h,V)}, \qquad \text{(B-1)}$$

where the phase function is given by

$$\Psi(t,k_\tau,\mathbf{k}_m,\mathbf{k}_h,V) = IDSQR(k_\tau,\mathbf{k}_m,\mathbf{k}_h,V)\; t, \qquad \text{(B-2)}$$

and the inverse of the double-square-root equation is equal to

$$IDSQR(k_\tau,\mathbf{k}_m,\mathbf{k}_h,V) = \sqrt{k_\tau^2 + \frac{V^2}{4}\left\{k_{mx}^2 + k_{hx}^2 + k_{my}^2 + k_{hy}^2\right\} + \frac{V^4}{16k_\tau^2}\left(k_{mx}k_{hx} + k_{my}k_{hy}\right)^2}. \qquad \text{(B-3)}$$

The common-azimuth data corresponding to the azimuth oriented along the in-line direction x can be extracted from the full prestack dataset by integrating the data over all the k_{hy} wavenumbers. The expression for computing a common azimuth dataset is

$$\begin{aligned} D_0(t,\mathbf{k}_m,k_{hx}) &= \\ &\int_{-\infty}^{+\infty} dk_{hy}\; D(t,\mathbf{k}_m,\mathbf{k}_h) = \\ &\int_{-\infty}^{+\infty} dk_{hy} \int_{-\infty}^{+\infty} dk_\tau \; M(k_\tau,\mathbf{k}_m) e^{i\Psi(t,k_\tau,\mathbf{k}_m,\mathbf{k}_h,V)} = \\ &\int_{-\infty}^{+\infty} dk_\tau \; M(k_\tau,\mathbf{k}_m) \int_{-\infty}^{+\infty} dk_{hy}\; e^{i\Psi(t,k_\tau,\mathbf{k}_m,\mathbf{k}_h,V)} = \\ &\int_{-\infty}^{+\infty} dk_\tau \; M(k_\tau,\mathbf{k}_m) I_1(k_\tau,\mathbf{k}_m,k_{hx},V). \end{aligned} \qquad \text{(B-4)}$$

In the stationary-phase regime, the second integral ($I_1(k_\tau,\mathbf{k}_m,k_{hx},V)$) can be asymptotically approximated by

$$\begin{aligned} I_1(k_\tau,\mathbf{k}_m,k_{hx},V) &\approx \\ &\frac{2\pi}{\sqrt{\Psi''_{stat}(k_\tau,\mathbf{k}_m,k_{hx},V)}}\, e^{i\Psi_{stat}(t,k_\tau,\mathbf{k}_m,k_{hx},V)+i\frac{\pi}{4}} = \\ &\bar{A}(t,\mathbf{k}_m,k_{hx},\tau) e^{i\Psi_{stat}(t,k_\tau,\mathbf{k}_m,k_{hx},V)+i\frac{\pi}{4}} = \\ &e^{i\Psi_{stat}(t,k_\tau,\mathbf{k}_m,k_{hx},V)+i\frac{\pi}{4}}, \end{aligned} \qquad \text{(B-5)}$$

where we dropped the amplitude factor $\bar{A}$ because it is function of time t, and where the phase function evaluated along the the stationary path Ψ_{stat} is of the form

$$\Psi_{stat}(t, k_\tau, \mathbf{k}_m, k_{hx}, V) = \omega(k_\tau, \mathbf{k}_m, k_{hx}, V)\, t. \tag{B-6}$$

The new dispersion relation $\omega(k_\tau, \mathbf{k}_m, k_{hx}, V)$ can be derived by finding the stationary path of equation (B-3) with respect to k_{hy}. The expression for this stationary path is

$$k_{hy} = \frac{-\frac{V^2}{4} k_{mx} k_{hx} k_{my}}{k_\tau^2 + \frac{V^2}{4} k_{my}^2}. \tag{B-7}$$

Substituting equation (B-7) into equation (B-3) and after a lot of algebra the dispersion relation $\omega(k_\tau, \mathbf{k}_m, k_{hx}, V)$ can be expressed as the cascade of the two changes of variables: the first corresponds to 2-D prestack migration along the azimuthal direction

$$\omega^2 = \frac{1}{k_{\tau 0}^2} \left(k_{\tau 0}^2 + \frac{V^2}{4} k_{mx}^2 \right) \left(k_{\tau 0}^2 + \frac{V^2}{4} k_{hx}^2 \right) \tag{B-8}$$

and the second corresponds to 2-D poststack migration along the cross-azimuth direction

$$k_{\tau 0}^2 = k_\tau^2 + \frac{V^2}{4} k_{my}^2. \tag{B-9}$$

3D Stolt migration of constant-azimuth prestack data

From the previous results on common-azimuth modeling we can derive an expression for Stolt common-azimuth migration. The reflectivity function can be estimated by

$$\hat{M}(\tau, \mathbf{k}_m) = \int_{-\infty}^{+\infty} dk_{hx} \int_{-\infty}^{+\infty} d\omega\; D_0(\omega, \mathbf{k}_m, k_{hx}) e^{i k_\tau(\omega, \mathbf{k}_m, k_{hx}, V)\ \tau + i\frac{\pi}{4}}. \tag{B-10}$$

The expression for Stolt migration is obtained by transforming the integral over ω into an inverse FFT by applying the change of variable

$$\omega = \omega(k_\tau, \mathbf{k}_m, k_{hx}, V), \tag{B-11}$$

and it is given by [equation (6) in the main text]

$$\hat{M}(\tau, \mathbf{k}_m) = \int_{-\infty}^{+\infty} dk_{hx} \int_{-\infty}^{+\infty} dk_\tau \left[\frac{d\omega}{dk_\tau} \right] D_0(\omega(k_\tau, \mathbf{k}_m, k_{hx}, V), \mathbf{k}_m, k_{hx}) e^{i k_\tau \tau + i\frac{\pi}{4}}. \tag{B-12}$$