

## **Two-term approximation to the transmission response through a set of thin layers**

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### **ABSTRACT**

We present a new, far-field solution for approximating the transmission response through a set of thin layers. The method relies on a modification to the O’Doherty & Anstey solution to the transmissivity through a layered medium. We tested the new method on a real log and compared the results to the ODA solution. We show that the approximate solution is quite accurate and thus can be used to provide connections with the equivalent medium theory.

### **INTRODUCTION**

The amplitude of reflections, especially relative amplitude, provides useful information for stratigraphic interpretation. Amplitude information, however, is not easily interpreted because of the numerous factors that affect it. In this work we are concerned with losses caused by transmission through a set of thin layers. A classical paper by O’Doherty and Anstey (1971) showed that the cascaded transmission loss through a great number of interfaces is significant. In geologic terms, they recognized two extreme types of stratification, shown in figures 1 and 2 as acoustic logs. The first log has high frequency variations in seismic impedance laid down by a cyclic pattern of sedimentation. The second log is the product of a transitional sedimentation process leading to a slowly-varying velocity function within basically thick layers. These two types of sedimentation patterns can be described through the autocorrelation function of their reflectivity series computed from the velocity and density logs. Figures 3 and 4 show the autocorrelation functions corresponding to the cyclic and transitional log, respectively. The important difference between the two types is that the first few values of the transitional autocorrelation are positive, whereas in the case of a cyclic sedimentation process, the second value of the auto-correlation is strongly negative. This reports presents a new way of approximating the transmission response through a set of thin layers. It is essentially a far-field approximation to the well-known result of O’Doherty & Anstey (hereinafter ODA). We compare the results of the ODA method and ours when tested on a real log of thin layers. First we describe the full ODA method and then show our modification of this method. We also describe the data preparation and explain the constraints of our model.

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Figure 1: Synthetic impedance log with cyclical deposition  
 nizar1-impedcyc [NR]

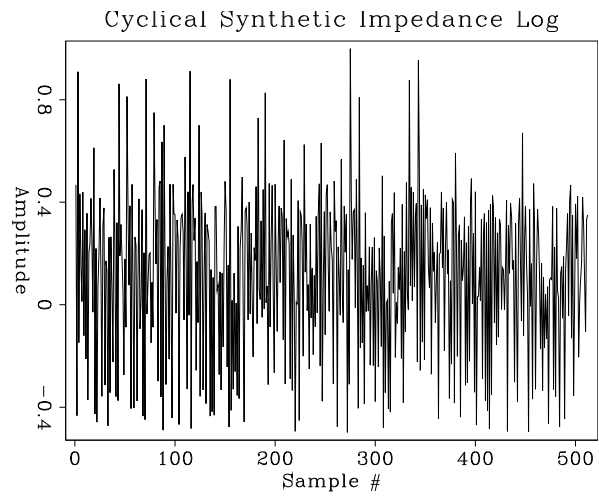


Figure 2: Synthetic impedance log with transitional deposition  
 nizar1-impedtrans [NR]

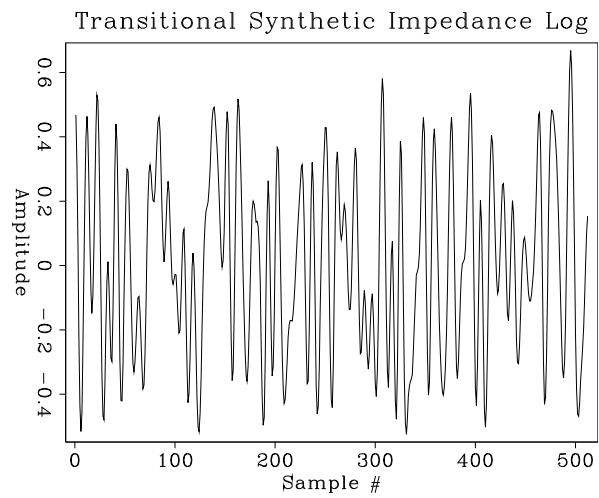
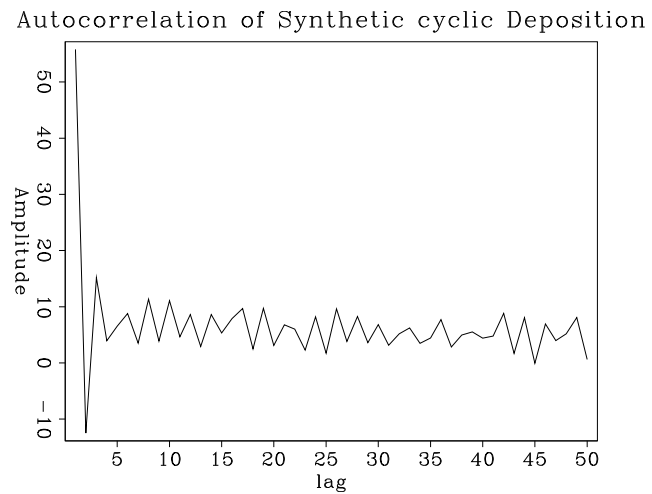


Figure 3: Autocorrelation of the synthetic impedance log with cyclical deposition  
 nizar1-autocyc [NR]



Autocorrelation of Synthetic Transitional Deposition

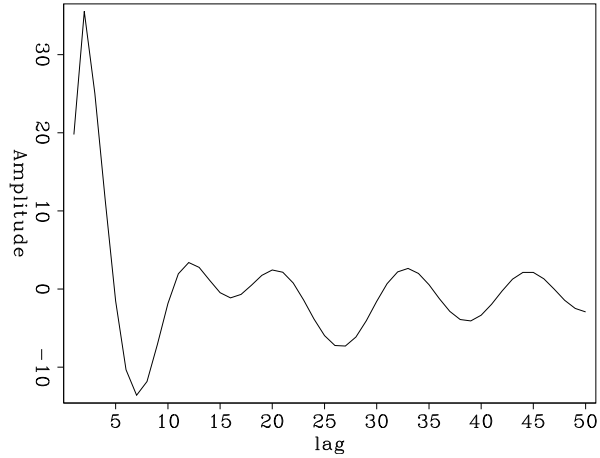


Figure 4: Autocorrelation of synthetic impedance log with transitional deposition `nizar1-autotrans` [NR]

## TRANSMISSION RESPONSE THROUGH THIN LAYERS

### A Two-term approximation

The core of the O'Doherty/Anstey paper is a simple equation relating the transmissivity of a set of equal travel-time layers to the reflection sequence the layers generate. This equation is usefully given in a Z-transform notation which keeps one foot in the time domain and the other in the frequency domain. The ODA result for the retarded transmissivity caused by propagation through a set of layers is:

$$T(z) = e^{-N \cdot R^+(z)}$$

where  $N$  is the number of layers and  $R^+(z)$  is the causal half of the normalized autocorrelation of the reflectivity function in a  $z$ -transform notation.  $R^+(z)$  can now be expanded out into its polynomial representation,

$$T(z) = e^{-N(R_0/2 + \sum_{j=1}^{\infty} R_j z^j)}$$

and we can replace  $z$  with its Fourier representation,

$$T(z) = e^{-N(R_0/2 + \sum_{j=1}^{\infty} R_j e^{i\omega j \Delta t})}$$

and then expand out the exponential in the exponent,

$$T(z) = e^{-N(R_0/2 + \sum_{j=1}^{\infty} R_j \sum_{k=0}^{\infty} (i\omega j \Delta t)^k / k!)}$$

and interchange the order of summation,

$$T(z) = e^{-N(R_0/2 + \sum_{k=0}^{\infty} (i\omega \Delta t)^k / k! \sum_{j=1}^{\infty} R_j j^k)}$$

and expand the summation over  $k$ ,

$$T(z) = e^{-N((R_0/2 + \sum_{j=1}^{\infty} R_j) + i\omega(\Delta t \sum_{j=1}^{\infty} j R_j) - \omega^2(\Delta t^2/2 \sum_{j=1}^{\infty} j^2 R_j) + \dots)}$$

Now recall that reflectivity is a differential process, and if the elastic parameters are stationary in time, then

$$R_0/2 = - \sum_{j=1}^{\infty} R_j$$

and our first, scaling, coefficient goes to zero leaving,

$$T(z) = e^{-N(i\omega(\Delta t \sum_{j=1}^{\infty} j R_j) - \omega^2(\Delta t^2/2 \sum_{j=1}^{\infty} j^2 R_j) + \dots)}$$

We have implemented both ODA solution and the the-term approximated power expansion and compared the solutions for accuracy and convergence on a real data example.

### Reflectivity log from real data

Here we treat the earth as a one-dimensional medium made from the superposition of many layers with varying acoustic properties. A seismic reflection is generated at every geological interface across which there is a contrast of acoustic impedance. The seismic impedance is defined as the product of density and velocity. Well log data can be used to construct a 1-D acoustic earth represented by a reflection coefficient series. For normal incidence, the reflection coefficient is given by

$$r(j) = \frac{\rho_{j+1}v_{j+1} - \rho_j v_j}{\rho_{j+1}v_{j+1} + \rho_j v_j} \quad (1)$$

where  $\rho$  is the density and  $v$  is the acoustic compressional velocity. The density and velocity logs were recorded as a function of depth, so we resampled them in time so that the ODA solution, and the corresponding Muir's approximation, would be valid (both assume an evenly time sampled series). We obtained well log data from a shaly sandstone formation, and we extracted a section that is 512 data points long. The section defines a geologic column of 512 very thin layers. Figures 5 and 6 show the impedance and reflectivity series constructed from the density and sonic logs.

Figure 5: Acoustic impedance series

../nizar1/./Figs/impedance512.pdf

Figure 6: Reflectivity series



Figure 7: Autocorrelation of calculated reflectivity series



### Constraints of the model

Physically, the autocorrelation function of a reflection series is related to the order of multiple reflections within the stratigraphic column. The value of the first lag represents the sum total of all the 2-bounce multiple reflections occurring in layers of one unit of thickness. The value at the second lag represents the sum total of all 2-bounce multiple reflections occurring in layers of two units of thickness. Values at higher lags represent 2-bounce multiples of longer periods (O'Doherty and Anstey, 1971). We have restricted our study to values of the autocorrelation functions at small lags. We applied a tapering function to the autocorrelation function, which leaves the very first few terms intact whereas, higher lag values are replaced by a decaying exponential function. Our earlier assumption of stationarity imposes the constraint that the sum of all reflection coefficients over negative and positive lags sums up to zero.

$$\sum_{-\infty}^{\infty} R_j = 0 \quad (2)$$

Therefore, the damping parameter  $\alpha$  has to be chosen to satisfy the relation

$$\frac{R_o}{2} + \sum_{j=1}^{j=L} R_j + \sum_{j=L+1}^{j=N} e^{-\alpha t} = 0 \quad (3)$$

where  $R_o$  is the value of the autocorrelation at zero lag and  $L$  represents the lag at which the exponential function is initially applied. Figures 7 and 8 show the first 50 lags of the autocorrelation before and after adding the exponential tail. Most of the information in the autocorrelation function is carried by the values at small lags.

Figure 8: Autocorrelation of reflectivity series after applying the exponential taper



### Results

After adding the exponential tail to the autocorrelation, we computed the transmissivity using both the classical ODA approach and Muir's approximation. Figure ?? clearly illustrates that

the approximation is sound. In both approaches the center of the pulse is around the 50th time sample and the Gaussian width/height ratio is approximately the same. The difference between the two approaches, shown in figure 9, is fairly small compared to the amplitude of the transitional responses (less than 5%) and has the shape of a first derivative of the Gaussian. The shape indicates a small time shift between the Muir's approximation and the ODA solution.



Figure 9: Difference between ODA method and Muir's approximation

### Remarks on the test data

When looking over the numerous papers on transmission through thin layers (Godfrey et al., 1980),(Muir, 1992),(Muir, 1993),(Oh, 1993), we noticed that almost all the autocorrelations of real logs had a negative value at the first lag. On the other hand, the log we worked on did not have a negative value until the fifth lag. At first we thought that we might be dealing with a log in an area that had predominantly transitional deposition. This hypothesis seemed to be disproven when we looked at the impedance log corresponding to the autocorrelation function (see figure 5). The impedance function did not appear to have a significant low-frequency component that would correspond to transitional layering. A second hypothesis is that we are dealing with an area whose primary depositional pattern is cyclical, but the break between its transitional and cyclical component occurs at a higher thickness than we normally see (4ft to 1ft). This might be caused by a higher than normal rate of deposition (such as a time period in which flooding occurred much more frequently). Or possibly by a period in which the climate and sea level remained relatively constant causing the rate of facies change to decrease. A further possibility can be seen by comparing the density and the velocity logs. Namely, the density and the velocity logs did not appear to be perfectly aligned. This misalignment could

explain some of the unusual behavior of our autocorrelation function. Figure 10 shows the autocorrelation function assuming constant density; we notice that the autocorrelation has a higher frequency content and a negative value is obtained at the third lag.



Figure 10: Autocorrelation calculated assuming constant density

## CONCLUSIONS

The loss of amplitude associated with transmission through a set of layers is directly related to stratigraphy. Cyclic sequences result in large transmission losses caused by the effects of very-short-delay multiple reflections. The transmission response of a set of thin layers is calculated using a two-term far-field solution. The method relies on a modification to the O'Doherty & Anstey solution to the transmissivity through a layered medium. Results from a test on a real log showed that the two-term approximation provides a very close approximation to the ODA solution. The advantage of this approximation is that it can provide connections with equivalent medium theory.

## ACKNOWLEDGEMENTS

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