

An investigation into eliminating surface multiples

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ABSTRACT

Since the earliest days of the Stanford Exploration Project, the elimination of free-surface marine multiples has been a target of processing research. In recent years the plane-wave methods of Riley and Claerbout (1974) have been superseded by fully two- and three-dimensional extensions of the method at Delft University (Verschuur et al., 1988). Alternative derivations of the newer approach have also been put forth by Dragoset and McKay (1993) using Kirchhoff integral theory and by Carvalho et al. (1991) based on scattering theory. Interestingly, there is an apparent disagreement among the various approaches regarding the correct formulation of the algorithm.

In this paper we first develop our own mathematical derivation in order to clarify the method. Using Green's identity, the solution of the inhomogeneous acoustic wave equation without free surface reflection is implicitly expressed as a Fredholm integral equation of the second kind. The associated kernels can be found by up-going and down-going wave decomposition. We then show by means of a simple but instructive example that solving the implicit integral equation by a Neumann series is numerically unwise, despite the fact that the series reduces to a finite sum, and argue for a treatment of the implicit relation by optimization methods. Finally we outline our plans to tackle this optimization within the framework of the C++ linear operators (CLOP) machinery recently developed at the SEP.

INTRODUCTION

In the very first SEP report, Riley and Claerbout proposed a seminal idea for elimination of free-surface generated multiple reflections from marine data (Claerbout, 1974; Riley, 1974). For a one-dimensional earth the idea is particularly simple

to grasp: the Z -transform response $C(Z)$ of the earth without the free-surface is illuminated in the real earth by the superposition of a downgoing source $S(Z)$ and the reflected upcoming signal $D(Z) = -U(Z)$. Mathematically this translates into a the purely *algebraic* relationship

$$U = (S - U)C \quad (1)$$

between the recorded data and the impulse response of the desired earth model without the free surface. We can check this formula for the simple case of a water layer over a uniform halfspace. If we denote the two-way vertical traveltime to the seafloor by Z^t , then the upcoming reverberation train caused by an impulsive (plane wave) source is given by

$$U = \frac{rZ^t}{1 + rZ^t} \quad (2)$$

where r is the seafloor reflection coefficient (Backus, 1959). Plugging U into equation (1) along with the impulsive source $S = 1$, we get the equation

$$\frac{rZ^t}{1 + rZ^t} = \left(1 - \frac{rZ^t}{1 + rZ^t}\right) C(Z) \quad (3)$$

which simplifies to the explicit solution

$$rZ^t = C(Z) \quad , \quad (4)$$

the expected primary reflection off the seafloor.

WAVEFIELD SEPARATION

Essential to the foregoing argument is the ability to decompose the recorded pressure data $P(Z)$ into the sum $U(Z) + D(Z)$ of upcoming and downgoing waves. The concept of upcoming and downgoing wave decomposition is powerful in analyzing surface multiples in higher dimensions as well. A number of authors have given algorithms for this decomposition in the special, but often useful, case of total field and/or gradient measurements on one or more parallel datum planes (Filho, 1992; Berkhout, 1985). In this case the derivatives of the up and down components are given by the constant-velocity, one-way wave operator in the frequency-wavenumber domain, and simple phase shifts are used to align the components to a reference datum plane.

Mathematically, the decomposition can be described by appropriate application of Green's identity. The advantage is that by properly choosing the reference wavefield, one can write down the decomposed wavefield by means of an integral equation; the integration only involves surface integrals on either the free surface or the source-receiver plane. While further measurements can improve the stability and accuracy of the decomposition (Sonneland et al., 1986; Monk, 1990), the scheme we use is applicable to conventional marine recording on a single datum plane.

Suppose that, in a source-free region, the recorded pressure data F on S_1 (Figure 1) are to be decomposed into downgoing wave D_1 and upcoming wave U_1 ; that is,

$$F = U_1 + D_1 \quad . \quad (5)$$

Kirchhoff integral theory says the upcoming wave U_0 on S_0 can be related to U_1 by

$$U_0 = g_n U_1 \quad , \quad (6)$$

where g_n is the Rayleigh integral operator of a dipole source distribution on S_1 (Berkhout and Palthe, 1979). (The detailed mathematical derivation will be given in this paper.) Since the total pressure vanishes at the free surface S_0 , the downgoing wave D_0 is equal to the opposite of upcoming wave U_0 . Mathematically,

$$D_0 = -U_0 \quad . \quad (7)$$

Again using the Rayleigh integral of the second kind, the downgoing wave D_1 on S_1 is related to D_0 as follows:

$$D_1 = G_n D_0 \quad (8)$$

with another integral operator G_n (which is actually g_n^* , the adjoint operator of g , for a horizontal datum). Solving for U_1 from relations (6)-(8), we have

$$U_1 = F + G_n g_n U_1 \quad , \quad (9)$$

which leads to

$$U_1 = [I - G_n g_n]^{-1} F \quad . \quad (10)$$

In the section on "Redatuming" we derive the associated quantities G_n and g_n and include the effects of a source in the region between S_0 and S_1 .

DELFT'S SURFACE-RELATED MULTIPLE ELIMINATION

To extend the one-dimensional Noah demultiple algorithm to two (or three) dimensions requires the complete seismic experiment, that is, the data as a function of shot location and receiver position at (or very near) the earth's surface. Heuristically, the response of the earth to upcoming energy reflected from the free surface at any given location can be obtained by convolving that downgoing trace with the recorded impulse response of the earth for a (suitably preprocessed) source at the exact same location. The result is again a simple relationship,

$$U(r, s) \doteq S(s) * C(r, s) - \int U(r', s) * C(r, r') dr' \quad , \quad (11)$$

coupling the impulse response $C(r, s)$ for receiver location r and source location s on the desired earth model with the upcoming wavefield recorded in the presence of the free surface.

A more precise derivation uses a Rayleigh integral of the second kind to perform pressure-to-pressure extrapolation. Denote the inhomogeneous Green's function for the earth without the free surface by $G_0(\mathbf{x}, t; \mathbf{x}', t')$. In this the primed coordinates reflect the source position, the unprimed the receiver location. The Rayleigh integral

tells us the response of the earth to a reflected upcoming wave $-U(\mathbf{x}, t)$ is, neglecting factors of π , given by

$$-\int \int_{S_0} U(\mathbf{x}', t') \frac{\partial G_0}{\partial n}(\mathbf{x}, t; \mathbf{x}', t') d\mathbf{x}' dt' \quad , \quad (12)$$

that is, the reflected wavefield is equivalent to a superposition of dipole sources. If we continue to restrict our analysis to the surface of the earth S_0 , the analogue of formula (1) becomes

$$U(\mathbf{x}, t) = \int S(t') G_0(\mathbf{x}, t; \mathbf{x}_s, t') dt' - \int \int_{S_0} U(\mathbf{x}', t') \frac{\partial G_0}{\partial n}(\mathbf{x}, t; \mathbf{x}', t') d\mathbf{x}' dt' \quad (13)$$

for the relation between the response U of a surface source at location \mathbf{x}_s on the free surface and the desired impulse response without the free surface. This is essentially the Delft formulation of surface-related multiple elimination. Surprisingly, a reciprocal result also holds when we interchange the roles of the wavefields with and without the free surface, as the next section shows.

WESTERN'S SURFACE-RELATED MULTIPLE ELIMINATION

Following the development in Morse and Feshbach (1953), let us consider an acoustic wavefield as shown in Figure 1. At $z = 0$, the sea level is denoted by S_0 ; and at $z = z_r$ the surface S_1 is where the sources and receivers are located. The medium between S_0 and S_1 is purely water of constant wave speed. Receivers collect upcoming signals from subsurface reflection and downgoing waves caused by surface reflection.

The inhomogeneous Green's function $G(\mathbf{x}, t; \mathbf{x}', t')$ satisfies the equation

$$\nabla^2 G(\mathbf{x}, t; \mathbf{x}', t') - \frac{1}{c^2(\mathbf{x})} \frac{\partial^2}{\partial t^2} G(\mathbf{x}, t; \mathbf{x}', t') = -4\pi \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') \quad (14)$$

with boundary conditions

$$G|_{S_0} = 0 \quad (15)$$

and

$$G|_{S_\infty} = 0 \quad . \quad (16)$$

Causality conditions imply that

$$G(\mathbf{x}, t; \mathbf{x}', t') = G_t(\mathbf{x}, t; \mathbf{x}', t') = 0 \quad \text{if } t < t' \quad . \quad (17)$$

The equation of wavefield without free surface reflection is

$$\nabla^2 u(\mathbf{x}, t) - \frac{1}{c^2(\mathbf{x})} \frac{\partial^2}{\partial t^2} u(\mathbf{x}, t) = -4\pi q(\mathbf{x}, t) \quad (18)$$

with the initial condition

$$u(\mathbf{x}, t) = 0 \quad \text{for } t < 0 \quad . \quad (19)$$

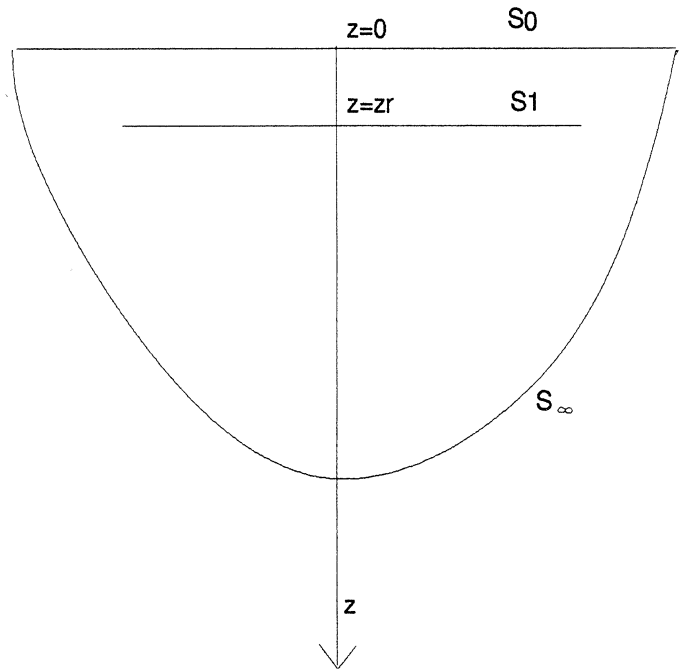


Figure 1: Geometry of the free surface, source, and receivers: S_0 and S_1 denote the free surface and source-receiver plane, respectively. [NR]

Here $u(\mathbf{x}, t)$ is the solution without surface multiples and is what we are interested in. To find u , we apply Green's identity to obtain

$$\begin{aligned}
 u(\mathbf{x}', t') &= \int_0^{t'^+} \int_V G(\mathbf{x}, t'; \mathbf{x}', t) q(\mathbf{x}, t) d\mathbf{x} dt \\
 &+ \frac{1}{4\pi} \int_0^{t'^+} \int_{S_0} (G(\mathbf{x}, t'; \mathbf{x}', t) \frac{\partial u(\mathbf{x}, t)}{\partial n} - u(\mathbf{x}, t) \frac{\partial G(\mathbf{x}, t'; \mathbf{x}', t)}{\partial n}) dS dt \\
 &+ \frac{1}{4\pi} \int_V \frac{1}{c^2(\mathbf{x})} [u(\mathbf{x}, t) G_t(\mathbf{x}, t'; \mathbf{x}', t) - G(\mathbf{x}, t'; \mathbf{x}', t) u_t(\mathbf{x}, t)]_0^{t'^+} d\mathbf{x} . \quad (20)
 \end{aligned}$$

Since the time interval of integration includes the final point t' , a plus sign is added for clarity.

The first term is the solution with free surface subjected to source function $q(\mathbf{x}, t)$. Its value on surface S_1 is simply the recorded data. The third term vanishes because of the initial conditions of $u(\mathbf{x}, t)$ and the causality conditions of $G(\mathbf{x}, t; \mathbf{x}', t')$. Since G is zero on free surface, there's only one term left in surface integral.

Approximately, the source function is of the form

$$q(\mathbf{x}, t) = f(t) \delta(\mathbf{x} - \mathbf{x}_s) , \quad (21)$$

where \mathbf{x}_s is the source location on S_1 . We can then rewrite equation (20) as

$$u(\mathbf{x}', t') = \int_0^{t'^+} G(\mathbf{x}_s, t'; \mathbf{x}', t) f(t) dt - \frac{1}{4\pi} \int_0^{t'^+} \int_{S_0} u(\mathbf{x}, t) \frac{\partial}{\partial n} G(\mathbf{x}, t'; \mathbf{x}', t) dS dt . \quad (22)$$

The first term on RHS of equation (22) is the solution with free surface. The second term, according to Rayleigh's integral, is the wave propagating down with surface source function $u(\mathbf{x}, t)$ on S_0 . Equation (22) thus states that the solution without free surface is the solution with free surface minus the surface reflection. This we identify as the formulation of Dragoset and McKay (1993).

It is useful to check the 1-D analogue of this relationship for the seismogram (2). The claim is that for an impulsive source $S(Z) = 1$

$$C(Z) = U(Z) - C(Z)U(Z) \quad , \quad (23)$$

which is simply a rearrangement of equation (1).

SCATTERING FORMULATION USING GREEN'S IDENTITY

A third derivation of surface-related multiple attenuation is obtained from scattering theory (Carvalho et al., 1991). In that approach, all multiples, both free-surface and interbed, appear as terms in an infinite expansion. Each individual term prescribes one or more volume integrations over the whole subsurface. The terms directly related to the free-surface, however, have the special property that they degenerate into surface integrals.

In this section, we present a streamlined derivation of surface-related multiple elimination from scattering theory. This leads to the Western formulation of surface-related multiple elimination. For notational ease, the development is taken in Fourier domain.

Define two acoustic operators associated with different space-dependent velocity functions to be

$$L_A = \nabla^2 + k_A^2(\mathbf{x}) \quad (24)$$

and

$$L_B = \nabla^2 + k_B^2(\mathbf{x}) \quad , \quad (25)$$

where

$$k_A(\mathbf{x}) = \frac{\omega^2}{c_A^2(\mathbf{x})} \quad (26)$$

and

$$k_B(\mathbf{x}) = \frac{\omega^2}{c_B^2(\mathbf{x})} \quad . \quad (27)$$

Let G_A and G_B be solutions of

$$L_A G_A(\mathbf{x}, \mathbf{x}', \omega) = -4\pi \delta(\mathbf{x} - \mathbf{x}') \quad (28)$$

and

$$L_B G_B(\mathbf{x}, \bar{\mathbf{x}}, \omega) = -4\pi \delta(\mathbf{x} - \bar{\mathbf{x}}) \quad . \quad (29)$$

Then by applying Green's identity to $G_B L_A G_A - G_A L_B G_B$ and taking the domain of integration to be the lower half plane bounded above by the free surface S_0 , we have

$$G_A - G_B = -\frac{1}{4\pi} \int_{S_0} \left(G_A \frac{\partial G_B}{\partial n} - G_B \frac{\partial G_A}{\partial n} \right) dS - \frac{1}{4\pi} \int_V G_A \hat{V} G_B dV \quad (30)$$

where

$$\hat{V} = k_B^2 - k_A^2 \quad . \quad (31)$$

\hat{V} is termed the scattering potential and is formally defined as

$$\hat{V} = L_B - L_A \quad . \quad (32)$$

Equation (30) relates two different wavefields by surface and subsurface reflections. This equation is very general. By properly choosing the wavefields G_A and G_B and the domain of integration, we can cancel selected terms. If the surface integral is eliminated, the equation reduces to the Lippmann-Schwinger equation. (Carvalho et al. work from this equation using the homogeneous Green's function with free surface and the inhomogeneous Green's function with free surface.) Eliminating the volume integral will lead us to Western's formulation and if we take the values on receiver plane S_1 , it is identical to the scheme we develop in this paper. There is a case where the individual terms G_A and G_B as well as the volume integral would disappear; the equation then becomes the reciprocity relation $\langle g_n, G \rangle = \langle G_n, g \rangle$ which is the one we derive in this paper. The following subsections explore these options in more detail.

Lippmann-Schwinger equation

Suppose we choose G_A to be the actual inhomogeneous Green's function with free surface S_0 , that is,

$$G_A(\mathbf{x}, \mathbf{x}', \omega) = \tilde{G}(\mathbf{x}, \mathbf{x}', \omega) \quad , \quad (33)$$

where \tilde{G} denotes the Fourier transform of $G(\mathbf{x}, t; \mathbf{x}', t')$. If we now set G_B to be a known reference wavefield \tilde{G}_r , also vanishing on surface S_0 , then the surface integral of equation (30) vanishes yielding

$$\tilde{G} = \tilde{G}_r - \frac{1}{4\pi} \int_V \tilde{G} \hat{V} \tilde{G}_r dV \quad , \quad (34)$$

or symbolically

$$\tilde{G} = \tilde{G}_r + \langle \tilde{G}_r | \hat{V} | \tilde{G} \rangle \quad . \quad (35)$$

(Note by specifying G_A and G_B , we mean the associated operators also change accordingly.) This is so-called the Lippmann-Schwinger equation which relates the actual wavefield and the reference wavefield. The integral term in equation (34) takes effect whenever \hat{V} is different from zero; that is, there will be subsurface contributions wherever the actual velocity profile is different from the reference one. It then states that the actual solution is the sum of the reference wavefield and subsurface interactions.

Western formulation

Let G_A be \tilde{G} and G_B the inhomogeneous Green's function $\tilde{G}_0(\mathbf{x}, \mathbf{x}', w)$ without free surface. Since there's no difference between these two velocity profiles, the volume integral in equation (30) vanishes. The free surface condition cancels the first term of the boundary integral, leaving the simple relation

$$\tilde{G}_0 = \tilde{G} - \frac{1}{4\pi} \int_{S_0} \tilde{G}_0 \frac{\partial \tilde{G}}{\partial n} dS \quad . \quad (36)$$

If we post-multiply this last equation by the source function $\tilde{f}(\omega)$ and denote by \tilde{u} the Fourier transform of the wavefield without the free surface u :

$$\tilde{u}(\mathbf{x}, \omega) = \tilde{G}_0(\mathbf{x}_s, \mathbf{x}, \omega) \tilde{f}(\omega) \quad , \quad (37)$$

then equation (36) becomes

$$\tilde{u} = \tilde{G} \tilde{f} - \frac{1}{4\pi} \int_{S_0} \tilde{u} \frac{\partial \tilde{G}}{\partial n} dS \quad . \quad (38)$$

This is the Fourier transform of equation (22), the Western formulation.

Reciprocity relation

To obtain our reciprocity relation, take G_A to be \tilde{G} and G_B to be \tilde{g} , the homogeneous Green's function g in frequency domain. Specify the values of \tilde{G} and \tilde{g} on S_0 and S_1 respectively to vanish on the boundaries. Since the medium is the same between S_0 and S_1 , the volume integral in equation (30) is zero when the domain of integration is taken to be between these two surfaces. What left is only the surface integral. Segregating the contributions of the two planes, we have

$$\langle G_n, g \rangle_{S_0} = \langle G, g_n \rangle_{S_1} \quad , \quad (39)$$

which is our reciprocity formula (54).

REDATUMING

We do not record our total pressure at the free surface, of course. Nor do we inject an impulsive source there either. This section reformulates our relationship (22) for pressure recorded on datum S_1 with a source located in the region between S_1 and S_0 , the typical marine configuration.

If we restrict \mathbf{x}' to the datum plane S_1 , the first term in equation (22) is the recorded data. (Note that $G(\mathbf{x}_s, t'; \mathbf{x}', t) = G(\mathbf{x}', t'; \mathbf{x}_s, t)$ and the first term is just the time convolution of Green's function and the source function.) Symbolically equation (22) is written as

$$u_1 = F + G_n u_0 \quad , \quad (40)$$

where u_1 and u_0 denote wavefields on S_0 and S_1 , F is known data, and G_n the associated integral operator.

The next step is to find u_0 in terms of u_1 . This can be done by applying Green's identity again.

The domain of interest is the upper half space bounded below by surface S_1 . The wave velocity c_0 is constant in the domain, and the homogeneous Green's function $g(\mathbf{x}, t; \mathbf{x}', t')$ of interest to us is defined as the solution of

$$\nabla^2 g - \frac{1}{c_0^2} \frac{\partial^2 g}{\partial t^2} = -4\pi \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') + 4\pi \delta(\mathbf{x} - \mathbf{x}^*) \delta(t - t') \quad , \quad (41)$$

where \mathbf{x}' is located in the region between S_0 and S_1 and \mathbf{x}^* is the image source location such that g vanishes on surface S_1 . Manipulating u and g following the same procedure yields

$$u(\mathbf{x}', t') = \frac{-1}{4\pi} \int_0^{t'+} \int_{S_1} u(\mathbf{x}, t) \frac{\partial g(\mathbf{x}, t; \mathbf{x}', t')}{\partial n} dS dt \quad . \quad (42)$$

If \mathbf{x}' is taken to be on S_0 , equation (42) can be symbolically written as

$$u_0 = g_n u_1 \quad . \quad (43)$$

The implicit integral equation then becomes

$$u_1 = F + G_n g_n u_1 \quad , \quad (44)$$

which is a Fredholm integral equation of the second kind.

Evaluation of G_n

The domain of interest in evaluating G_n is the region bounded by surface S_0 and S_1 . Since the source locations are restricted to be on S_1 , the wave equation of G becomes

$$\nabla^2 G(\mathbf{x}, t; \mathbf{x}', t') - \frac{1}{c_0^2} \frac{\partial^2 G(\mathbf{x}, t; \mathbf{x}', t')}{\partial t^2} = 0 \quad (45)$$

with boundary conditions

$$G|_{S_0} = 0 \quad (46)$$

$$G|_{S_1} = G_{S_1} \quad (47)$$

and causality conditions

$$G(\mathbf{x}, t; \mathbf{x}', t') = G_t(\mathbf{x}, t; \mathbf{x}', t') = 0 \text{ if } t < t' \quad . \quad (48)$$

Note that $\mathbf{x}' \in S_1$ and G_{S_1} is a known function.

Next we take g defined in equation (41) as the reference field and rewrite it as

$$\nabla^2 g - \frac{1}{c_0^2} \frac{\partial^2 g}{\partial t^2} = -4\pi \delta(\mathbf{x} - \bar{\mathbf{x}}) \delta(t - \bar{t}) + 4\pi \delta(\mathbf{x} - \bar{\mathbf{x}}^*) \delta(t - \bar{t}) \quad . \quad (49)$$

The image source at $\mathbf{x} = \bar{\mathbf{x}}^*$ ensures that g vanishes on S_1 . Note also that $\bar{\mathbf{x}}$ is in between S_0 and S_1 , and $\bar{\mathbf{x}}^*$ is below S_1 . In addition to the causality conditions

$$g(\mathbf{x}, t; \bar{\mathbf{x}}, \bar{t}) = g_t(\mathbf{x}, t; \bar{\mathbf{x}}, \bar{t}) = 0 \text{ if } t < \bar{t} \quad , \quad (50)$$

we impose the condition that the initiation time of g is later than the time of G , that is,

$$g(\mathbf{x}, t; \bar{\mathbf{x}}, \bar{t}) = 0 \text{ if } \bar{t} < t' \quad . \quad (51)$$

We then apply Green's identity and take the integration domain V' between S_0 and S_1 . This produces the integral relation

$$\begin{aligned} & \int_{t'-}^{\bar{t}^+} \int_{V'} g(\mathbf{x}, \bar{t}; \bar{\mathbf{x}}, t) (\nabla^2 G(\mathbf{x}, t; \mathbf{x}', t') - \frac{1}{c_0^2} \frac{\partial^2 G}{\partial t^2}) - G(\nabla^2 g - \frac{1}{c_0^2} \frac{\partial^2 g}{\partial t^2}) d\mathbf{x} dt \\ &= \int_{t'-}^{\bar{t}^+} \int_S g \frac{\partial G}{\partial n} - G \frac{\partial g}{\partial n} dS dt - \frac{1}{c_0^2} \int_{V'} [g G_t - G g_t]_{t'-}^{\bar{t}^+} d\mathbf{x} \quad . \end{aligned} \quad (52)$$

The last term vanishes because of the causality conditions and equation (51). S is composed of S_1 and S_0 . Rewriting equation (52), noting the zero boundary conditions of G and g on S_0 and S_1 , we have

$$\begin{aligned} & \int_{t'-}^{\bar{t}^+} \int_{S_0} g(\mathbf{x}, \bar{t}; \bar{\mathbf{x}}, t) \frac{\partial G}{\partial n}(\mathbf{x}, t; \mathbf{x}', t') dS dt \\ &= 4\pi G(\bar{\mathbf{x}}, \bar{t}; \mathbf{x}', t') + \int_{t'-}^{\bar{t}^+} \int_{S_1} G(\mathbf{x}, t; \mathbf{x}', t') \frac{\partial g}{\partial n}(\mathbf{x}, \bar{t}; \bar{\mathbf{x}}, t) dS dt \quad . \end{aligned} \quad (53)$$

Since \mathbf{x}' is on S_1 , for clarity we replace it by \mathbf{x}_1 . If we restrict $\bar{\mathbf{x}}$ to be on S_0 and denote it by \mathbf{x}_0 , $G(\mathbf{x}_0, \bar{t}; \mathbf{x}_1, t')$ is zero. In that case equation (53) becomes

$$\int_{t'-}^{\bar{t}^+} \int_{S_0} g(\mathbf{x}, \bar{t}; \mathbf{x}_0, t) \frac{\partial G}{\partial n}(\mathbf{x}, t; \mathbf{x}_1, t') dS dt = \int_{t'-}^{\bar{t}^+} \int_{S_1} G(\mathbf{x}, t; \mathbf{x}_1, t') \frac{\partial g}{\partial n}(\mathbf{x}, \bar{t}; \mathbf{x}_0, t) dS dt \quad . \quad (54)$$

We can obtain $\frac{\partial G}{\partial n}$ if we notice that the inverse operator of the homogeneous Green's function g is the wave operator itself. Thus

$$\frac{\partial G}{\partial n}(\mathbf{x}_0, \bar{t}; \mathbf{x}_1, t') \delta_{S_0} = \int_{t'-}^{\bar{t}^+} \int_{S_1} G(\mathbf{x}, t; \mathbf{x}_1, t') L_{(\mathbf{x}_0, \bar{t})} \frac{\partial g_1}{\partial n}(\mathbf{x}, \bar{t}; \mathbf{x}_0, t) dS dt \quad , \quad (55)$$

where g_1 is the homogeneous free space Green's function without the image source, and $L = \nabla^2 - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2}$.

BINOMIAL EXPLOSION

One direct way to solve for the data without free-surface multiples is to use a Neumann series expansion in terms of the (deconvolved) data with free-surface reflections. The key observation of Verschuur et al. is that, owing to the finite recording length of the data and the delay between the source initiation and the first reflected

arrival, the infinite series is actually finite. What is perhaps less clear is that the Neumann expansion is a severely unstable process!

To see this, we again consider the 1-D example from equation (2). The Neumann series for the solution of equation (1) takes the form

$$C = \hat{U} + \hat{U}^2 + \hat{U}^3 + \hat{U}^4 + \dots \quad (56)$$

where \hat{U} is the deconvolved seismogram $S^{-1}U$. Suppose we want to use the first few terms of this series to suppress multiple energy. Expanding the series in terms of the seismogram given in equation (2) produces the sum of seismograms:

$$\begin{array}{rcccccccc} rZ^t & -r^2Z^{2t} & +r^3Z^{3t} & -r^4Z^{4t} & +r^5Z^{5t} & -r^6Z^{6t} & +r^7Z^{7t} & -\dots \\ + & r^2Z^{2t} & -2r^3Z^{3t} & +3r^4Z^{4t} & -4r^5Z^{5t} & +5r^6Z^{6t} & -6r^7Z^{7t} & +\dots \\ & & +r^3Z^{3t} & -3r^4Z^{4t} & +6r^5Z^{5t} & -10r^6Z^{6t} & +15r^7Z^{7t} & -\dots \\ & & & +r^4Z^{4t} & -4r^5Z^{5t} & +10r^6Z^{6t} & -20r^7Z^{7t} & +\dots \\ & & & & +r^5Z^{5t} & -5r^6Z^{6t} & +5r^7Z^{7t} & -\dots \\ & & & & & +r^6Z^{6t} & -6r^7Z^{7t} & +\dots \\ & & & & & & +r^7Z^{7t} & -\dots \end{array} \quad (57)$$

whose partial sums are

$$\begin{array}{rcccccccc} S_0 & = & rZ^t & -r^2Z^{2t} & +r^3Z^{3t} & -r^4Z^{4t} & +r^5Z^{5t} & -r^6Z^{6t} & +r^7Z^{7t} & -\dots \\ S_1 & = & rZ^t & & -r^3Z^{3t} & +2r^4Z^{4t} & -3r^5Z^{5t} & +4r^6Z^{6t} & -5r^7Z^{7t} & +\dots \\ S_2 & = & rZ^t & & & -r^4Z^{4t} & +3r^5Z^{5t} & -6r^6Z^{6t} & +10r^7Z^{7t} & -\dots \\ S_3 & = & rZ^t & & & & -r^5Z^{5t} & +4r^6Z^{6t} & -10r^7Z^{7t} & +\dots \\ S_4 & = & rZ^t & & & & & -r^6Z^{6t} & +5r^7Z^{7t} & -\dots \\ S_5 & = & rZ^t & & & & & & -r^7Z^{7t} & +\dots \\ S_6 & = & rZ^t & & & & & & & -\dots \end{array} \quad (58)$$

which eventually produces the desired seismogram rZ^t , but only after an initial wildly oscillating amplification of higher order multiples. In general, the column of coefficients for $r^m Z^{mt}$ in (57) is the series of binomial coefficients $(-1)^{m-1-j} \binom{m-1}{j}$. The sums can be evaluated explicitly using the relation

$$\sum_{k=0}^m (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m} . \quad (59)$$

Applying this relation to the m 'th order reflection, we find, using the central limit theorem, that the maximum partial sum has the approximate magnitude $2^{m-2} r^m / \sqrt{2\pi(m-2)}$, which has overcompensated the actual multiple magnitude r^m by a gigantic factor.

Vincent Broman (broman@nosc.mil), in response to a na-net query on whether an alternative polynomial expansion could be used, e.g. Chebyshev polynomials, showed that *no* data-independent polynomial expansion can keep the higher order multiples from being overestimated. He calculated that given any proposed polynomial

$$P(U) = a_0 U^0 + a_1 U^1 + a_2 U^2 + \dots + a_m U^m , \quad (60)$$

the coefficient of Z^{nt} is given by

$$r^n \sum_{k=1}^m a_k - 1^{n-k} \binom{n-1}{k-1} \quad (61)$$

which clearly increases for large n .

This result tells us that although the Fredholm equation can be solved by Neumann's series, doing so is not a particularly good way to approach the problem. Instead a data-adaptive method must be used. In addition, the finite aperture of our recorded data introduces further artifacts. Therefore we suggest that instead of attacking an equation, it is better to find u_1 minimizing $\|(G_n g_n - I)u_1 + F\|$. This optimization problem can be tackled using any of a variety of algorithms. Right now, though, we do not know which one will be most robust. The thing to do first, then, is to implement the basic linear operations in our formulas. By casting them as C++ building blocks, we can use the CLOP tools (Nichols et al., 1993) to implement any of a wide variety of linear solvers and optimization methods without rewriting the geophysical subroutines each time. To this end, a student in Gene Golub's SCCM program will be working with SEP during the spring quarter to resurrect and enhance the CLOP framework and toolkit.

CONCLUSIONS

In this study we have clarified the theoretical basis for surface-related multiple elimination. We have shown that the Delft and Western formulations are actually different, not contradictory, and have adapted them to handle the case where the sources and receivers are on a datum plane below the free surface.

By means of a simple 1-D example we showed that the Neumann series algorithm for surface-related multiple suppression is not a good choice for data processing and suggest alternatives that we intend to explore.

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