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Short Note

Prestack migration by split-step DSR

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INTRODUCTION

The DSR prestack migration equation, though defined for depth variable velocity, can be used to image media with strong velocity variations using a phase-shift plus interpolation (PSPI) or split-step correction. The split-step method is based on applying a phase-shift correction to the extrapolated wavefield, a correction that attempts to compensate for the lateral velocity variations. I show how to extend DSR prestack migration to lateral velocity media and exemplify the method by applying the new algorithm to the Marmousi dataset.

Split-step prestack migration

The depth-variable-velocity **prestack migration** in offset-midpoint coordinates (Yilmaz, 1979) is formulated as

$$p(t=0,k_y,h=0,z) = \int d\omega \int dk_h \; e^{i\sum_i k_{zi}(\omega,k_y,k_h)\Delta z_i}\, p(\omega,k_y,k_h,z=0), \tag{1}$$

where $p(\omega,k_y,k_h,z=0)$ is the 3-D Fourier transform of the field $p(t,y,h,z=0)$ recorded at the surface, using Claerbout's (1985) sign convention:

$$p(\omega,k_y,k_h,z=0) = \int dt\; e^{i\omega t} \int dy\; e^{-ik_y y} \int dh\; e^{-ik_h h}\, p(t,y,h,z=0).$$

The phase $k_z(\omega,k_y,k_h)$ is defined in the dispersion relation as

$$k_z(\omega,k_y,k_h) \equiv -\mathrm{sign}(\omega)\left[\sqrt{\frac{\omega^2}{v^2}-\frac{1}{4}(k_y+k_h)^2}+\sqrt{\frac{\omega^2}{v^2}-\frac{1}{4}(k_y-k_h)^2}\right]. \tag{2}$$

The two integrals in ω and k_h in equation (1) represent the imaging condition for zero-offset and zero time ($h=0, t=0$). In parallel, the depth-variable-velocity **zero-offset migration** (Gazdag, 1978) is formulated as

$$p(t=0,k_y,z) = \int d\omega_0\; e^{i\sum_i k_{zi}(\omega_0,k_y)\Delta z_i}\, p(\omega_0,k_y,z=0) \tag{3}$$

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where $p(\omega_0, k_y, z=0)$ is the 2-D Fourier transform of the field $p(t, y, z=0)$. The phase $k_z(\omega_0, k_y)$ is defined in the dispersion relation as

$$k_z(\omega_0, k_y) \equiv -2\ \mathrm{sign}(\omega_0)\sqrt{\frac{\omega_0^2}{v^2} - \frac{k_y^2}{4}}. \tag{4}$$

For zero-offset migration, the idea of a split-step correction appeared first in Gazdag and Sguazzero's (1984) PSPI algorithm, but it did not become a standalone technique until Stoffa et al. (1990) reversed the order of the algorithm, permitting the correction to be applied only once. The schematic flow of the two algorithms is shown in Figure **??**, and the split-step correction is represented by the box containing the $e^{-\frac{\omega}{v(x,z)}\Delta z}$ term. Gazdag and Sguazzero (1984)

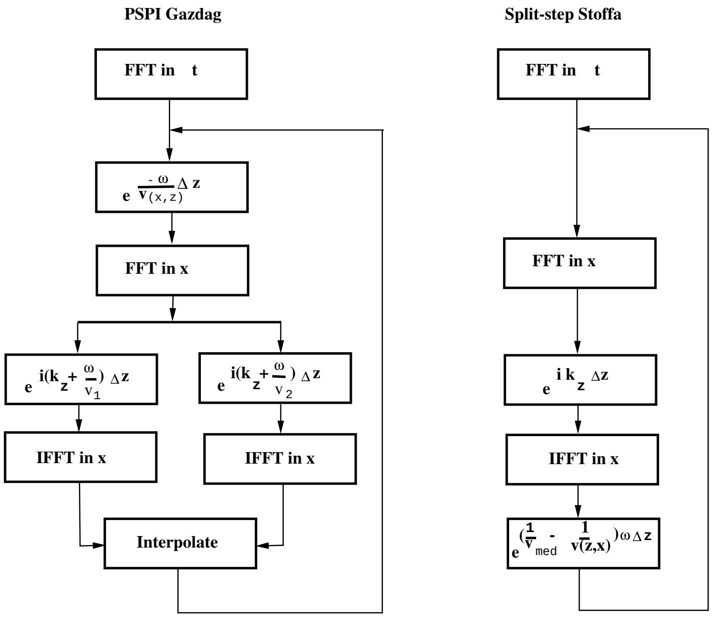

Figure 1: The two Fourier migration algorithms.
a) Phase Shift Plus Interpolation (PSPI).
b) Split-step Fourier Migration. mihai1-gazsplit [NR]

implement a technique in the PSPI algorithm to ensure that all the zero dips (corresponding to the case $k_x = 0$) are downward continued without distortion. The technique consists of multiplying the wavefield $P(x, z, \omega)$ with the phase-shift correction factor

$$e^{-i\frac{\omega}{v(x,z)}\Delta z}$$

prior to the Fourier transformation along the X axis. The downward extrapolation phase incorporates another correction factor

$$e^{i\frac{\omega}{v_1}\Delta z}$$

where the subscript v_1 denotes one of the constant velocities used in the downward extrapolation step. When the velocity varies only in depth $v(z)$, the two terms cancel each other. When the reflectors are flat ($k_x = 0$), the extrapolation term becomes

$$e^{-i\sqrt{\frac{\omega^2}{v^2}-k_x^2}\Delta z} = e^{-i\frac{\omega}{v}\Delta z}$$

and is cancelled by the multiplication with the phase-shift correction factor

$$e^{i\frac{\omega}{v_1}\Delta z}.$$

As a result, the zero-dip reflections are downward continued with the correct laterally varying phase-shift

$$e^{-i\frac{\omega}{v(x,z)}\Delta z}.$$

The same result can be obtained via the Stoffa et al. (1990) algorithm, where the phase-correction term is contained in a single term, as follows:

$$e^{i(\frac{1}{vm}-\frac{1}{v(x,z)})\omega\Delta z}.$$

The difference is that the downward extrapolation is done with a single velocity and the perturbation slowness phase shift is performed after the downward extrapolation step. For prestack migration, the two square roots in the phase term represent specifically the extrapolation term for the source and for the receiver. In other words if we downward extrapolate first the source, and second the receiver, we need in each case a different phase-correction term. The phase-correction term includes a laterally varying velocity function, $v_g(x_g,z)$ for the geophone-field continuation and $v_s(x_s,z)$ for the source-field continuation. A minor problem arises in the fact that the DSR prestack migration equation (1) is defined in midpoint and offset coordinates, while the velocity is defined in shot and geophone coordinates. We can easily go from one system to another using the transformation relations

$$\begin{aligned} y &= \frac{x_g+x_s}{2} \\ h &= \frac{x_g-x_s}{2}, \end{aligned} \tag{5}$$

where y and h are the midpoint and offset coordinates and x_s and x_g are the shot and geophone surface coordinates. Note that the variable h represents half the total distance between the source and geophone. The split-step correction term becomes

$$e^{i(\frac{2}{vm}-\frac{1}{vg(x_s,z)}-\frac{1}{vg(x_g,z)})\omega\Delta z}.$$

In midpoint-offset coordinates the split-step correction term becomes

$$e^{i(\frac{2}{vm}-\frac{1}{v(y-h,z)}-\frac{1}{v(y+h,z)})\omega\Delta z}.$$

In other words, for each midpoint y, we use the velocity located at $y-h$ (the source location) and the velocity located at $y+h$ (the receiver location). A diagram of the prestack migration

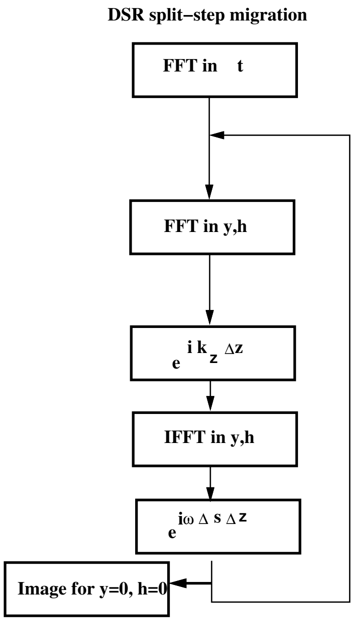

Figure 2: DSR split-step prestack migration algorithm. mihai1-DSRsplit [NR]

split-step algorithm is represented in Figure **??**. The last box represents the multiplication with the split-step correction exponential term, where the slowness difference Δs is defined as

$$\Delta s(y,h,z) = \frac{2}{v_m} - \frac{1}{v(y-h,z)} - \frac{1}{v(y+h,z)}.$$

The DSR split-step migration algorithm was applied to the Marmousi data, a well known prestack synthetic dataset generated by the Institut Français du Pétrole. The Marmousi dataset is based on a real geologic model from the Cuanza basin in Angola (Aline Bourgeois et al, 1991). The geological model of the basin consists of a deltaic sediment interval deposited upon a saliferous evaporitic series. The sediments are affected by normal growth faults caused by the salt creep. Under the salt there is a folded carbonate sedimentation series forming a structural hydrocarbon trap. The challenge presented to the exploration geophysicists it to image the hydrocarbon trap. The complex velocity model, with strong lateral velocity variations, is shown in Figure **??**a. Figure **??**b shows the near-offset section with a half-offset $h = 100m$. Figure **??**a shows the result of zero-offset split-step migration applied to the near-offset section. The full near offset is 200 meters (h=100 m). Applying a zero-offset algorithm to a non-zero offset section obviously introduces errors in the imaging results, and the target area is poorly imaged. In comparison, the DSR split-step prestack migration algorithm produces an excellent image of the hydrocarbon trap, shown in Figure **??**b.

The size of the prestack migration dataset used was 720x256x64 and was increased to 1024x256x64 by zero padding for the fast Fourier transform. The prestack migration took about 20 minutes on a 1024x256x64 dataset. The prestack migration took about 20 minutes on our CM-5 when using the entire frequency spectrum (1024 frequencies). The runtime can be effectively cut by a factor of two by using only half of the total number of frequencies. My implementation also sums implicitly over the offset. By using the artifact reduction techniques described in Popovici (1993) and stationary phase tables, the algorithm can be modified to handle separate offsets.

CONCLUSIONS

A very simple split-step modification to DSR prestack migration leads to a powerful migration algorithm that can handle strong lateral velocity variations. The new algorithm produces very good images of the Marmousi dataset and is computationally very efficient on our CM-5.

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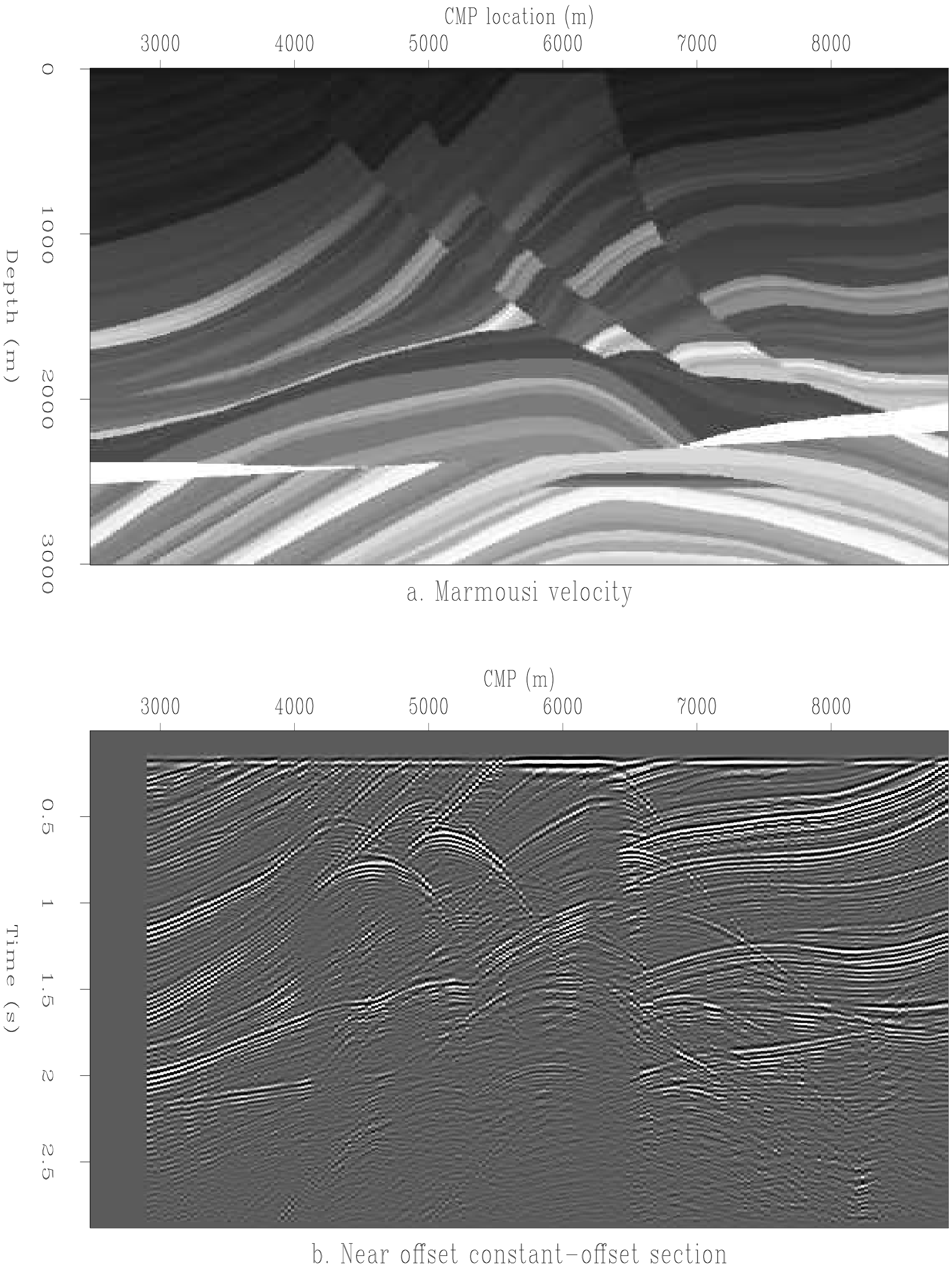
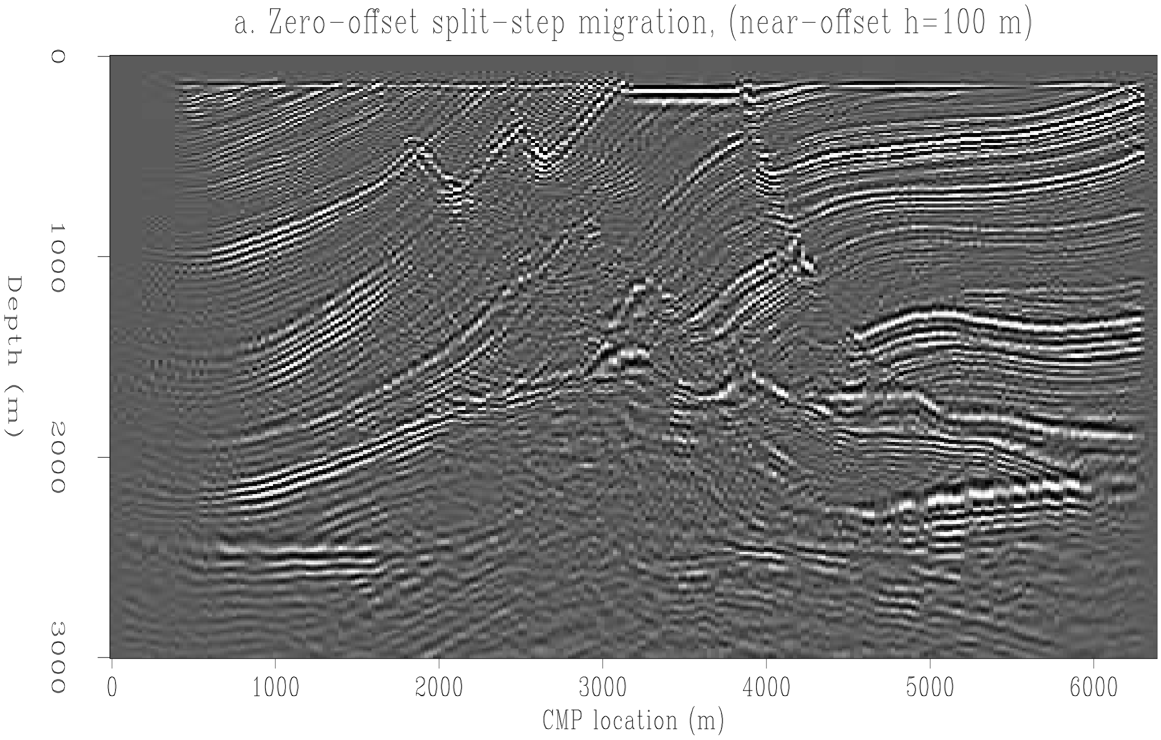


Figure 3: Marmousi velocity model and near-offset section.
a. Marmousi velocity model.
b. Near-offset section (h=100 m). mihai1-marmVelandNO [NR]



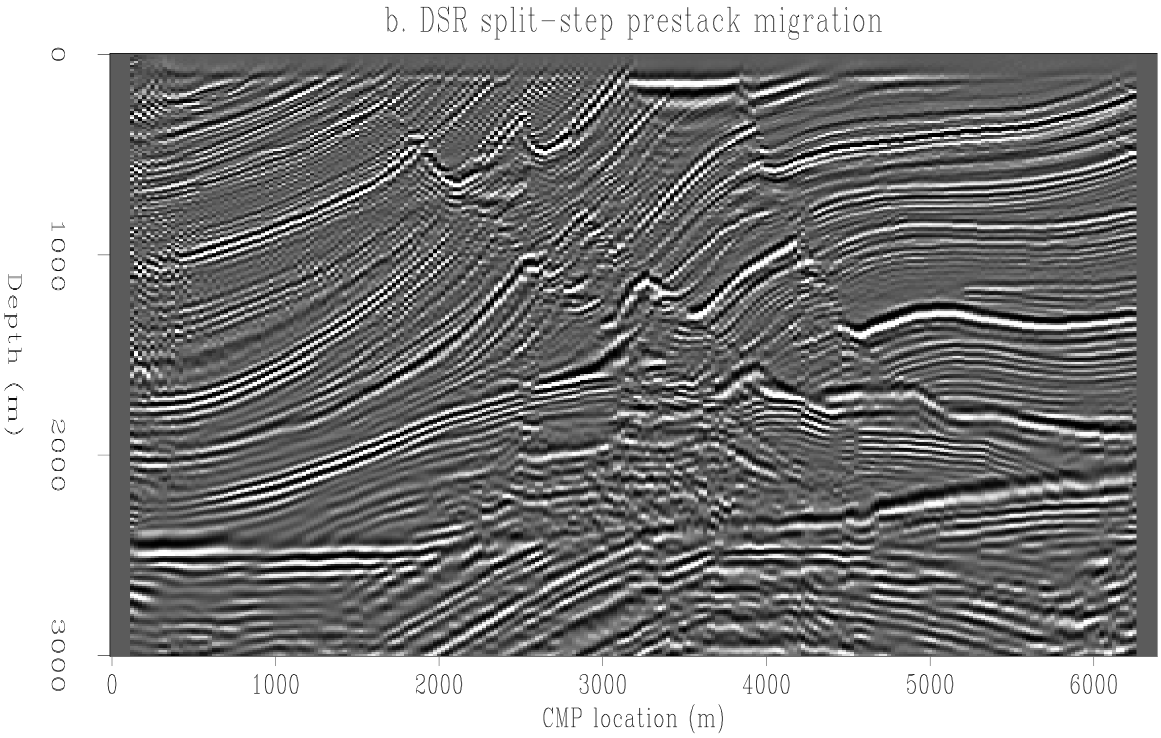


Figure 4: Zero-offset and prestack migration of the Marmousi.
a. Zero-offset migration of the near-offset section.
b. Prestack migration using DSR with a split-step term. mihai1-ZOandDSRsplit [NR]