Stanford Exploration Project, Report 94, November 11, 1997, pages 73–80

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Iterative estimation of depth migration operators

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keywords: *velocity, inversion, prestack, downward-continuation, 3-D*

ABSTRACT

We divide the velocity estimation problem into two steps: focusing operator estimation and back projection to the velocity field. We propose estimating perturbations in the focusing operators by analyzing homothetic scans of the travel-time field. In the case of systematic bias in the velocity model, the correct perturbations to the focusing operator are indicated by the homothetic scans.

INTRODUCTION

3-D prestack depth migration is becoming a common tool in regions with complex geology (Ratcliff and Weber, 1995). In order to derive benefits from prestack migration an accurate velocity model is required. Time domain focusing methods (Yilmaz and Chambers, 1984) are effective because of the direct relation between the best focusing operator and the migration velocity model. Unfortunately, when the velocity function is complex, move-out varies significantly from hyperbolic and cannot be effectively described by a single parameter (V_{rms}). In these cases depth migration is required.

In contrast with time-migration, depth migration requires the definition of an interval velocity model. A ray-tracing step links the interval velocity model and the focusing operators. Because of this indirect link, interval velocity can not be directly measured from the data, but is estimated through an inversion process that involves back-projecting travel-time information into the velocity model. This inversion process is underdetermined, and requires a fair amount of human interaction and interpretive judgment to constrain the model and converge to a useful solution. The interpretive process is crucial, but expensive, especially when compared with computational costs which keep failing. To reduce the need for interpretive judgment we split the velocity estimation problem into two steps. First, determine the kinematics of the best focusing operators. Second, estimate the interval velocity from the focusing operators. In this paper we present a methodology to address the first of these two tasks.

Berkhout (1996) recently proposed a method to estimate the best focusing opera-

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tors in the prestack domain. However, he suggests an updating scheme that requires picking (manual or automatic) of timing discrepancies between images and operators on a trace-by-trace basis. We feel that this approach can be unreliable in complex regions and aim for a much sparser parameterization of the perturbations to the focusing operators. For this purpose, we use the concept developed by Audebert (1996) of homothetic scaling of the travel-time field. The optimal scaling can be reliably and efficiently measured from the data at each reflector location by scanning over appropriate ranges of the scaling factor (γ). In addition, by using focusing rather than coherency over offsets, there is a potential to do residual migration, making scanning computationally efficient (Fomel, 1996). Consistent with our previously stated goal of deferring the estimation of interval velocity as much as possible, we prefer to use the measured scaling factor to update the focusing operators, rather than the velocity model. A general theory needs to be developed that relates measured scaling factors to focusing operators. In this paper we show that for a simple case of a constant perturbations, the best focusing γ point towards the correct perturbation in the focusing operators.

SCANNING HOMOTHETIC PARAMETER

To do efficient focusing analysis we must employ a method that does not require constant recalculation of ray-paths. In addition, for the method to be robust, we must limit the number of parameters that describe perturbations in the focusing operator. A homothetic scaling of the travel-times meet both these criteria.

Homothetic scaling of travel-times

Audebert (1996) shows that if the slowness field is globally scaled, ray geometry is unchanged. Audebert (1996) points out that starting from slowness field $S(\bar{x})_{orig}$, the travel-time from $\bar{x}_1$ to $\bar{x}_2$ is:

$$T_{orig}(\bar{x}_1, \bar{x}_2) = \int_{L_(\bar{x}_1, \bar{x}_2)} S_{orig} dl \tag{1}$$

where $L_(\bar{x}_1, \bar{x}_2)$ is the ray-path between $\bar{x}_1$ and $\bar{x}_2$. If we scale S_{orig} by a positive constant γ:

$$\begin{aligned} T_\gamma(\bar{x}_1, \bar{x}_2) &= \int_{L_(\bar{x}_1, \bar{x}_2)} \gamma S_{orig} dl && (2) \\ &= \gamma T_{orig}(\bar{x}_1, \bar{x}_2) && (3) \end{aligned}$$

$L_(\bar{x}_1, \bar{x}_2)$ is the same in both cases because the original Snell's law at each interface can be reduced from the scaled travel-time version:

$$\begin{aligned} \gamma v_1 sin\theta_1 &= \gamma v_2 sin\theta_2 && (4) \\ v_1 sin\theta_1 &= v_2 sin\theta_2 && (5) \end{aligned}$$

As a result, scaled versions of the travel-time field represent valid focusing operators. By finding the scaled (by γ) version of the travel-time field that best focuses the data we get an estimate of the error in our interval velocity model.

Synthetic Example

To illustrate the homothetic scaling approach we built a synthetic model (Figures 1 and 2), and downward continued to the reflector. If we start with an initial constant velocity model equal to the background velocity we see good coherency at zero time at CMP locations far away from the anomaly (Figure 3). For CMP's closer to the anomaly, focusing deteriorates and deviates from zero time.

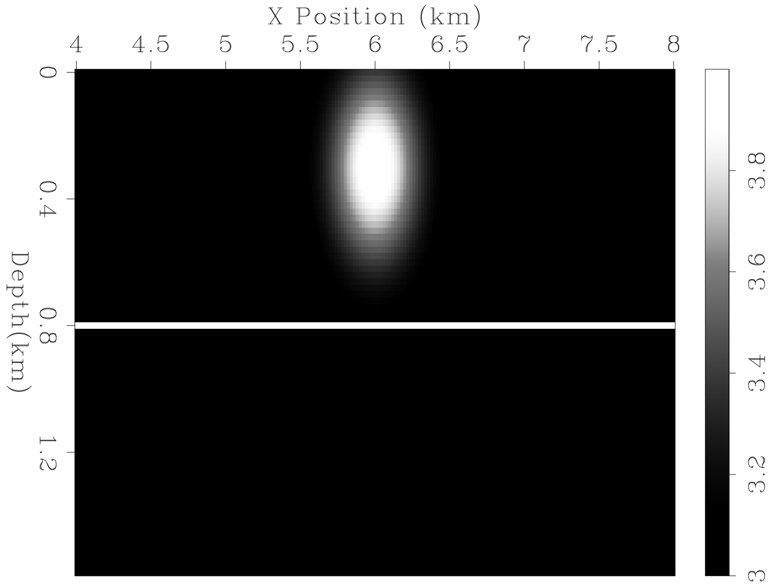

Figure 1: Input velocity model. Gaussian anomaly at 4 km/s in an otherwise homogeneous half-space (3km/s). bob1-layer.gauss [ER]

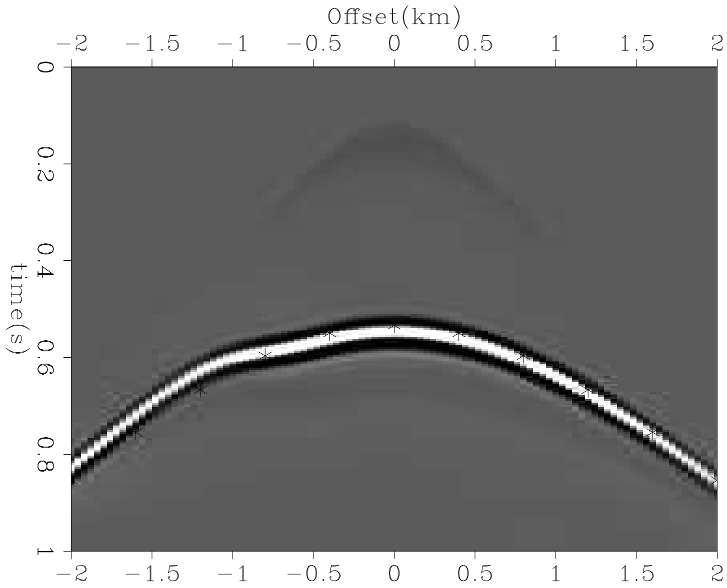

Figure 2: A sample shot gather illustrating the non-hyperbolic move-out (hyperbolic move-out indicated by '*') caused by the velocity anomaly. bob1-layer.shot [ER]

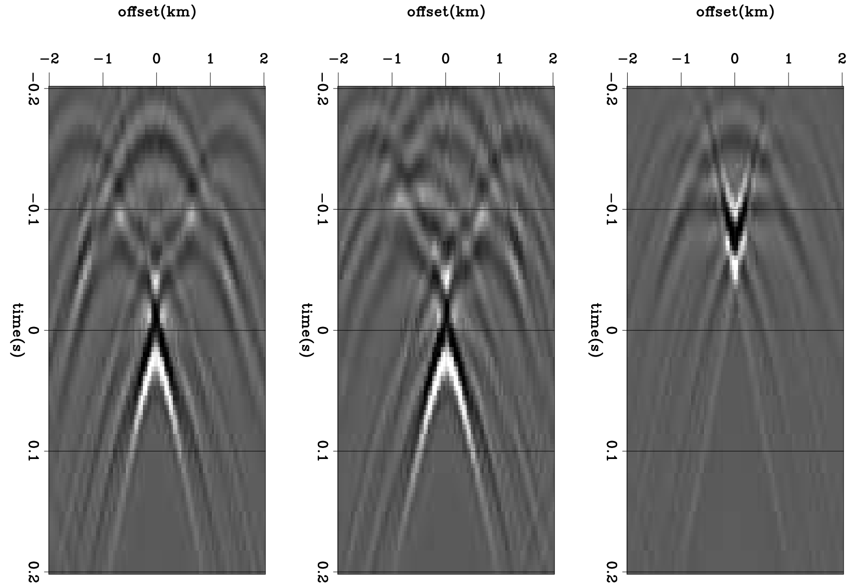

Figure 3: Result of downward continuing at three different locations. The left panel is far away from anomaly(CMP 8); the center, at the edge of the anomaly(CMP 7); and the right directly above anomaly(CMP 6). bob1-layer.gama.1 [CR]

We then found which scaled version of the travel-time field best focused the data at various CMP locations. Figure 4 shows the optimal focusing result for the same three CMP's as Figure 3. As expected, away from the anomaly the best focusing occurs at zero time when $\gamma = 1$. At the edge of the anomaly, best focusing is achieved at positive time, with $\gamma < 1$. (outer offset ray-paths pass through the anomaly, decreasing curvature, indicating a faster velocity). While directly above the anomaly we see maximum focusing at negative time, at $\gamma > 1$. (inner offsets faster, more curvature, indicating lower velocity and increased travel-times).

DEPTH MIGRATION OPERATOR UPDATE

The obvious question: *is it possible to limit the need for costly and complicated back projection by using our picks for γ to update our focusing operator?* In the case where we have a significant (DC) error in our initial velocity model, the answer is yes. A systematic error in our velocity model will result in a systematic bias in our maximum focusing γ. If we apply a constant scaling to our travel-time field, such as the median focusing γ, we are not changing ray-paths, so back projection is not required. We can then rescan the γ parameter, further refining our focusing operator.

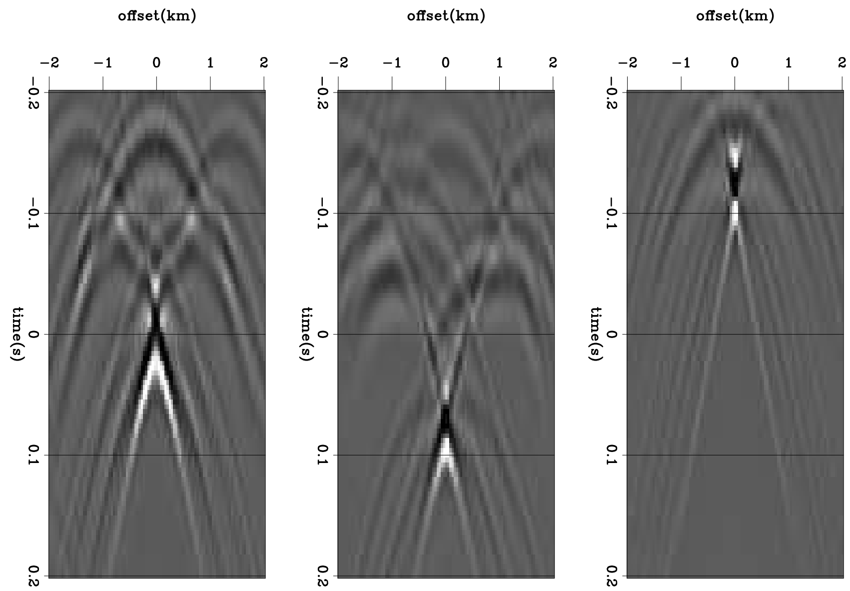

Figure 4: The best focusing γ. The left, CMP 8, $\gamma = 1.$; center, CMP 7, $\gamma = .88$; and right, directly above, CMP 6 $\gamma = 1.10$. $\boxed{\text{bob1-layer.gama.best}}$ [CR]

Synthetic Test

Using the same synthetic model, and starting with a constant velocity 10% less than the background model velocity, we get a systematic bias in our best focusing γ, Figure 5 shows the same three CMP locations as Figures 3 and 4. In each case the best focusing γ is significantly lower than when we started with the correct background velocity.

More complex velocity anomalies

Inverting for more complex perturbations in focusing operators from γ perturbations poses more of a problem. Some insight into the problem can be gained by observing the γ response to the Gaussian anomaly. The pattern of best focusing γ ($>$ 1. for reflector positions directly above the anomaly, $<$ 1. for positions above the anomaly) similar to the well documented 'w' shape observed in stacking velocity (Toldi, 1985). Looking at the difference between the correct focusing operator and our initial focusing operator (Figure 6) leads to some insight on how to project these γ perturbations to the focusing operator. The flat reflector and constant velocity nature of the model lead to the Kjartansson's (1979) 'V' pattern response.

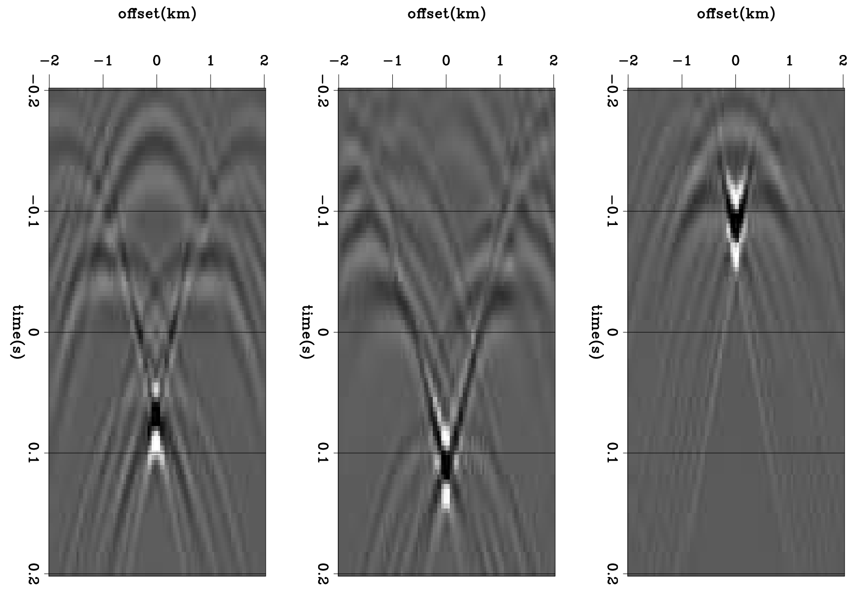

Figure 5: The best focusing γ at same 3 CMP locations. Left panel (CMP 8), focuses at 1.06, center panel (CMP 7) at .85, and right (CMP 6) at .93. bob1-layer.wrong.vel [CR]

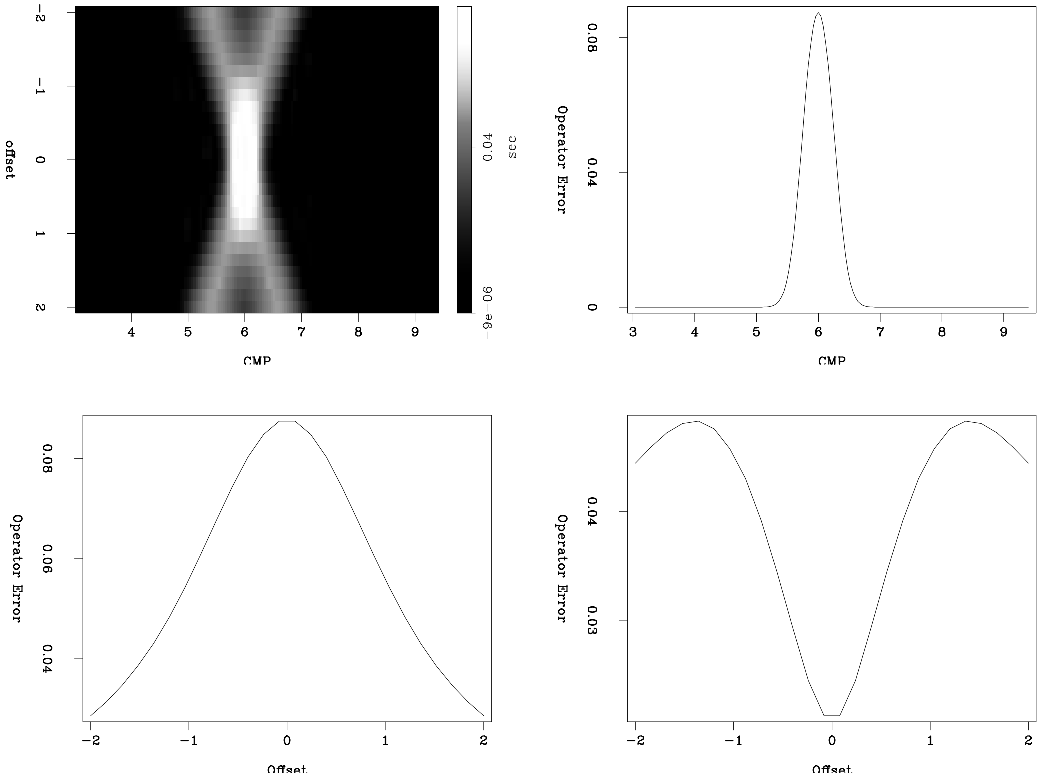

Figure 6: Top left, difference between correct focusing operator and initial operator; top right, zero-offset deviations; bottom left, operator deviations directly above the anomaly; and bottom right, response .4 km from center of the anomaly. bob1-layer.difference [CR]

FUTURE WORK AND CONCLUSIONS

Preliminary results are encouraging. The data focuses at the predicted γ when given an inaccurate velocity model. In addition, for the case of a DC error in global travel-times, γ can be found iteratively, reducing the need for costly back projections. A theory relating how to actually to map γ perturbations to changes in the focusing operator needs to be developed. Building a linear operator relating travel-time pertubations to change in γ reponse seems to be the next step. Changing the focusing operator is at best a first order approximation. At some stage we must back project the focusing operator to the the velocity field, recalculate travel-times and repeat the focusing analysis. An open question is *what is the best method for back projection, is the central ray approach of Audebert(1996) sufficient?* Another potential area of research is to see if we can avoid going back to the data when scanning over γ. *Is it possible to apply a small convolution operator, similar to residual migration, on the image space that simulates changes in γ?*

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