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Depth focusing analysis for 3-D migration velocity estimation

Hermes Malcotti and Biondo Biondi[1]

keywords: *focusing, migration, velocity*

ABSTRACT

We present partial results of the implementation of 3D depth focusing velocity analysis, based on 2-D and 3-D prestack downward continuation algorithm in the CMP domain. We discuss the focusing analysis methodology for point diffractors in a constant velocity media. In addition, we discuss the 2-D and 3-D kinematic bases of 3D focusing analysis and the implicit approximations of depth focusing analysis technique. We show the depth error panels resulting from using diffractors points and discuss the differences between the depth error gather obtained by a 2-D and 3-D downward continued operator.

INTRODUCTION

Velocity is the link between time and depth; therefore, velocity estimation is the most fundamental and important problem in seismic imaging. In the literature, different migration velocity estimation methods can be found; one method is depth focusing analysis. This method is based on the fact that the wave-propagation enegy is best focused when migration velocity is correct. The focusing analysis ideas was introduced by Doherty and Claerbout (1974), and presented the fundamental equation for focusing analysis:

$$V_r Z_r = V_m Z_f, \tag{1}$$

where V_r is the real wave propagation velocity, Z_r is the real reflect or depth, V_m is the migration velocity and Z_f is the the focusing depth. Therefore, the wave-field downward continued with the real propagation velocity V_r to reach the real depth is equivalent to the wave-field downward continued with an erroneous velocity V_m to reach the focusing depth.

Yilmaz and Chambers (1984) presented the time version of equation (1) and applied the focusing analysis method with a time migration operator. Later, working with a depth migration operator the focus panel was modified into a depth error

[1]**email:** hermes@sep.stanford.edu, biondo@sep.stanford.edu

focusing panel and depth focusing analysis (Faye and Jeannot, 1986). The depth error panel provided a direct relationship between the real reflector depth (Z_r), the focusing depth (Z_f) and the reflector migrated depth (Z_m) obtained at the imaging condition (t=0 s) with the wrong migration velocity V_m. Therefore the real velocity is estimated assuming flat horizons and small velocity errors. MacKay and Abma (1992) discussed the details of this technique and presented a modified version of equation (1) for the case of dipping reflectors, assuming a constant velocity.

Depth error panels in complex areas have interpretation problems. The depth error panels are made of traces downward continued with the wrong velocity; therefore the focusing energy has an erroneous positioning in depth and space. Consequently, there is energy partially migrated, considered a spurious event that complicates the depth error panel interpretation. MacKay and Abma (1993) proposed the use of the residual curvature like a weighting function on the focus panels. Audebert and Diet (1993) and Jeannot and Berranger (1994) proposed and a residual zero-offset migration of the depth error panels, where the energy is mapped to the migrated CMP coordinates.

One of the most important characteristics of depth focusing analysis is its independence from the dimensionality of the data. A depth focus error gather does not change for 2-D or 3-D prestack data; it is a 2-D gather of zero-offset traces extracted in every step of the downward continuation process. This feature of the depth error panels make possible evaluate migration velocity by a 2-D gather built from 3-D downward continuation data.

The goal of this paper is to present a depth migration velocity estimation tool based on a 3-D downward continuation operator in common-azimuth domain as well as to present depth focusing ideas. Partial results of the 3-D depth focusing velocity analysis analysis based on a 2-D and 3-D prestack downward continuation operator and the 2-D and 3-D kinematic bases of 3D focusing analysis are discuss.

BUILDING AND INTERPRETING THE DEPTH ERROR PANEL

The focusing principles can be understood by thinking of a point diffractor. The seismic response of a point diffractor is an hyperboloid in 3-D or an hyperbola in 2-D ($Z = 0$). If a CMP gather is downward continued with the real propagation velocity ($V_m = V_r$) to the real point diffractor depth, z_r (imaging condition: t=0s), the diffracted energy is collapsed to the hyperbola apex and the energy is maximum. The point diffractor position (x_m, y_m, Z_m) is equal to the point diffractor point at (x_r, y_r, z_r) (see Figure 1). If the velocity is changed ($V_m \neq V_r$) and the CMP is downward continued, the image obtained at the imaging condition (t=0 s) is not well focused and its energy is not maximum at the hyperbola apex. For this new velocity, there is a focusing point, where the energy is maximum at the zero-offset trace, but its spatial position and depth differ from the real position.

Based on the work of Doherty and Claerbout (1974) and MacKay and Abma (1992), the focusing point depth Z_f obtained with migration velocity is equal to the real depth Z_r if the migration velocity is equal to the real propagation velocity V_r:

$$2\frac{Z_r}{V_r} = 2\frac{Z_m}{V_m}. \tag{2}$$

Equation (2) is valid on the imaging line (t=0 s) (see Figure 2) and is still valid for velocities V_m close to V_r. Replacing V_m in equation (1), the real depth is expressed as function of the focusing depth:

$$Z_r = \sqrt{Z_m Z_f} \equiv \frac{Z_m + Z_f}{2}, \tag{3}$$

The vertical depth error is defined by

$$\delta = \frac{Z_f - Z_m}{2}, \tag{4}$$

where Z_f is estimated from the error depth gather defined by the zero-offset trace chosen at every downward continued operator step. Moreover, the depth error formula (Equation (4)) is used to define the depth error gather. It is important to remember that focusing analysis is a technique for making estimations of the real RMS velocity V_r.

Figure 1 shows the focusing panel and depth error panel. A long the x-axes of the focusing panel is plotted the downward continuation step, where every zero-offset trace is chosen. In the vertical axis is the retarded time τ defined by the recording time, and the vertical time to the current downward continuation step, divided by the velocity V_m:

$$\tau = t + \frac{2Z}{V_m}. \tag{5}$$

On the diagonal of the focusing panel equation (2) is defined and is the imaging line where the focusing energy is located when the CMP data is downward continued with the real propagation velocity (i.e., $V_m = V_r$).

Using the focusing panel and applying the lateral shift defined by the equation (4), the depth error panel is built. Now, the imaging line is the zero error line and the vertical axes is the retarded time. Working with the retarded time allows us to compensate the time shifting between traces of different downward continuation steps (Claerbout, 1984).

In heterogeneous media focusing analysis fails due to the assumption that the Z_m, Z_r and Z_f are aligned along the same ray path (see Figure 3). To solve this problem Audebert and Diet (1993) suggests correcting the depth focusing panel by residual zero-offset migration with the migration velocity, to obtain a depth focusing panel in agreement with normal incidence ray (in the horizontal layered media it is a vertical path, (see Figure 2). Jeannot and Berranger (1994) suggest making a ray residual migration or a velocity scan of the depth focus panel instead of applying a more expensive zero offset residual migration.

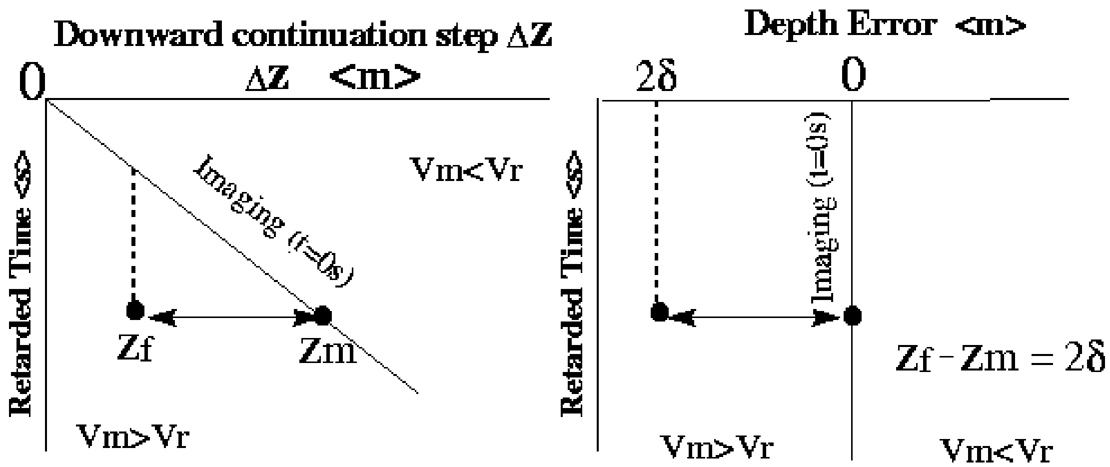

Figure 1: Focusing gather (right) and error gather (left) hermes-panel [NR]

3-D KINEMATIC FOR THE FOCUSING ANALYSIS

Depth focusing analysis is a methodology for velocity estimation based on the fact that the total wave propagation time is invariant: the total traveltime from the real reflection point obtained with the real propagation velocity is equivalent to the total traveltime obtained from the focus reflection image obtained with a wrong propagation migration velocity plus a residual time related with the error in velocity. This residual time, in a horizontal stratified media case with constant velocity V_r (see Figure 2), is equal to a vertical time δt associated with a depth error δ a paraxial approximation (MacKay and Abma, 1992):

$$\delta t = \frac{(Z_f - Z_m)}{2V_m}. \tag{6}$$

Figure 3 presents the migration of a dipping reflector with the media velocity and with a wrong velocity V_m. Keeping the notation for the migration velocity V_m, real velocity V_r, and the downward continued point with the migration velocity to the imaging condition $m = (x_m, y_m, Z_m)$, the real position of the dipping reflector point is $r = (x_r, y_r, z_r)$ and the focusing point is $f = (x_f, y_f, Z_f)$. The total time t_T can be related to the focus time t_f by the following equation (MacKay and Abma, 1992):

$$t_T = t_f + \delta t. \tag{7}$$

This time equation in 2-D is rewritten using the offset, h, and the dipping angle, θ, in the law of cosines for the triangle obtained by a image source (see Figure 3):

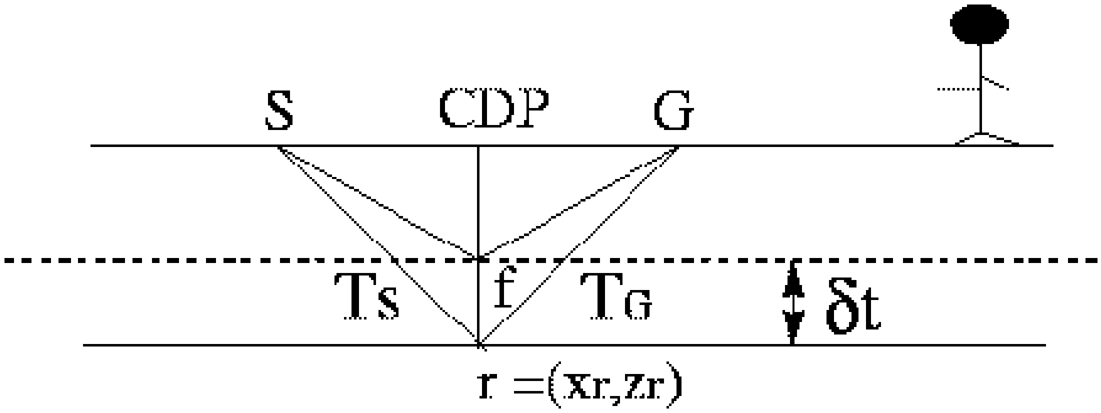

Figure 2: The residual time obtained when the data is migrated with a migration velocity, V_m, greater than the real propagation velocity V_r. hermes-homo [NR]

$$t_T^2 = (2\frac{z_r}{V_r})^2 + (2\frac{h_r}{V_r})^2 - 8z_f\frac{h_r}{(V_r)^2}cos(\theta_r) \tag{8}$$

$$t_f^2 = (2\frac{z_f}{V_m})^2 + (2\frac{h_f}{V_m})^2 - 8z_f\frac{h_f}{(V_m)^2}cos(\theta_m). \tag{9}$$

Taking into account the previous formulas, we can conclude that in a subsurface with dipping reflectors and lateral gradient velocity, the position of the focusing point (x_f, Z_f) is not between the real position and the migrated position, as it is in a lateral homogeneous media (i.e., constant velocity media or horizontal layered media). Therefore, in heterogeneous media the depth error panel needs to be corrected using a residual zero-offset migration to move the depth error panel energy to its correct position (Audebert and Diet, 1993). However, there is still error in depth. Another way to understand this mispositioning of the focusing point is to consider in the normal ray as between the focusing imaging and CMP point at the surface. This normal ray is different from the normal ray obtained with the real velocity field, thus it has a different surface incident angle.

The focusing analysis methodology for 3D data is equivalent to 2-D methodology. The depth error panel is built by extracting the zero offset trace from every downward continuation step of prestack data recorded at the surface. In this paper, the downward operator is a 3-D phase-shift in CMP domain operator.

It is important to point out, that in 3-D the focusing point has associated a depth error and lateral position error in in-line and cross line directions. Therefore, to apply equation (6) and (2), it is necessary to perform a residual migration of depth error gather, then the focusing point, the migrated point and the real point are along the same normal ray.

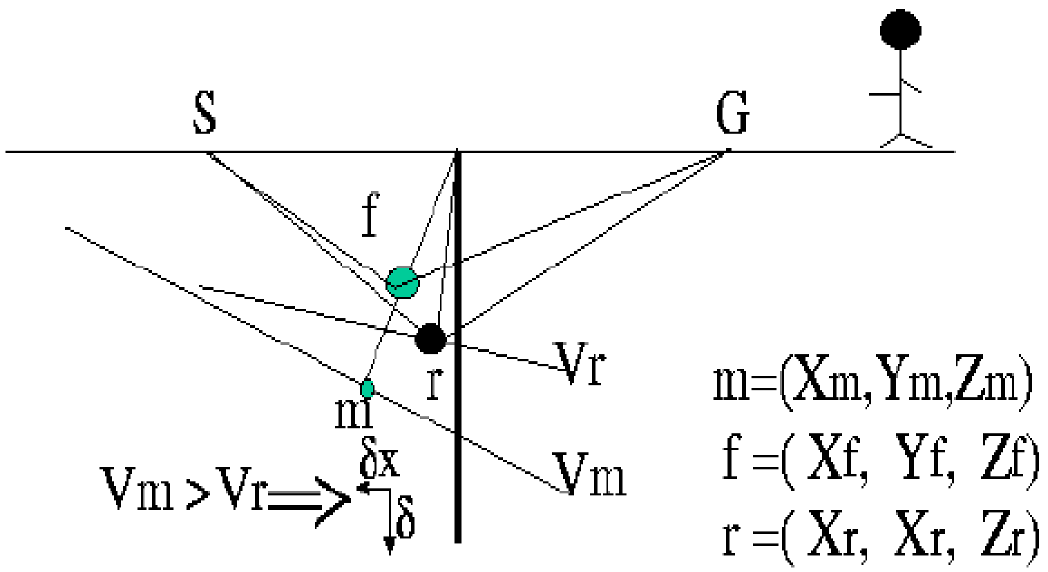

Figure 3: The residual time obtained when the data is migrated with a migration velocity, V_m, greater than the real propagation velocity V_r. hermes-heter [NR]

RESULTS

To illustrate the focusing analysis methodology, we chose a 2-D and 3-D CMP phase-shift migration operator, the simplest operator deduced from the double square root operator. Working with two filtered spikes with a central frequency 30Hz and a constant velocity of 3000 m/s, the operator modeled a CMP gather recorded at the surface with a 25 m between traces and 4ms for temporal sampling, as shown in the Figure 4 for the 2-D modeling and Figure 8 for the 3-D modeling.

This CMP gather is downward continued with the 2-D and 3-D operator (adjoint operator)with a constant velocity to build the depth error gather for this CMP gather (see Figures 6 and 7 for the 2-D case and Figure 9 for the 3-D case). For every downward continuation step of migration (adjoint) operator, the zero-offset trace is stored. In the 2-D diffractor migration the depth error panels are plotted in Figures (6) and 7. Because we are using a phase-shift operator there is aliasing noise in the depth error gathers.

Figure 6 shows the depth error gather obtained when the CMP is downward continued with constant velocity $V_m = 3000m/s$ equal to the real propagation velocity, V_r. This result shows when $V_m = V_r$, the energy at the zero-offset trace is maximum when depth error, δ, is equal to zero for both point diffractors. In contrast, the depth error is different of zero in the depth error gathers obtained with a downward continuation velocity, V_m, is not equal to the real propagation velocity. Moreover, the

2-D depth error gather shows a phase difference of 90 degrees indicating the maximum focusing energy, as is predicted by the 2-D migration theory (Doherty and Claerbout, 1974).

Another interesting feature of the error gather is the linear trend in the condition of maximun focusing, because of the far-offsets. In contrast, the depth error gather build with a 3-D downward continuation operator does not present that change in phase (see Figure 9) as predicts the theory.

The depth error gather is usually plotted by taking the envelope of every trace in the gather (see Figure 7). This is because that depth focusing analysis method is based on a 2-D operator and to eliminate the phase difference mentioned before. We show depth error panels with the trace and with the envelope to emphasize the difference between the the depth error gather built with a 2-D downward continuation operator and the gather built with a 3-D operator.

Comparing the error panels built with a 2-D and 3-D downward continuation phase-shift operator in the CMP domain for constant velocity, shows the independence on the dimensionality of the data (see Figures 6, 7, 9 and 10). This is an important result in the implementation the depth focusing methodology in 3-D for velocity estimations. Moreover, it is a 2-D gather making it easy to interpret and to estimate the real propagation velocity by picking the depth error defined by the focal point on the depth error gather and using the equations 9 and 4 the real propagation velocity V_r and the real depth Z_r are estimated from a wrong migration velocity V_m and a wrong focusing depth Z_f as equation (1).

Analyzing the 2-D spike migrations with different velocities of the Figure 5, we can interpret them as three differents steps in the downward continued process of the CMP with just one velocity higher than 4000m/s. Hence a depth error gather can be interpreted as a set of zero-offsets traces with different velocities, increasing from negative error to positive errors (1994).

Figures 9 and 10 presents the error gather built from a 3-D phase-shift downward continuation operator in CMP domain for two points diffractor, as in the 2-D case.We tested three differents constant migration velocities. The depth error panel obtained with $V_m = 2000m/s$, we observe that the energy is focusing for a positive depth error. It shows that the migration velocity was low and the focusing depth and the real depth, are greater than the migration depth. For a migration velocity equal to the real propagation velocity, as spected there is not error in depth. When the migration velocity is 4000m/s the focusing point has a negative depth error negative because the velocity is higher than the real velocity. In addition, we can observe a banded behavior on the 3-D depth error panels and it is smooth when we displayed the same panel by the envelope (see Figure 10).

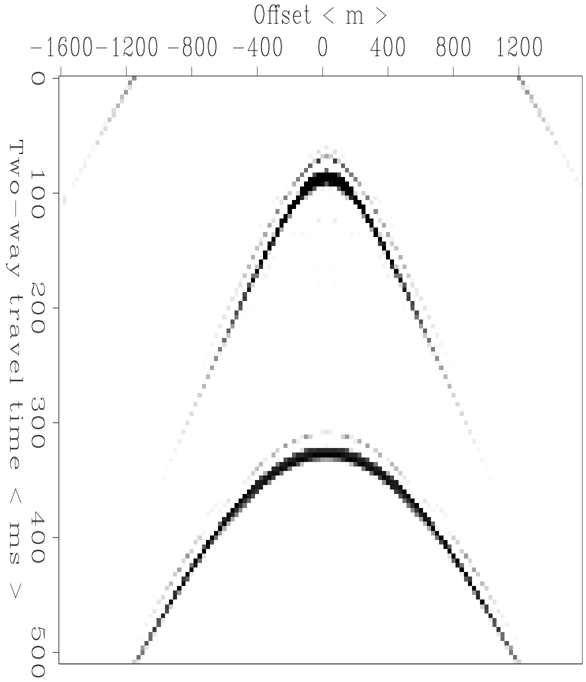

Figure 4: Impulse response of phase-shift 2-D modeling operator for two filtered spikes. $\boxed{\text{hermes-SPKmod128}}$ [CR]

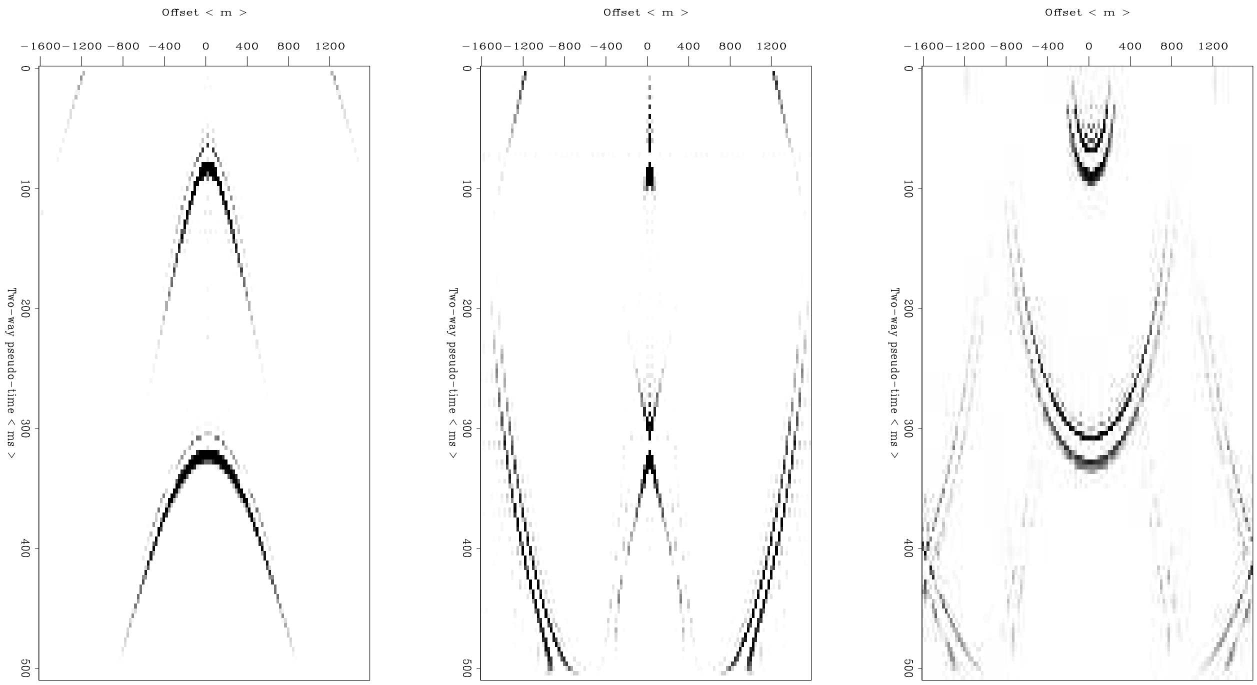

Figure 5: 2-D Phase-Shift migration of two difractors with velocities: 2000m/s (left), 3000m/s (center) and 4000m/s (right). These migrated gather can be interpreted as diferents setps in a downward continuation process with migration velocity greater or equal to 4000m/s and the focusing depth is define by the gather in the middle. $\boxed{\text{hermes-SPKmigs}}$ [CR]

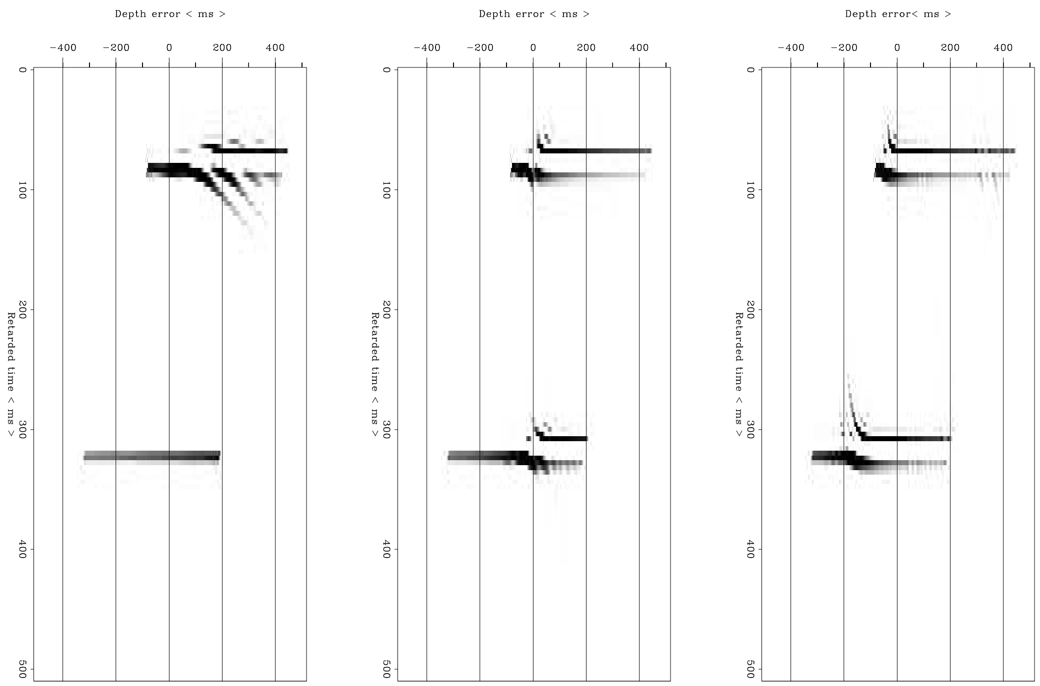

Figure 6: Depth error panels for three differents velocities: 2000m/s (left), 3000m/s (center) and 4000m/s. They are gathers of zero-offset traces chosen in every downward continuation step of a 2-D Phase-Shift operator in CMP domain. hermes-FOCUspks [CR]

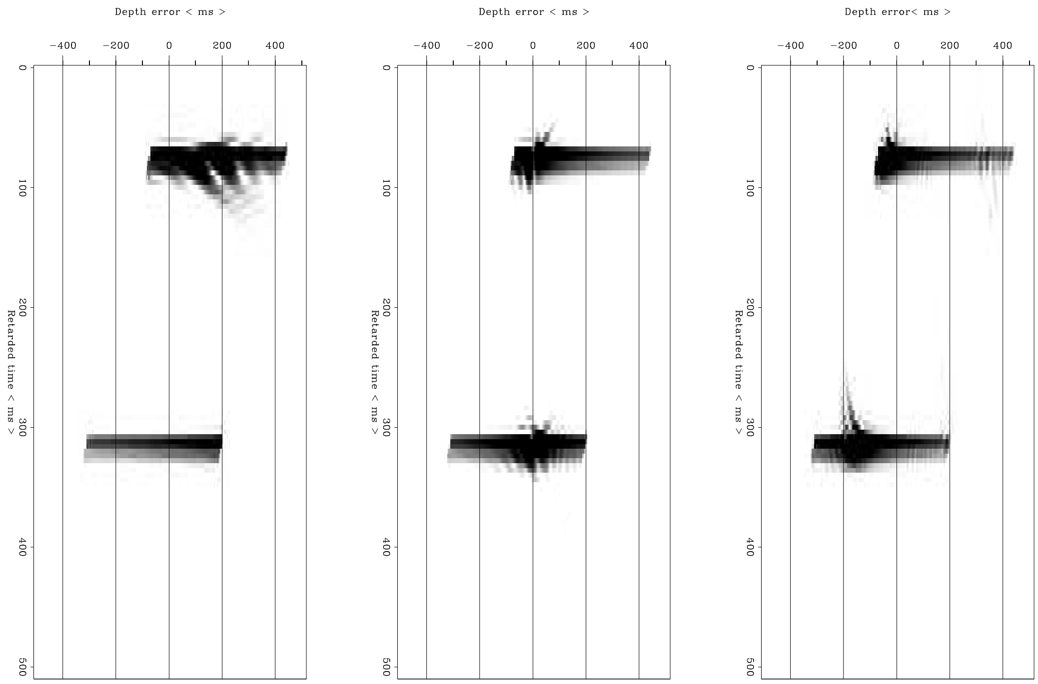

Figure 7: Depth error panels for three differents velocities: 2000m/s (left), 3000m/s (center) and 4000m/s. They are gathers of zero-offset trace envelope chosen in every downward continuation step of a 2-D Phase-Shift operator in CMP domain. hermes-FOCUspksENV [CR]

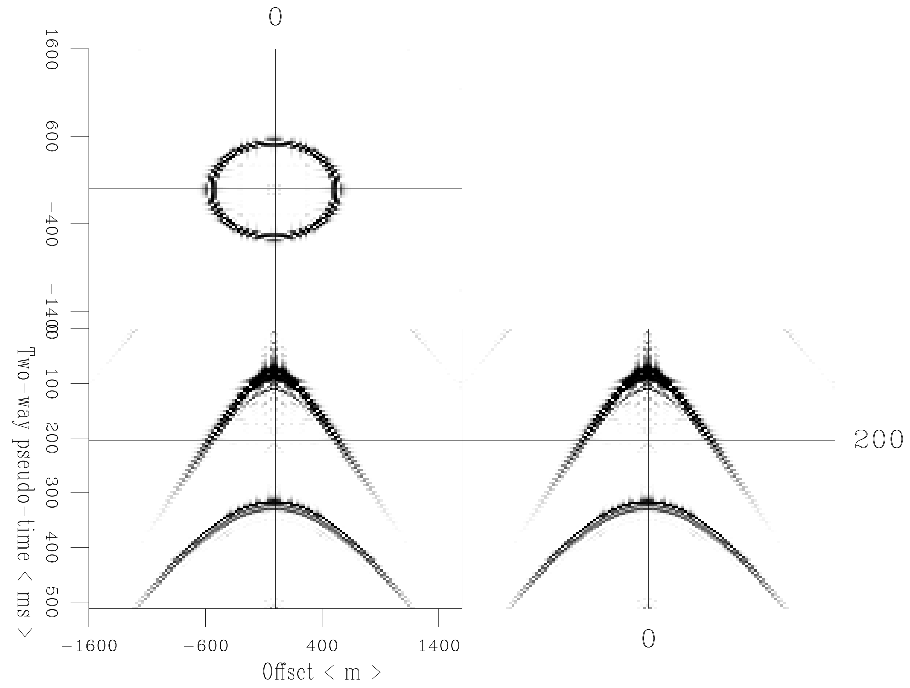

Figure 8: Impulse response of phase-shift 3-D modeling operator for two filtered spikes. hermes-SPK3Dmod3000 [CR]

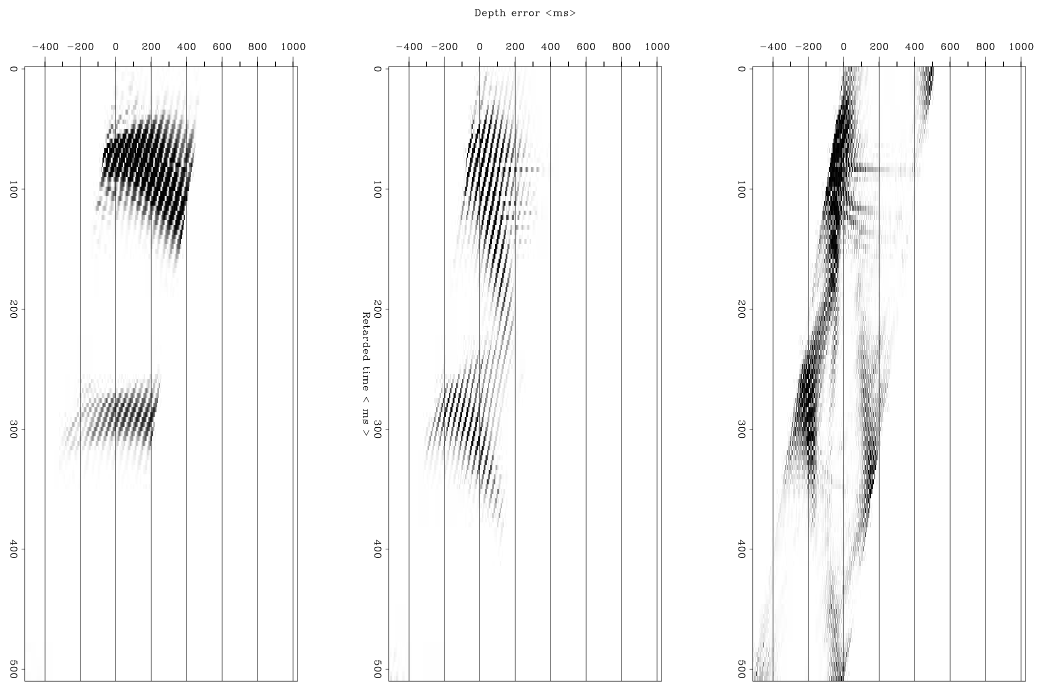

Figure 9: Depth error panels for three differents velocities: 2000m/s (left), 3000m/s (center) and 4000m/s. They are gathers of zero-offset traces chosen in every downward continuation step of a 3-D Phase Shift operator CMP domain. hermes-FOCU3D [CR]

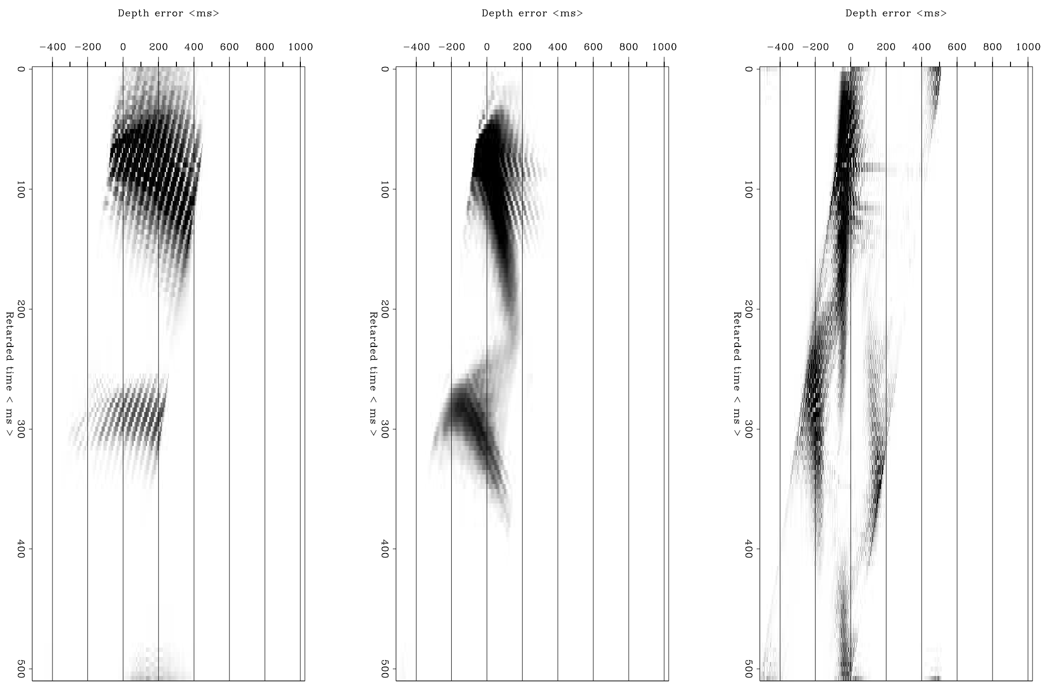

Figure 10: Depth error panels for three differents velocities: 2000m/s (left), 3000m/s (center) and 4000m/s. They are gathers of zero-offset trace envelope chosen in every downward continuation step of a 3-D Phase Shift hermes-FOCU3DENV [CR]

CONCLUSIONS

We have presented the methodology to built the depth error gather from a 3-D phase-shift downward operator. the 2-D methodology presented in the literature.

The basic difference between the depth error gather built from a 2-D downward operator and the the depth error gather, is a 90 degrees phase-shift of the wavelet in the depth step trace predicted by the 2-D theory. The focusing condition in the focusing depth the depth error panel in 3-D still remains the same as in the 2-D case.

The depth error gather built using a 3-D downward continuation is a 2-D gather of doward continuated zero-offset traces.

The depth error gather built using a 3-D downward continuation operator needs a residual correction in complex velocity models, because the focusing point and the real diffractor position are not along the same path, which in the constant velocity case is a vertical path.

FUTURE WORK

The next step in this work is the implementation of depth focusing analysis methodology with a 3-D downward continuation operator in common azimuth. We will also implement a graphic tool for the depth error panels picking.

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REFERENCES

Audebert, F., and Diet, J. P., 1993, Migrated focus panels: focusing analysis reconciled with prestack depth migration:, pages 961–964.

Claerbout, J. F., 1984, Imaging the Earth's Interior: SEP–**40**, 0–0.

Doherty, S., and Claerbout, J., 1974, Velocity analysis based on the wave equation: Stanford Exploration Project, **1**, 160–171.

Faye, J. P., and Jeannot, J. P., 1986, Prestack migration velocities from focusing depth analysis: 54th Ann. Int. Mtg., Soc. Expl. Geophys., Expanded Abstracks, 438–440.

Jeannot, J. P., and Berranger, I., 1994, Ray-mapped focusing: A migration velocity analysis for Kirchhoff prestack depth imaging: 64th Ann. Int. Mtg., Soc. Expl. Geophys., Expanded Abstracks, 1326–1329.

MacKay, S., and Abma, R., 1992, Imaging and velocity estimation with depth-focusing analysis: Geophysics, **57**, 1608–1622.

MacKay, S., and Abma, R., 1993, Depth-focusing analysis using a wavefront-curvature criterion: Geophysics, **58**, 1148–1156.

Yilmaz, O., and Chambers, R., 1984, Migration velocity analysis by wave-field extrapolation: Geophysics, **49**, 1164–1174.