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# Cross product operator detects plane reflectors

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## ABSTRACT

I propose an estimation of the dip of a plane layer volume by minimizing the output of a cross product differential expression. This rather peculiar expression allows reliable estimates in small expectation volumes and it is easily extended to Prediction Error (PE) filters. I believe the method yields reasonable dip estimates even for cases that only approximate a plane layer volume (e.g., after the addition of noise). However, the approach does not yield a straightforward technique for subtracting the dominant plane-layered contribution from the original image. The cross product expression yields a vectorial, not a scalar output. The back-projection of the vectorial output by the adjoint operation results in a scalar function which does not have a simple, meaningful interpretation. The usefulness of the cross product expression is uncertain.


## INTRODUCTION

I propose a filter technique that yields a dip estimation for data volumes that consists mostly of a set of parallel planes (plane layer volume). The filter is based on the cross product of vector algebra and improves and clarifies Claerbout's earlier Lomoplan formulation (Claerbout, 1994). I developed the dip estimation technique for the computation of coherency attributes of seismic images (Schwab et al., 1996). The method may also be useful in the estimation of seismic data quality, the optimization of migration velocities, or in general inversion theory.

In this article, I first argue that a seismic image almost everywhere resembles a simple plane layer volume. Next, I suggest a simple cross product expression to detect and estimate the dip of a plane layer volume. I attempt an interpretation of the filter output. Finally, I clarify and enhance Claerbout's original Lomoplan idea and discuss the advantages to extend the technique to PE filters.

In summary, I have not tested the reliability of the dip estimation. The scheme's extension to PEF has failed to yield convincing results in the calculation of coherency attributes (partially since PEF are more powerful in removing events than the simple

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Finite Difference filter of the cross product formulation). Alternative approaches, such as direct estimate of coherency or estimates of a normed gradient, exist. I hope to apply the method to dip estimation, e.g., for migration velocity estimation. Coherency attributes could possibly use the dip estimation. But the simple filter approach to removing the plane layer volume contribution seems to fail.

## DETECTION OF A PLANE LAYER VOLUME OF A GIVEN SLOPE

Sedimentary structures in seismic sections are sometimes compared to a cake whose every layer lies neatly upon another layer. Most Austrian pastry chiefs, however, would be horrified by the jumble of folded and broken layers that we find in most seismic sections. Nevertheless, at almost every location a small local image volume does resemble a cake (see Figure 1). The effect of large scale folding is negligible within such detailed image volumes and, despite the dip of the layers and their variable thickness, every culinary connoisseur will quickly recognize the fundamentally parallel-plane nature of such a subsurface piece.

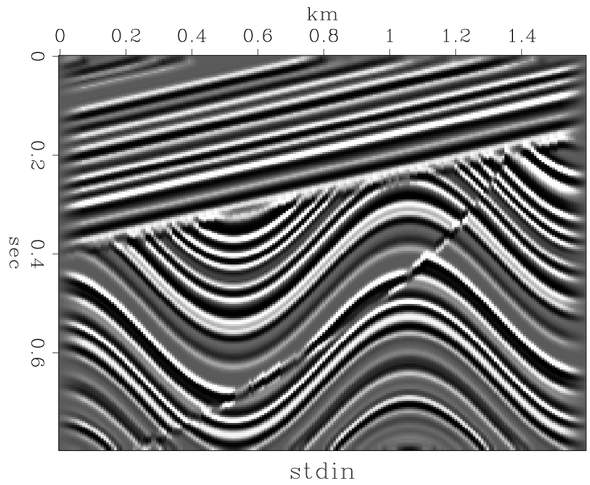


Figure 1: The seismic section is folded and broken. But a small enough subvolume will approximate a plane layer volume almost everywhere. matt2-sigmoid [ER]

The plane layer volume (our cake model) can be expressed mathematically as a 1-D function $f(x, y, z) = g(n_x x + n_y y + n_z z)$ where $\mathbf{n} = (n_x, n_y, n_z)$ is the unit normal vector to all planes. Such a one-dimensional representation is much simpler than the general volume representation $f(\mathbf{x})$.

The two-dimensional example in Figure 2 and 3 illustrates the relationship between the normal vector $\mathbf{n}$ of a plane layer volume and its gradient function. The image $f(x, y)$ in Figure 2 consists of parallel lines of constant amplitude. Consequently, the image can be expressed as $g(\mathbf{n} \cdot \mathbf{x})$ where $\mathbf{n}$ is orthogonal to all lines of constant amplitude[2]. Figure 3 displays the gradient vector $\nabla f(\mathbf{x}) = \nabla g(\mathbf{n} \cdot \mathbf{x})$ by small arrows. The chain rule applied to $\nabla g(\mathbf{n} \cdot \mathbf{x})$ proves what Figure 3 illustrates: the constant normal vector $\mathbf{n}$ is everywhere parallel to the gradient $\nabla f(\mathbf{x})$. In general, we can

[2]The image was computed by increasingly delaying a single one-dimensional function $g(t)$.

formulate a simple (and maybe obvious) detection criterion: an image volume $f(\mathbf{x})$ is a plane layer volume with normal $\mathbf{n_0}$, if (and only if) at all $\mathbf{x}$

$$\mathbf{n_0} \times \nabla \mathbf{f} = \mathbf{0}. \tag{1}$$

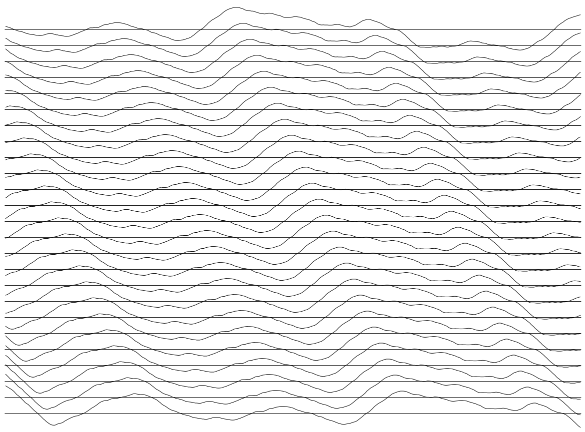

Figure 2: Input synthetic image: parallel lines of constant amplitudes. matt2-puckin [ER]

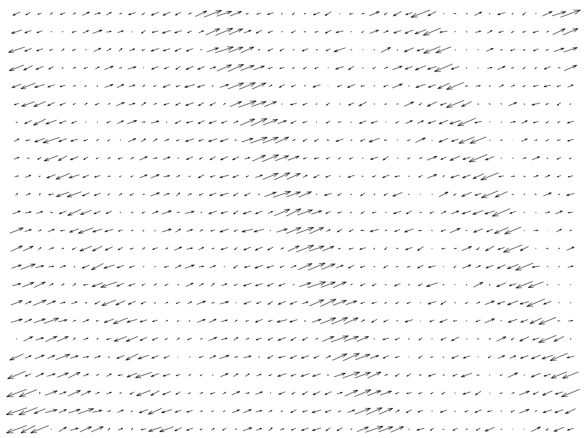

Figure 3: Gradient vectors at various locations of the input image. (All gradient vectors should be parallel. I believe it is an artifact of my gradient plotting program that they are not.) matt2-puckout [ER]

Equation (1) might remind the reader of the vector analysis identity $\nabla \times \nabla f(\mathbf{x}) = \mathbf{0}$. This identity states that the curl of any conservative vector field (a field that can be represented as the gradient of a scalar function $f$ (potential function)) is zero everywhere. All scalar fields $f$, including plane layer volumes, satisfy this vector analysis identity. However, not all scalar fields $f$ satisfy the cross product equation (1). In other words, the replacement of the left differential operator $\nabla$ by a constant vector $\mathbf{n}$ limits the set of scalar functions $f$ to the functions that are plane layer volumes (with normal $\mathbf{n}$).

For example, the image volume shown in Figure 4 satisfies the conservative field identity (since it is a scalar field $f$), but not the cross product expression (1) (as I will demonstrate in a later section). Similarly the radially-symmetric gravitational field of a point mass is a conservative field and satisfies the general curl expression, but defies the plane layer criterion (1).

However, the conservative field and the plane layer criterion are related if the scalar field is a plane layer volume. In the case of a plane layer volume, the left differential $\partial_i$ of the conservative field criterion yields the i-th component of the plane layer gradient $\mathbf{n}$.

## ESTIMATION OF THE SLOPE OF A PLANE LAYER VOLUME

If a given image volume is approximated by a plane layer volume, we can find the normal vector $\mathbf{n_1}$ by minimizing following expectation over the volume:

$$\min_{\mathbf{n_1}} E((\mathbf{n_1} \times \nabla f(\mathbf{x}))^2) = \min_{\mathbf{n_1}} E((\nabla f \sin\theta)^2) \tag{2}$$

where $\theta$ denotes the angle between $\nabla f$ and $\mathbf{n_1}$. We will have to decide on an orientation for $\mathbf{n_1}$, since both $\mathbf{n_1}$ and $-\mathbf{n_1}$ minimize the expectation.

Since **n1** consists of only two independent variables, small expectation volumes will yield reliable estimates of the normal vector, or the corresponding dip. In particular, the minimum is independent of amplitude and polarity changes of the gradient $\nabla f$. For example, in Figure 3, the gradient varies within the window while the cross product $\mathbf{n_1} \times \nabla f$ is zero everywhere.

## APPLICATIONS

An obvious application for the estimation of the normal vector is the automatic measurement of the local dip in a seismic image. Such dip information should be useful when optimizing for the correct migration velocity. For example, the dip information could be useful for tying seismic reflections to dip meter measurements in a well.

Additionally, the cross product expression can be used to remove a dominant plane layer contribution from an image volume. First, we estimate the normal vector $\mathbf{n}$ for the given cube using equation (2). Second, we remove the plane layer contribution by computing equation (1) for the estimated $\mathbf{n}$. As we will see, this removal does not, unfortunately, yield a simple scalar residual.

Let us assume the dominant component of parallel planes in an image volume $f(\mathbf{x})$ is $h(\mathbf{x})$ and its normal is $\mathbf{n}$. Consequently, $f$ can be expressed as a sum of the parallel planes $h$ and the remaining image $r$ where $f(\mathbf{x}) = h(\mathbf{x}) + r(\mathbf{x})$. The application of the cross product filter (1) to $f$ yields

$$\begin{aligned} \mathbf{n} \times \nabla f &= \mathbf{n} \times \nabla(h + r) \\ &= \mathbf{n} \times \nabla h + \mathbf{n} \times \nabla r \\ &= \mathbf{n} \times \nabla r \end{aligned} \tag{3}$$

since $\mathbf{n} \times \nabla h = 0$.

We would prefer to yield $r(\mathbf{x})$, a scalar volume of the difference $f$ and its dominant set of parallel planes. Instead, we computed the expression $\mathbf{n} \times \nabla r$, which consists of a three component vector at each output location $(\mathbf{x})$. Not only is the output's size tripled, its meaning is rather obscure: the vector at each image point $(\mathbf{x})$ is the cross product of the local gradient $\nabla f(\mathbf{x})$ and the normalized, global gradient $\mathbf{n}$.

$$\mathbf{n} \times \nabla f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0 & n_z & -n_y \\ -n_z & 0 & n_x \\ n_y & -n_x & 0 \end{bmatrix} \begin{bmatrix} \partial_x \\ \partial_y \\ \partial_z \end{bmatrix} f \tag{4}$$

Each row is a finite difference operator in a plane, e.g., the third row is $(n_y \partial_x - n_x \partial_y)$.

One possible approach (but not the only one) to transform $\mathbf{n} \times \nabla f(\mathbf{x})$ back to the original space of $f(\mathbf{x})$ is to use the adjoint operator of $\mathbf{n} \times \nabla$:

$$\begin{aligned} f_{fil}(\mathbf{x}) &= (\mathbf{n} \times \nabla)'(\mathbf{n} \times \nabla) f(\mathbf{x}) \\ &= \begin{bmatrix} \partial_x & \partial_y & \partial_z \end{bmatrix} \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix} \begin{bmatrix} 0 & n_z & -n_y \\ -n_z & 0 & n_x \\ n_y & -n_x & 0 \end{bmatrix} \begin{bmatrix} \partial_x \\ \partial_z \\ \partial_z \end{bmatrix} f \\ &= [(n_z^2 + n_y^2) d_{xx} - 2 n_y n_z d_{yz} + (n_x^2 + n_z^2) d_{yy} - 2 n_x n_z d_{xz} + (n_x^2 + n_y^2) d_{zz} - 2 n_x n_y d_{xy}] f \end{aligned}$$

I do not know of an interpretation of this expression, such as a rotated and weighted Laplacian operator. A geometric interpretation of the equation does not offer any additional insights, either.

## OVERCOOKED SPAGHETTI – OR WHAT WAS WRONG WITH LOMOPLAN

In his book *Three Dimensional Filtering*(1994), Claerbout suggests the filter

$$\begin{bmatrix} \frac{\partial}{\partial x} + p_x \frac{\partial}{\partial t} \\ \\ \frac{\partial}{\partial y} + p_y \frac{\partial}{\partial t} \end{bmatrix} \tag{5}$$

to destroy a "family of dipping planes" $f(t, x, y) = g(\tau - p_x x - p_y y)$.

The cross product approach described above extends Claerbout's original 2-D-filter pair by a third filter in the third plane. Figure 4 shows a rare case in which Claerbout's two annihilation filters would not suffice to detect a plane layer volume. The image volume is a set of identical $(y, z)$-planes. The $(y, z)$-plane is a superposition of random Gaussian functions. You may visualize the volume as a box full of parallel

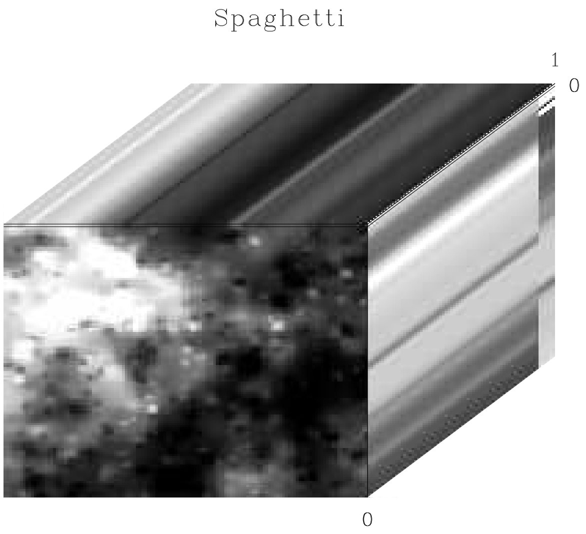


Figure 4: Synthetic data that corresponds to the sum of random Gaussian events but are constant with regard of $x$ and $y$. The side panels can be annihilated by PE filters. The front panel cannot. matt2-spaghetti [ER]

overcooked spaghetti. Claerbout's filter pair yields zero output when applied to the $(x, y)$– and $(x, z)$–plane and would lead to the incorrect conclusion that the image is a plane layer volume.

The third filter of the cross product expression (4) operates on the $(y, z)$-planes and would yield a nonzero output. Consequently, the cross product approach correctly detects that the volume is not a plane layer volume.

## ALTERNATIVE METHODS TO ESTIMATE DIP

The cross product expression may not be the most obvious approach for the dip estimation of a layered volume. I emphasize the cross product, because it allows a formulation of dip estimation by Prediction Error filters (see next section). However, other methods do exist. For example, Claerbout (1992) shows a method based on the volume's gradient and crosscorrelation function.

In general, estimating the gradient of a layered volume is harder than estimating its dip. The gradient information includes the dip information (the direction of the gradient vector) but also includes the gradient amplitude. As Figures 2 and 3 illustrate, the gradient amplitude varies within a plane layer volume even though the direction of the gradient vector remains constant. Consequently, we can estimate a single meaningful direction on an expectation volume encompassing the entire window, while the average gradient in that window would be close to zero. To average the gradient in the given window, we would have to restrict the window size to a fairly small area in which the amplitude of the gradient vector is nearly constant.

A normalized version of the amplitude vector, $f(x)/|f(x)|$, involves a division by zero at locations $\mathbf{x}$ where $f(\mathbf{x}) = 0$.

On the other hand, the cross product formulation to measure the dip of a 3-

D function is insensitive to the gradient amplitude and depends only on the gradient direction.

## PE FILTERS VERSUS DIFFERENTIAL OPERATORS

We could implement the filters in the cross product expression (4) as finite difference operators. Finite difference operators are susceptible to frequency dispersion and cannot deal with aliased data. Alternatively, we could implement Prediction Error (PE) filters that are related to these finite difference filters, but that overcome aliasing and frequency dispersion(Claerbout, 1994).

A planar or 2-D PE filter,

$$p_z \partial_x - p_x \partial_z \sim \begin{bmatrix} f & a \\ g & b \\ 1 & c \\ \cdot & d \\ \cdot & e \end{bmatrix} \tag{6}$$

corresponds to a finite-difference operator $n_z \partial_x - n_x \partial_z$. The vertical axis is $z$, the horizontal axis is $x$ and the "$\cdot$"s are zeros. Imagine all the coefficients but $d = -1$ and the given 1 have vanished. Such a filter would annihilate the appropriately sloping plane wave. Slopes that are not exact integers are also approximately extinguishable because the adjustable filter coefficients can interpolate in time.

Consequently, an alternative (but related) method for analyzing the dip of an image volume is to replace the finite difference operators implied in equation (1) by PE filters. We then find the PE filter coefficients that minimize the output of the filters convolved with the image volume. The optimal PE filter coefficients implicitly contain the dip information of the image volume, as we can easily show, by displaying the filter's amplitude spectrum.

The shape of a 2-D PE filter is constrained by its objective: the filter's height can be changed, but its width has to remain two columns wide. Two columns can destroy a single plane, which corresponds to our model of a single plane reflector dip within a sub-cube. Any additional column could destroy an additional plane and, consequently, could remove more energy than the single plane reflector model would justify. The length of the 2-D PE filter has to be larger than the maximal step-out (dip) of the input data. The coefficients in the first column of the PE filter (6) accomplish a deconvolution that is not implied in the original cross product formulation. However, these coefficients are necessary to ensure that the filter output is white (Claerbout, 1994). If the image does not resemble the plane reflector model, but, for example, the superposition of two plane layered volumes, the filter will probably try to remove both partially, rather than exclusively removing the dominant contribution.

By forcing a 1-D, two-term filter to predict both backward and forward, we avoid perfect predictability on growing 1-D exponentials(Claerbout, 1976). To prevent predictions of exponentially growing planes, we require the 2-D PE filter to extinguish two copies of the data: the original copy and a second copy with both $x$ and $z$ reversed. Dips on the reversed copy are the same as on the original, but amplitude growth on one is decay on the other.

## REFERENCES

Claerbout, J. F., 1976, Fundamentals of geophysical data processing: Blackwell.

Claerbout, J. F., 1992, Earth Soundings Analysis: Processing Versus Inversion: Blackwell Scientific Publications.

Claerbout, J. F., 1994, Applications of Three-Dimensional Filtering: Stanford Exploration Project.

Schwab, M., Holden, C., and Claerbout, J., 1996, Revealing geological discontinuities by local plane wave suppression: SEP–**92**, 29–48.