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Prestack depth migration in anisotropic heterogeneous media

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ABSTRACT

This paper presents a 2-D and 3-D prestack depth migration in anisotropic media for P-waves. Assuming an acoustic VTI medium, the double square root (DSR) equation becomes dependent only on the migration velocity field and the parameter η. In order to handle lateral velocity variation, I use the extended split-step approximation of the double square root. I tested this algorithm with two different 2-D synthetic seismic data sets in a VTI medium, and the results are encouraging. The first VTI synthetic model consisted of a set of dipping reflectors, from 0° to 90° with Thomsen's parameters $\epsilon = 0.2$ and $\delta = 0$. The second model is the anisotropic Marmousi model characterized by strong lateral velocity and η variation. In order to handle the lateral η variation, I define a number of reference η's that in the same fashion that reference velocities are defined. The resulting anisotropic prestack Marmousi section correctly imaged dipping events. In addition, I show other possible implementations of this anisotropic migration, in order to handle η variation as a function of depth and lateral coordinates. In the case where the anisotropic medium has non-zero δ, the extended split-step migration algorithm works in pseudo-depth to avoid the explicit dependency of the DSR operator on the vertical velocity.

INTRODUCTION

It is well known that in an anisotropic medium, an isotropic migration with the right velocity locates dipping reflections in an erroneous vertical and lateral position (Uzcategui, 1995; Alkhalifah, 1997b). Moreover, the final migrated image looks undermigrated. Alkhalifah (1997b) presented a 2-D prestack time migration based on the stationary phase approximation of the in-line offset wavenumber. In addition, he presented a 3-D poststack time DSR operator for P-waves as a function of η and the vertical NMO velocity V_{NMO}. This last result is very important because a vertical transversely depends on only two parameters.

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Alkhalifah et al. (1997) introduce the basis of time processing for heterogeneous anisotropic media based on the NMO-velocity and the η parameter. They show that the vertical velocity is necessary for the time-to-depth conversion but it is not necessary for imaging (or mapping). In order to avoid working in depth, Alkhalifah et al. (1997) redefine the P-wave equation in vertical time or pseudo-depth.

This paper extends the DSR operator to handle lateral velocity variations for anisotropic media for P-waves presented by Alkhalifah (1997). Using the extended split-step method, the DSR operator is approximated to include lateral velocity variations, through the definition of reference velocities for every downward continuation step (Malcotti and Biondi, 1998). The results in this paper are obtained by keeping constant the η parameter during the downward continuation and by defining a number of reference η in the same fashion as the velocity. A more general approach will be addressed in future papers.

The anisotropic migration algorithm presented in this paper is a modified version of the extended isotropic split-step depth migration that I also present in this report (Malcotti and Biondi, 1998). The main modifications are the inclusion of the η parameter, and the downward continuation is in pseudo-depth, in the same fashion as isotropic migrations work (Claerbout, 1985).

DSR FOR ANISOTROPIC MEDIA

The 3-D prestack DSR operator in frequency and wave number domain is (Claerbout, 1985; Biondi and Palacharla, 1996)

$$DSR(K_m, K_h, z, \omega) = \sqrt{\frac{w^2}{v^2_{(mS,z)}} + \frac{1}{4}[(K_{mx} - K_{hx})^2 + (K_{my} - K_{hy})^2]} + \sqrt{\frac{w^2}{v^2_{(mG,z)}} + \frac{1}{4}[(K_{mx} + K_{hx})^2 + (K_{my} + K_{hy})^2]}, \qquad (1)$$

where K_{mx} is the CDP in-line wavenumber, K_{my} is the CDP cross-line wavenumber, K_{hx} is the offset in-line wavenumber, K_{hy} is the offset cross-line wavenumber, and $v_{(mG,z)}$ and $v_{(mS,z)}$ are the velocities expressed as a function of the survey geometry.

Alkhalifah (1997) derives the dispersion equation for transversely anisotropic media. He assumes that the vertical S-wave velocity is equal to zero ($V_{S0} = 0$) and that the dispersion relation for the 3-D prestack DSR can be rewritten in a more general equation as follows:

$$DSR(K_m, K_h, z, \omega) = K_z =$$
$$\frac{v_{(mS,z)}}{v_{v(mS,v)}} \sqrt{w^2 - \frac{1}{4}\left(\frac{v^2_{(mS,z)}[(K_{mx} - K_{hx})^2 + (K_{my} - K_{hy})^2]}{1 - 2\eta_{(mS,z)}(\frac{v_{(mS,z)}}{w})^2[(K_{mx} - K_{hx})^2 + (K_{my} - K_{hy})^2]}\right)}$$

$$+\frac{v_{(mG,z)}}{v_{v(mG,z)}}\sqrt{w^2+\frac{1}{4}(\frac{v^2_{(mG,z)}[(K_{mx}+K_{hx})^2+(K_{my}+K_{hy})^2]}{1-2\eta_{(mG,z)}(\frac{v_{(mG,z)}}{w})^2[(K_{mx}+K_{hx})^2+(K_{my}+K_{hy})^2]}}), \quad (2)$$

where $v_{v(mS,v)}$ and $v_{v(mG,v)}$ are the vertical velocity as a function of the shot and receiver (survey geometry). Notice that if $\delta = 0$ the vertical velocity (V_v) is equal to the migration velocity, there is no problem plotting the results in the depth coordinate.

Equation (2) is expressed in pseudo-depth by the following equation:

$$K_z = K_\tau \frac{1}{v_v}. \tag{3}$$

Rewriting the DSR equation equation (2) in pseudo-depth (τ) for the 2-D case, we have :

$$K_\tau = \sqrt{w^2 - \frac{v^2(K_{mx}-K_{hx})^2}{1-2\frac{v^2}{w^2}\eta(K_{mx}-K_{hx})^2}} + \sqrt{w^2 - \frac{v^2(K_{mx}+K_{hx})^2}{1-2\frac{v^2}{w^2}\eta(K_{mx}+K_{hx})^2}} \tag{4}$$

Applying the split-step approximation to equation (2), gives the DSR used in a 3-D prestack depth migration (Malcotti and Biondi, 1998) as follows:

$$\begin{aligned}
&DSR(k_m, k_h, z, \omega) \cong \\
&\frac{v_{(mS,z)}}{v_{v(mS,z)}}[\sqrt{\frac{\omega^2}{v^2_{Sref(z)}} - \frac{\frac{1}{4}((K_{mx}-K_{hx})^2+(K_{my}-\overline{K}_{hy})^2)}{1-2\eta_{(z)}(\frac{v_{Sref(z)}}{w})^2((K_{mx}-K_{hx})^2+(K_{my}-K_{hy})^2)}} \\
+ \quad &\left(\frac{\omega}{v_{(mS,z)}} - \frac{\omega}{v_{Sref(z)}}\right)] \\
+ \quad &\frac{v_{(mG,z)}}{v_{v(mG,z)}}[\sqrt{\frac{\omega^2}{v^2_{Gref(z)}} - \frac{\frac{1}{4}((K_{mx}+K_{hx})^2+(K_{my}+\overline{K}_{hy})^2)}{1-2\eta_{(z)}(\frac{v_{Gref(z)}}{w})^2((K_{mx}-K_{hx})^2+(K_{my}-K_{hy})^2)}} \\
+ \quad &\left(\frac{\omega}{v_{(mG,z)}} - \frac{\omega}{v_{Gref(z)}}\right)].
\end{aligned} \tag{5}$$

where $v_{Sref(z)}$ and $v_{Gref(z)}$ are the reference velocities defined by the geometry of the survey and depth. In the case of 3-D prestack migration algorithm, this DSR is rewritten using the common-azimuth approximations (Biondi and Palacharla, 1996).

Equation (5) shows that in anisotropic media, the split-step approximation is obtained by assuming that η is constant for every depth step. In the seismic synthetic examples that I show in the results section, I use two different approaches to migrate the anisotropic Marmousi data set. In the first approach, I use a constant η. In the second approach I define a number of reference η's, in order to obtain a better migrated image of the dipping reflectors. This is an approach that gives impressive results but it should be generalized to handle lateral η variations, where η variations are the velocity variation.

Other possible solution to include lateral η variation is using different η's defined in the same fashion that reference velocities are defined but migrate with just one reference velocity (Alkhalifah 1998, personal communication).

Alkhalifah and Tsvankin (1994), assuming that the vertical S-wave velocity is equal to zero $v_{S0} = 0$, introduce an anisotropic parameter called η. This anisotropic parameter η can be written as a function of Thomsen's parameters (δ and ϵ) as follows:

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta}, \tag{6}$$

In this paper I set $\delta = 0$ to avoid working with the ratio of the vertical velocity and migration velocity in equation (5).

The variable η is expressed as a function of the NMO velocity (V_{NMO}) as follows:

$$\eta = \frac{1}{2}\frac{v_h^2}{V_{NMO}^2} - 1, \tag{7}$$

where the v_h is the horizontal velocity. For isotropic medium, $\eta = 0$ and $V_{NMO} = v_v$, where V_{NMO} is short spread NMO velocity. This parameter η contains the information about the ratio between the horizontal velocity v_h and vertical velocity.

RESULTS

In order to test the anisotropic code based on equation (5), I use two different 2-D synthetic models. The first synthetic model is characterized by a vertical and lateral linear gradient of $0.5s^{-1}$. Dipping and flat reflectors are embedded in the velocity field. The second model is the anisotropic Marmousi model generated by Alkhalifah (1997a).

The extended anisotropic split-step algorithm is based on a linear interpolation of the different downward continued wavefields with every reference velocity. Therefore, this algorithm by definition is able to handle linear lateral velocity gradient that characterized the synthetic seismic in Figure 1. In contrast, the Marmousi model has a complex velocity field that represents a challenge to the split-step anisotropic migration.

The seismic synthetic data for the first model was modeled using an analytic ray tracer for a factorized transversely isotropic medium, given by Alkhalifah (1995). The resulting zero-offset section modeled with this program is shown in Figure 1. The reflectors have a slope of 0°, 30°, 45°, 75° and 90°, and the Thomsen's parameters are $\delta = 0$ and $\epsilon = 0.2$ and $\eta = \epsilon$ (equation 6). Working with $\eta = \epsilon$ guarantees that the vertical velocity is equal to the migration velocity [equation (7)]

Figures 2 and 3 show the migrated zero-offset section with the extended anisotropic split-step depth migration for $\eta = 0$ and $\eta = 0.2$, respectively, and the correct migration velocity. This value of η correspond to a ratio between horizontal and vertical P-wave velocity of about 20% (Alkhalifah, 1997a). As expected, an isotropic migration ($\eta = 0$) looks under-migrated and it needs higher migration velocities in order to correctly image the anisotropic seismic data (Figure 2). Figure 3 shows the resulting image using the correct migration velocity and $\eta = 0.2$. In this case, I use five reference velocities to represent linear lateral change in velocity. It can be observed that the reflectors with dips 30° and 45° are well imaged. In contrast, reflectors with greater dips (75° and 90°) are not imaged because the number of reference velocities is insufficient to handle those dips.

Figure 4 shows the prestack migration image resulting from applying the extended anisotropic split-step migration for TI media. Like the zero-offset anisotropic migration (Figure 3), this prestack image was obtained by using 5 reference velocities. The first two reflectors are well imaged, although for deeper reflectors it is necessary to increase the number of reference velocities in the split-step migration.

Figure 5 shows the Marmousi velocity field used to model the anisotropic seismic data set (Alkhalifah, 1997a) . Overall, this finite difference modeling has the same survey geometry used by IFP. This includes the same source and receiver locations, an identical sampling interval and recording time, and the same minimum offset. The central frequency of this anisotropic data set is 30Hz and the maximum offset is 3575 m.

In the anisotropic Marmousi model, the parameter η is a function of lateral coordinates and depth. The η field was created following the original velocity field (Figure 5) and honoring a linear variation of the horizontal velocity in depth. Therefore, velocities greater than $2500 m/s$ and lesser than the water velocity have associated a $\eta = 0$.

Figure 6 shows the prestack anisotropic migration with 5 reference velocities and 5 reference η's. The reference η's are calculated in every depth step during the downward continuation. This prestack migrated section is a good image. It is important for imaging to take care of the vertical and lateral variation of η is important because η anomalies can cause small triplications in the wavefront (Alkhalifah et al., 1997). In addition, if the anisotropic migration with constant η is performed on this data set, dipping reflectors would be imaged with a smaller dip than in the original model

CONCLUSION

I presented a P-wave depth migration for transversely anisotropic media based on an extended split-step operator. This TI migration operator resulted to be kinematically accurate on the synthetic model with constant $\eta = \delta$. I presented a generalization of the extended split-step migration in VTI media that handle lateral variations of the parameter η by defining different reference η's in a similar way reference velocities are defined. This approach allowed the anisotropic migration image steep events in the Marmousi model. In the same way that the accuracy of the isotropic split-step operator depends on the number of reference velocities used, the anisotropic depth migration depends on the number of reference velocities and η's. The resulting migrated images with the Marmousi data set show that migrating with different reference η is a good approach to handle lateral variation of this parameter.

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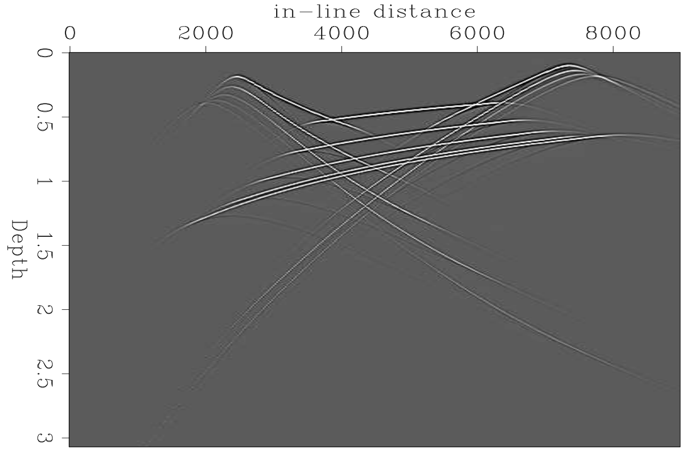

Figure 1: Anisotropic synthetic data: zero-offset data section. The velocity is represented by a strong vertical and lateral linear gradient of $0.5s^{-1}$. hermes2-SYNT [CR]

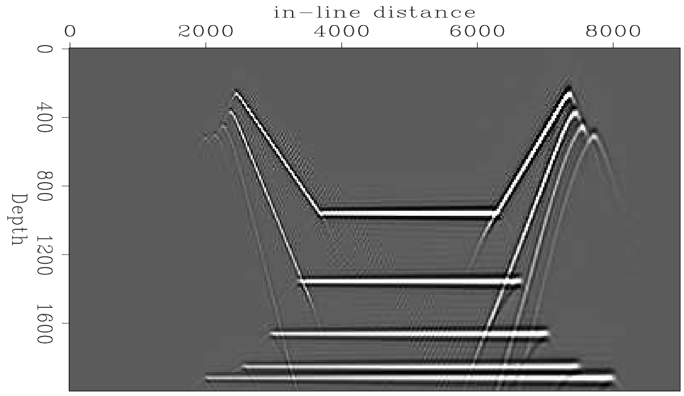

Figure 2: Extended split-step Zero-offset isotropic migration of the anisotropic synthetic data (Figure 1). In this case $\eta = \epsilon = 0.2$ and $\delta = 0$. hermes2-SUNTZEROisomig [CR]

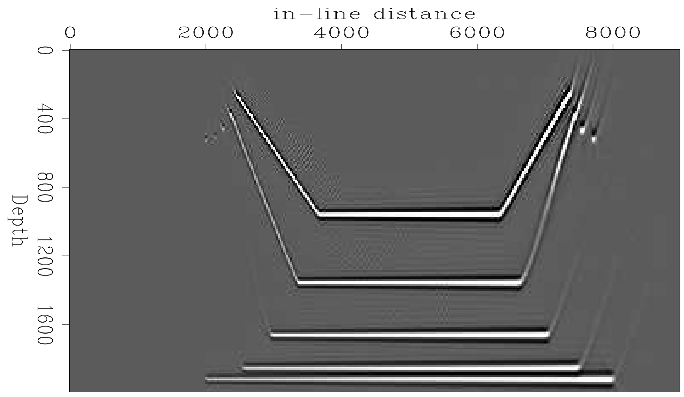

Figure 3: Extended split-step zero-offset anisotropic migration of the synthetic data modeled with $\eta = 0.1$ (Figure 1). In this case $\eta = \epsilon = 0.2$ and $\delta = 0$. hermes2-SUNTZEROANISOmig [CR]

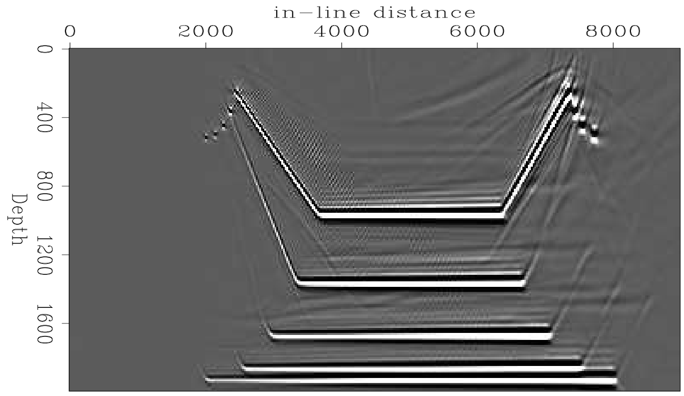

Figure 4: Extended split-step anisotropic prestack migration of the anisotropic synthetic data (Figure 1). This prestack imaged is obtained by using 5 reference velocities and constant $\eta = 0.2$. hermes2-SYNTPRE [CR]

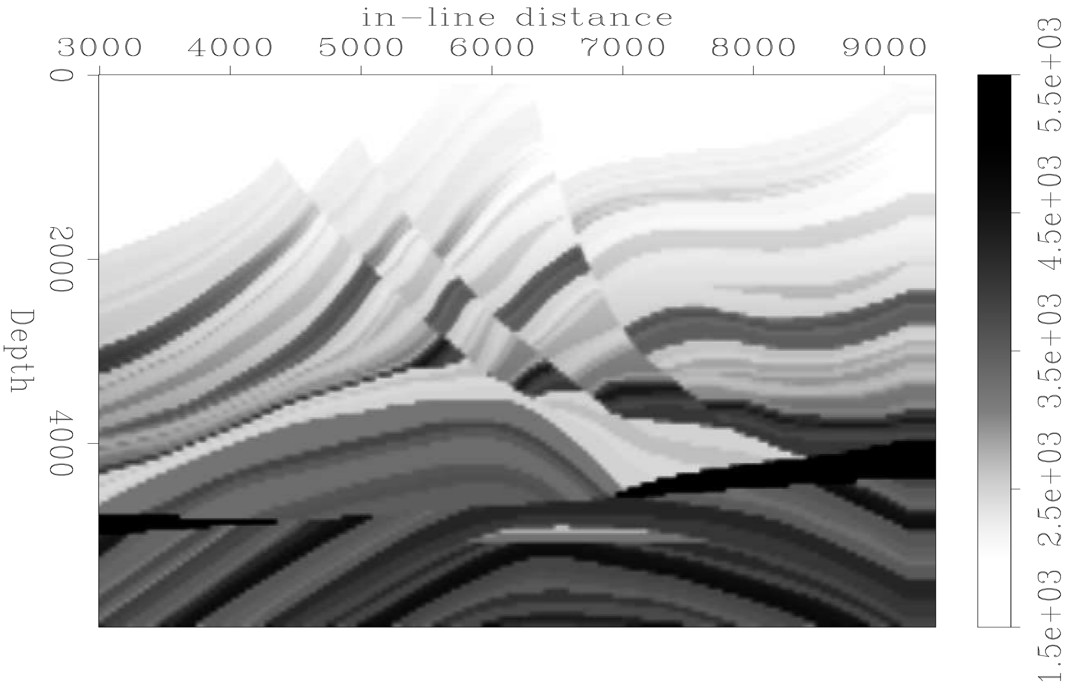

Figure 5: Anisotropic synthetic data: Marmousi velocity model (focusing velocity) (Versteeg, 1993; Alkhalifah, 1997a). hermes2-marmvel [CR]

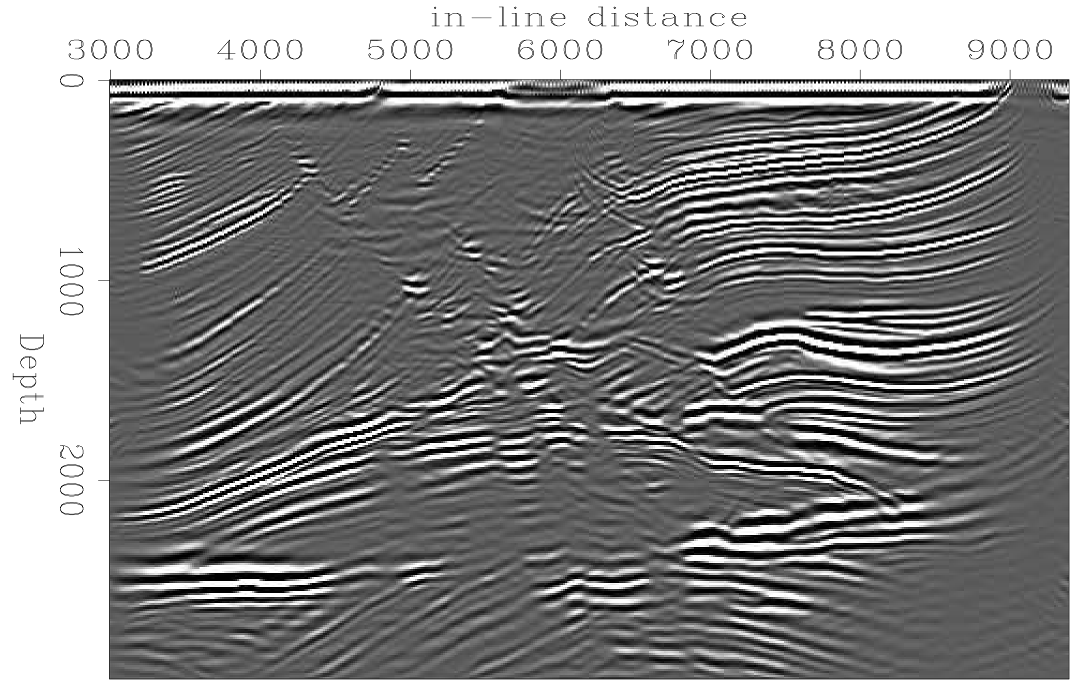

Figure 6: Anisotropic prestack depth migration using 5 reference velocities and 5 reference η's. hermes2-MAR15TIPRE [CR]