SYMMETRY - SO WHAT?

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Space-time acoustic imaging algorithms appear at first glance very different from their f-k counterparts. Actually, space-time processes can be constructed which are very similar to f-k processes, apparently because hyperbolas in space-time are associated with hyperbolas in f-k.

This similarity is most apparent for the NMO-stack operation. In normal space-time processing this operation has two parts. We first have a time shift

$$t \rightarrow t_n = \sqrt{t^2 - \frac{4h^2}{v^2}} \tag{1}$$

then a sum over offsets h. In f-k space, the same operation can be accomplished by first a frequency shift

$$\omega \rightarrow \omega_n = \sqrt{\omega^2 - \frac{v^2 k_h^2}{4}} \tag{2}$$

then a sum over offset spatial frequency k_h. A space-time hyperbolic moveout correction and stack translates into a hyperbolic moveout and stack in f-k space also. Granted, the two operations should not produce identical results on real data. Noise, multiples, statics, discrete and finite data sampling all give rise to differences, and in fact, the f-k algorithm above will give any data a 45^o phase shift.

Nevertheless, the same basic effect is achieved by the same operation in other space.

Much of this symmetry carries over to zero-offset migration. For this operation the f-k representation is a hyperbolic frequency shift

$$\omega \rightarrow \omega_n = \sqrt{\omega^2 - \frac{v^2 k_y^2}{4}} \tag{3}$$

The time domain representation may assume many guises. An analog to (3) can be constructed by noting that zero-offset traveltime for a point scatterer at $(y_o, z_o = vt_o/2)$ is

$$t(y) = \sqrt{t_o^2 + \frac{4(y - y_o)^2}{v^2}} \tag{4}$$

The migration of point (y_o, t_o) may be accomplished by summing over its diffraction hyperbola, that is,

$$t \rightarrow t_o = \sqrt{t^2 - \frac{4(y - y_o)^2}{v^2}} \tag{5}$$

then summing over y. As for the f-k NMO-stack representation, a 45^o phase shift is left in the data. The space-time migration requires a sum (over midpoint) with no f-k analog, but in both cases the hyperbolic shift [(3), (5)] appears.

Finally, consider the imaging process for unstacked data. In f-k space, migration is governed by the "double square root" dispersion relation

$$k_z = \sqrt{\frac{\omega^2}{v^2} - \left(\frac{k_h + k_y}{2}\right)^2} + \sqrt{\frac{\omega^2}{v^2} - \left(\frac{k_h - k_y}{2}\right)^2} \tag{6}$$

This equation hardly looks like a hyperbola. However, if we solve for ω

we discover a single square root equation

$$\omega = \frac{k_z v}{2} \sqrt{\left(1 + \frac{k_h^2}{k_z^2}\right)\left(1 + \frac{k_y^2}{k_z^2}\right)} \tag{7}$$

If k_y/k_z (dip) is fixed, the remaining variables define a family of hyperbolas. Another family is defined if k_h/k_z is fixed.

It should come as no great surprise that a "double square root" equation exists in space-time too. The two-way traveltime for a point scatterer at (y_o, t_o) is

$$t = \frac{1}{2}\left[\sqrt{t_o^2 + \frac{4(y - y_o + h)^2}{v^2}} + \sqrt{t_o^2 + \frac{4(y - y_o - h)^2}{v^2}}\right] \tag{8}$$

Solving for t_o, we get a single square root equation

$$t_o = t\left[\left(1 - \frac{4h^2}{v^2 t^2}\right)\left(1 - \frac{4(y - y_o)^2}{v^2 t^2}\right)\right]^{\frac{1}{2}} \tag{9}$$

If h/vt is fixed (a radial trace - see Ottolini, this report) the remaining variables, just as in f-k space, define a family of hyperbolas.

We can accomplish f-k imaging of unstacked data by first a frequency shift

$$\omega \rightarrow \omega_n = \frac{k_z v}{2} \tag{10}$$

then a sum over offset spatial frequency k_h, and a 45^o phase shift. The same process in space-time could be a time shift

$$t_o \leftarrow t \ , \tag{11}$$

then summations over both offset h and basement point y, and a phase shift. Note that the arrows in (10) and (11) point the opposite way. Inside the double square root appears unmigrated frequency [equation (6)] but migrated time [equation (8)].

The symmetry between f-k and space-time is summarized in the table on the following page.

operation	time (before) time (after)	frequency (before) frequency (after)
NMO + stack	$t = \sqrt{t_o^2 + \frac{4h^2}{v^2}}$ $t_o = \sqrt{t^2 - \frac{4h^2}{v^2}}$	$\omega = \sqrt{\omega_n^2 + \frac{v^2 k_h^2}{4}}$ $\omega_n = \sqrt{\omega^2 - \frac{v^2 k_n^2}{4}}$
migration (zero-offset)	$t = \sqrt{t_o^2 + \frac{4(y - y_o)^2}{v^2}}$ $t_o = \sqrt{t^2 - \frac{4(y - y_o)^2}{v^2}}$	$\omega = \sqrt{\omega_n^2 + \frac{v^2 k_y^2}{4}}$ $\omega_n = \sqrt{\omega^2 - \frac{v^2 k_y^2}{4}}$
migration (unstacked data)	$t = \frac{1}{2}\left[\sqrt{t_o^2 + \frac{4}{v^2}(y - y_o - h)^2} + \sqrt{t_o^2 + \frac{4}{v^2}(y - y_o - h)^2}\right]$ $t_o = t\sqrt{\left(1 - \frac{4h^2}{v^2 t^2}\right)\left(1 - \frac{4(y - y_o)^2}{v^2 t^2}\right)}$	$\omega = \omega_n\sqrt{\left(1 + \frac{v^2 k_n^2}{4\omega_n^2}\right)\left(1 + \frac{v^2 k_y^2}{4\omega_n^2}\right)}$ $\omega_n = \frac{1}{2}\left[\sqrt{\omega^2 - \frac{(k_h + k_y)^2 v^2}{4}} + \sqrt{\omega^2 - \frac{(k_h - k_y)^2 v^2}{4}}\right]$

all of which says that the wave equation really is hyperbolic