Chapter 1

Introduction to imaging

1.1 EXPLODING REFLECTORS

The basic equipment for reflection seismic prospecting is a source for impulsive sound waves, a geophone (something like a microphone), and a multichannel waveform display system. A survey line is defined along the earth's surface. It could be the path for a ship, in which case the receiver is called a hydrophone. About every 25 meters the source is activated, and the echoes are recorded nearby. The sound source and receiver have almost no directional tuning capability because the frequencies that penetrate the earth have wavelengths longer than the ship. Consequently, echoes can arrive from several directions at the same time. It is the joint task of geophysicists and geologists to interpret the results. Geophysicists assume the quantitative, physical, and statistical tasks. Their main goals, and the goal to which this book is mainly directed, is to make good pictures of the earth's interior from the echoes.

1.1.1 A powerful analogy

Figure 1.1 shows two wave-propagation situations. The first is realistic field sounding. The second is a thought experiment in which the reflectors in the earth suddenly explode. Waves from the hypothetical explosion propagate up to the earth's surface where they are observed by a hypothetical string of geophones.

Notice in the figure that the ray paths in the field-recording case seem to be the same as those in the exploding-reflector case. It is a great conceptual advantage to imagine that the two wavefields, the observed and the hypothetical, are indeed the same. If they are the same, the many thousands of experiments that have really been done can be ignored, and attention can be focused on the one hypothetical experiment. One obvious difference between the two cases is that in the field geometry waves must first go down and then return upward along the same path, whereas in the hypothetical experiment they just go up. Travel time in field experiments could

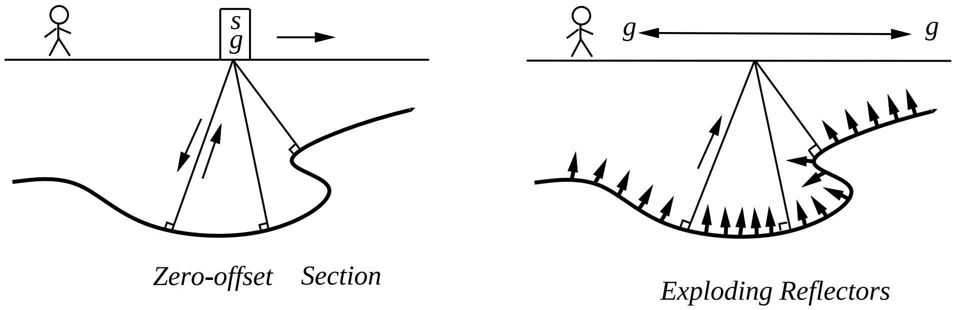

Figure 1.1: Echoes collected with a source-receiver pair moved to all points on the earth's surface (left) and the "exploding-reflectors" conceptual model (right). xrf-expref [NR]

be divided by two. In practice, the data of the field experiments (two-way time) is analyzed assuming the sound velocity to be half its true value.

1.1.2 Huygens secondary point source

Waves on the ocean have wavelengths comparable to those of waves in seismic prospecting (15-500 meters), but ocean waves move slowly enough to be seen. Imagine a long harbor barrier parallel to the beach with a small entrance in the barrier for the passage of ships. This is shown in Figure 1.2. A plane wave incident on the barrier from the open ocean will send a wave through the gap in the barrier. It is an observed fact that the wavefront in the harbor becomes a circle with the gap as its center. The difference between this beam of water waves and a light beam through a window is in the ratio of wavelength to hole size.

Linearity is a property of all low-amplitude waves (not those foamy, breaking waves near the shore). This means that two gaps in the harbor barrier make two semicircular wavefronts. Where the circles cross, the wave heights combine by simple linear addition. It is interesting to think of a barrier with many holes. In the limiting case of very many holes, the barrier disappears, being nothing but one gap alongside another. Semicircular wavefronts combine to make only the incident plane wave. Hyperbolas do the same. Figure 1.3 shows hyperbolas increasing in density from left to right. All those waves at nonvertical angles must somehow combine with one another to extinguish all evidence of anything but the plane wave.

A Cartesian coordinate system has been superimposed on the ocean surface with x going along the beach and z measuring the distance from shore. For the analogy with reflection seismology, people are confined to the beach (the earth's surface) where they make measurements of wave height as a function of x and t. From this data they

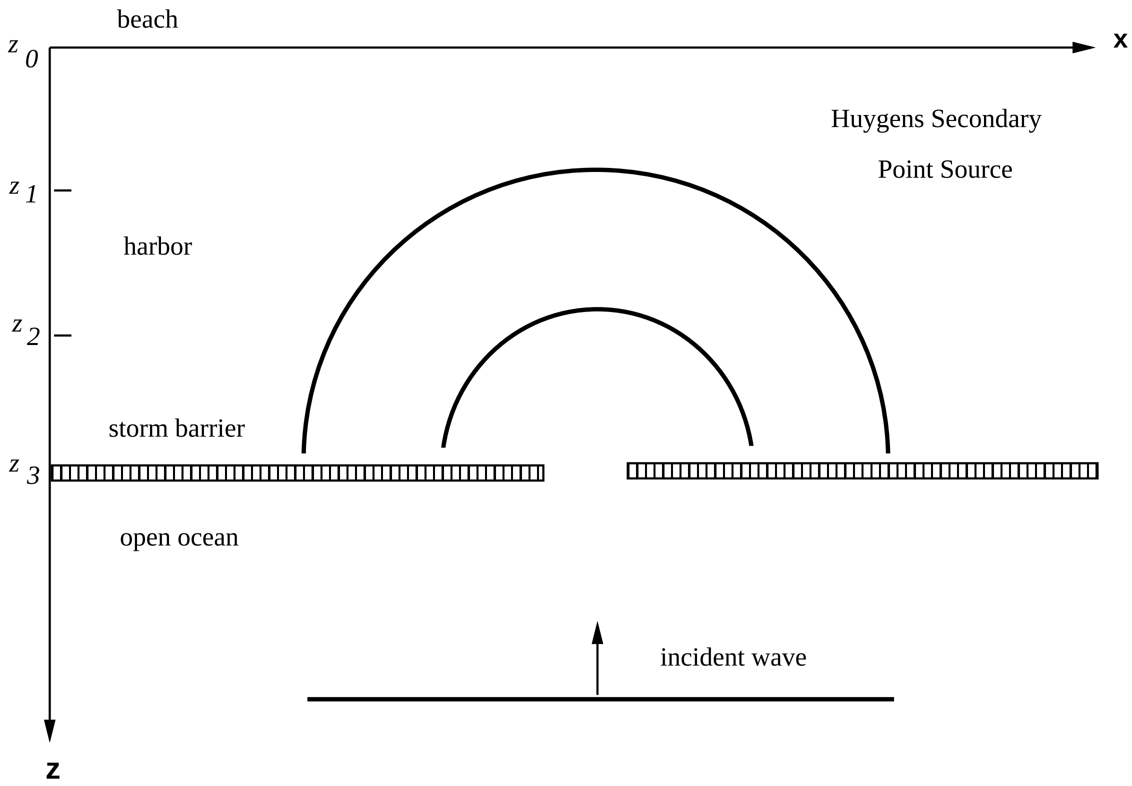

Figure 1.2: Waves going through a gap in a barrier have semicircular wavefronts (if the wavelength is long compared to the gap size). xrf-storm [NR]

Figure 1.3: A barrier with many holes (top). Waves, (x, t)-space, seen beyond the barrier (bottom). xrf-storm2 [ER]

can make inferences about the existence of gaps in the barrier out in the (x, z)-plane. Figure 1.4a shows the arrival time at the beach of a wave from the ocean through a gap. The earliest arrival occurs nearest the gap. What mathematical expression

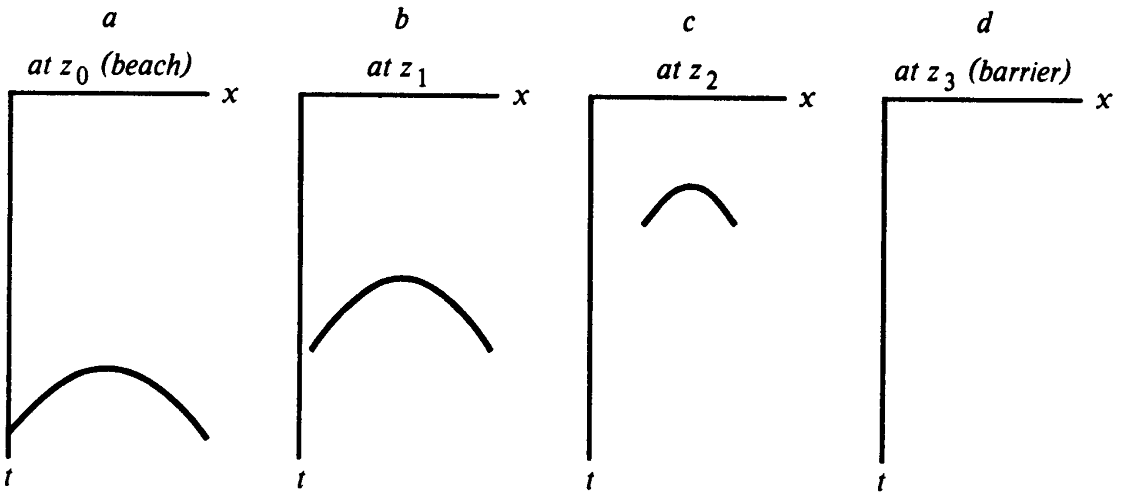

Figure 1.4: The left frame shows the hyperbolic wave arrival time seen at the beach. Frames to the right show arrivals at increasing distances out in the water. The x-axis is compressed from Figure 1.2. (Gonzalez) $\boxed{\text{xrf-dc}}$ [NR]

determines the shape of the arrival curve seen in the (x, t)-plane?

The waves are expanding circles. An equation for a circle expanding with velocity v about a point (x_3, z_3) is

$$(x - x_3)^2 \quad + \quad (z - z_3)^2 \quad = \quad v^2 t^2 \tag{1.1}$$

Considering t to be a constant, i.e. taking a snapshot, equation (1.1) is that of a circle. Considering z to be a constant, it is an equation in the (x, t)-plane for a hyperbola. Considered in the (t, x, z)-volume, equation (1.1) is that of a cone. Slices at various values of t show circles of various sizes. Slices of various values of z show various hyperbolas. Figure 1.4 shows four hyperbolas. The first is the observation made at the beach $z_0 = 0$. The second is a hypothetical set of observations at some distance z_1 out in the water. The third set of observations is at z_2, an even greater distance from the beach. The fourth set of observations is at z_3, nearly all the way out to the barrier, where the hyperbola has degenerated to a point. All these hyperbolas are from a family of hyperbolas, each with the same asymptote. The asymptote refers to a wave that turns nearly 90° at the gap and is found moving nearly parallel to the shore at the speed dx/dt of a water wave. (For this water wave analogy it is presumed—incorrectly—that the speed of water waves is a constant independent of water depth).

If the original incident wave was a positive pulse, the Huygens secondary source must consist of both positive and negative polarities to enable the destructive interference of all but the plane wave. So the Huygens waveform has a phase shift. In the next section, mathematical expressions will be found for the Huygens secondary source. Another phenomenon, well known to boaters, is that the largest amplitude of the Huygens semicircle is in the direction pointing straight toward shore. The amplitude drops to zero for waves moving parallel to the shore. In optics this amplitude drop-off with angle is called the *obliquity factor.*

1.1.3 Migration defined

A dictionary gives many definitions for the word *run.* They are related, but they are distinct. Similarly, the word *migration* in geophysical prospecting has about four related but distinct meanings. The simplest is like the meaning of the word *move.* When an object at some location in the (x, z)-plane is found at a different location at a later time t, then we say it *moves.* Analogously, when a wave arrival (often called an *event*) at some location in the (x, t)-space of geophysical observations is found at a different position for a different survey line at a greater depth z, then we say it *migrates.*

To see this more clearly, imagine the four frames of Figure 1.4 being taken from a movie. During the movie, the depth z changes beginning at the beach (the earth's surface) and going out to the storm barrier. The frames are superimposed in Figure 1.5a. Mainly what happens in the movie is that the event migrates upward toward

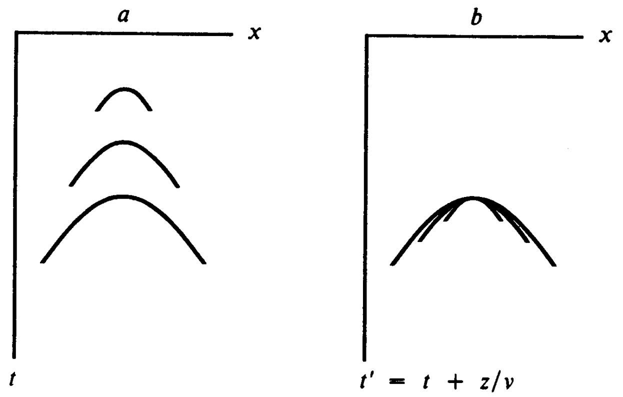

Figure 1.5: Left shows a superposition of the hyperbolas of Figure 1.4. At the right the superposition incorporates a shift, called retardation $t' = t + z/v$, to keep the hyperbola tops together. (Gonzalez) xrf-dcretard [NR]

$t = 0$. To remove this dominating effect of vertical translation make another superposition, keeping the hyperbola tops all in the same place. Mathematically, the time t axis is replaced by a so-called *retarded* time axis $t' = t + z/v$, shown in Figure 1.5b. The second, more precise definition of *migration* is the motion of an event in (x, t')-space as z changes. After removing the vertical shift, the residual motion is mainly a shape change. By this definition, hyperbola tops, or horizontal layers, do not migrate.

The hyperbolas in Figure 1.5 really extend to infinity, but the drawing cuts each one off at a time equal $\sqrt{2}$ times its earliest arrival. Thus the hyperbolas shown depict only rays moving within 45° of the vertical. It is good to remember this, that the ratio of first arrival time on a hyperbola to any other arrival time gives the cosine of the angle of propagation. The cutoff on each hyperbola is a ray at 45°. Notice that the end points of the hyperbolas on the drawing can be connected by a straight line. Also, the slope at the end of each hyperbola is the same. In physical space, the angle of any ray is $\tan\theta = dx/dz$. For any seismic event, the slope $v\,dt/dx$ is $\sin\theta$, as you can see by considering a wavefront intercepting the earth's surface at angle θ. So, energy moving on a straight line in physical (x,z)-space migrates along a straight line in data (x,t)-space. As z increases, the energy of all angles comes together to a focus. The focus is the exploding reflector. It is the gap in the barrier. This third definition of migration is that it is the process that somehow pushes observational data—wave height as a function of x and t —from the beach to the barrier. The third definition stresses not so much the motion itself, but the transformation from the beginning point to the ending point.

To go further, a more general example is needed than the storm barrier example. The barrier example is confined to making Huygens sources only at some particular z. Sources are needed at other depths as well. Then, given a wave-extrapolation process to move data to increasing z values, exploding-reflector images are constructed with

$$\text{Image } (x,z) \quad = \quad \text{Wave } (t=0,x,z) \tag{1.2}$$

The fourth definition of migration also incorporates the definition of *diffraction* as the opposite of migration.

$$\begin{array}{ccc} \text{observations} & & \text{model} \\ \boxed{\begin{array}{c} z=0 \\ \\ \text{all } t \end{array}} & \begin{array}{c} \text{migration} \\ \longrightarrow \\ \longleftarrow \\ \text{diffraction} \end{array} & \boxed{\begin{array}{c} t=0 \\ \\ \text{all } z \end{array}} \end{array}$$

Diffraction is sometimes regarded as the natural process that creates and enlarges hyperboloids. *Migration* is the computer process that does the reverse.

Another aspect of the use of the word *migration* arises in chapter 6, where the horizontal coordinate can be either shot-to-geophone midpoint y, or offset h. Hyperboloids can be downward continued in both the (y,t)- and the (h,t)-plane. In the (y,t)-plane this is called *migration* or *imaging,* and in the (h,t)-plane it is called *focusing* or *velocity analysis.*

1.1.4 An impulse in the data

The Huygens diffraction takes an isolated pulse function (delta function) in (x,z)-space and makes it into a hyperbola in (x,t)-space at $z = 0$. The converse is to start from a delta function in (x,t)-space at $z = 0$. This converse refers to a seismic survey in which no echoes are recorded except at one particular location, and at that location only one echo is recorded. What earth model is consistent with such observations? As shown in Figure 1.6 this earth must contain a spherical mirror whose center is at the anomalous recording position.

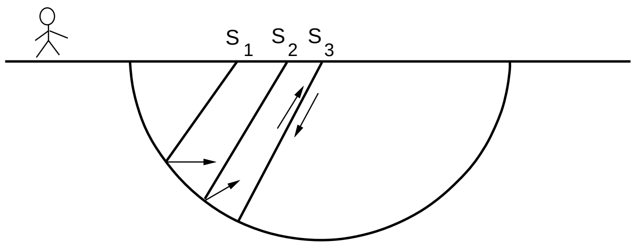

Figure 1.6: When the seismic source S is at the exact center of a semicircular mirror, then, and only then, will an echo return to the geophone at the source. This semicircular reflector is the logical consequence of a dataset where one echo is found at only one place on the earth. xrf-semicirc [NR]

It is unlikely that the processes of nature have created many spherical mirrors inside the earth. But when we look at migrated geophysical data, we often see spherical mirrors. Obviously, such input data contains impulses that are not consistent with the wave-propagation theory being explained here. This illustrates why petroleum prospectors study reflection seismic data processing, even though they personally plan to write no processing programs. The raw data is too complex to comprehend. The processed data gives an earth model, but its reliability is difficult to know. You may never plan to build an automobile, but when you drive alone far out into the desert, you should know as much as you can about automobiles.

1.1.5 Hand migration

Given a seismic event at (x_0, t_0) with a slope $p = dt/dx$, let us determine its position (x_m, t_m) after migration. Consider a planar wavefront at angle θ to the earth's surface traveling a distance dx in a time dt. Assuming a velocity v we have the wave angle in terms of measurable quantities.

$$\sin\theta \quad = \quad \frac{v\ dt}{dx} \quad = \quad p\,v \tag{1.3}$$

The vertical travel path is less than the angled path by

$$t_m \quad = \quad t_0 \cos\theta \quad = \quad t_0\,\sqrt{1 \ - \ p^2\,v^2} \tag{1.4}$$

A travel time t_0 and a horizontal component of velocity $v\sin\theta$ gives the lateral location after migration:

$$x_m \quad = \quad x_0 \ - \ t_0\,v\,\sin\theta \quad = \quad x_0 \ - \ t_0\,p\,v^2 \tag{1.5}$$

Consideration of a hyperbola migrating towards its apex shows why (1.5) contains a minus sign. Equations (1.4) and (1.5) are the basic equations for manual migration of reflection seismic data. They tell you where the point migrates, but they do not tell you how the slope p will change.

1.1.6 Reflector steepening

Consider a vertical wall, a limiting case of a dipping bed. Its reflections, the asymptotes of a hyperbola, have a nonvertical steepness. This establishes that migration increases the apparent steepness of dipping beds. I use the words *apparent steepness* because it is the slope as seen in the (x, t)-plane that has steepened. Migration really produces its output in z, but z/v is often overlain on t to create a *migrated time section* . When we say a hyperbola migrates to its apex, we are of course thinking of the migrated time section. Let us determine the steepening as a function of angle.

Consider a point $(x_{0+}, t_{0+}) = (x_0 + \Delta,\, t_0 + p\Delta)$ neighboring the original point (x_0, t_0). By equations (1.4) and (1.5), this neighbor migrates to

$$
\begin{aligned}
t_{m+} &= (t_0 + p\,\Delta)\,\sqrt{1 - p^2\, v^2} && (1.6)\\
x_{m+} &= x_0 + \Delta - (t_0 + p\,\Delta)\, p\, v^2 && (1.7)
\end{aligned}
$$

Now we compute the stepout p_m of the migrated event

$$
\begin{aligned}
p_m &= \frac{dt_{m+}}{dx_{m+}} = \frac{dt_{m+}/d\Delta}{dx_{m+}/d\Delta} \\
p_m &= \frac{p\,\sqrt{1 - p^2\, v^2}}{1 - p^2\, v^2} = \frac{p}{\sqrt{1 - p^2\, v^2}} = \frac{\tan\,\theta}{v} && (1.8)
\end{aligned}
$$

So slopes on migrated time sections, like slopes in Cartesian space, imply tangents of angles while slopes on unmigrated time sections imply sines.

It may seem paradoxical that dipping beds change slope on migration whereas flanks of hyperbolas do not change slope during downward continuation. One reason is that migration is downward continuation *plus* imaging (selecting $t = 0$). Another reason is that a hyperbola is a special event that comes from a single source at a single depth whereas a dipping bed is a superposition of point sources from different depths. Figure 1.7 shows how points making up a line reflector diffract to a line reflection, and how points making up a line reflection migrate to a line reflector.

1.1.7 Limitations of the exploding-reflector concept

The exploding-reflector concept is a powerful and fortunate analogy. For people who spend their time working entirely on data interpretation rather than on processing, the exploding-reflector concept is more than a vital crutch. It's the only means of transportation! But for those of us who work on data processing, the exploding-reflector concept has a serious shortcoming. No one has yet figured out how to extend the concept to apply to data recorded at nonzero offset. Furthermore, most data is recorded at rather large offsets. In a modern marine prospecting survey, there is not one hydrophone, but hundreds, which are strung out in a cable towed behind the ship. The recording cable is typically 2-3 kilometers long. Drilling may be about 3

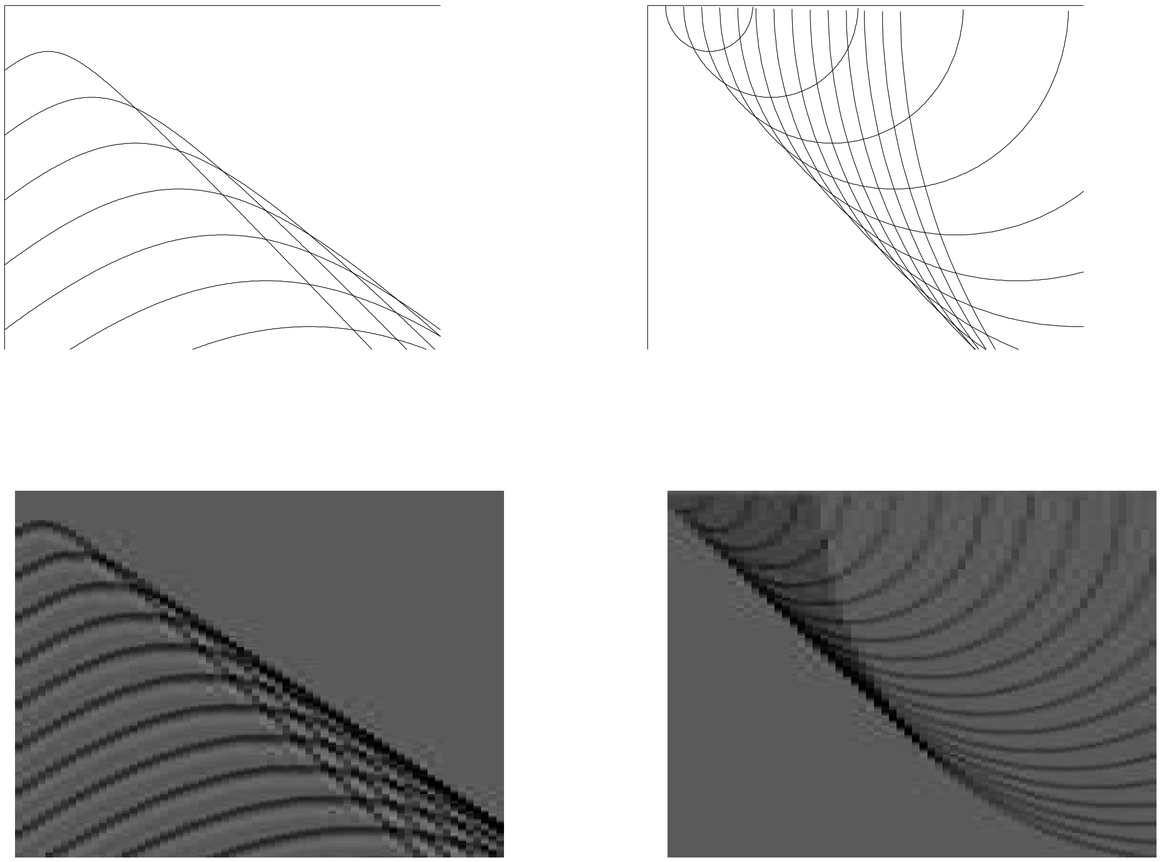

Figure 1.7: Left is a superposition of many hyperbolas. The top of each hyperbola lies along a straight line. That line is like a reflector, but instead of using a continuous line, it is a sequence of points. Constructive interference gives an apparent reflection off to the side. Right shows a superposition of semicircles. The bottom of each semicircle lies along a line that could be the line of an observed plane wave. Instead the plane wave is broken into point arrivals, each being interpreted as coming from a semicircular mirror. Adding the mirrors yields a more steeply dipping reflector. xrf-dip [ER]

kilometers deep. So in practice the angles are big. Therein lie both new problems and new opportunities, none of which will be considered until chapter 6.

Furthermore, even at zero offset, the exploding-reflector concept is not quantitatively correct. For the moment, note three obvious failings: First, Figure 1.8 shows rays that are not predicted by the exploding-reflector model. These rays will be

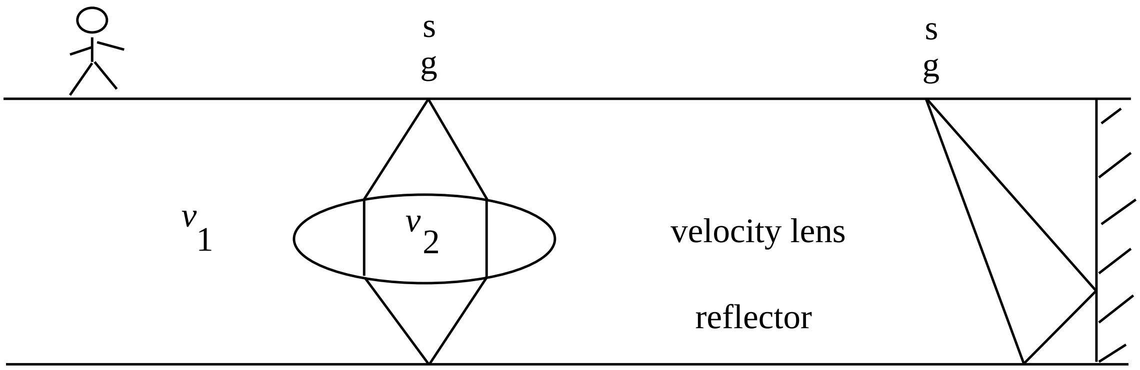

Figure 1.8: Two rays, not predicted by the exploding-reflector model, that would nevertheless be found on a zero-offset section. xrf-fail [NR]

present in a zero-offset section. Lateral velocity variation is required for this situation to exist.

Second, the exploding-reflector concept fails with multiple reflections. For a flat sea floor with a two-way travel time t_1, multiple reflections are predicted at times $2t_1$, $3t_1$, $4t_1$, etc. In the exploding-reflector geometry the first multiple goes from reflector to surface, then from surface to reflector, then from reflector to surface, for a total time $3t_1$. Subsequent multiples occur at times $5t_1$, $7t_1$, etc. Clearly the multiple reflections generated on the zero-offset section differ from those of the exploding-reflector model.

The third failing of the exploding-reflector model is where we are able to see waves bounced from both sides of an interface. The exploding-reflector model predicts the waves emitted by both sides have the same polarity. The physics of reflection coefficients says reflections from opposite sides have opposite polarities.

1.1.8 Plate tectonics example

Plate tectonic theory says the ocean floors are made of thin plates that are formed at volcanic ridges near the middle of the oceans. These plates move toward trenches in the deepest part of the ocean where they plunge back down into the earth. The best evidence for the theory is the lack of old rocks on the floors of the earth's oceans. Generally, continents are older rocks jostled by the younger moving oceanic plates. The formation of plates by mid-ocean ridge volcanism is readily observed in a variety of ways. Whether the plates really do plunge at the trenches is not so clear observationally. The evidence comes from earthquake locations and from reflection seismology. Figure 1.9 shows some reflection data from the Japan trench. Two reflections dominate, the sea floor reflection and a deeper layer dipping down to

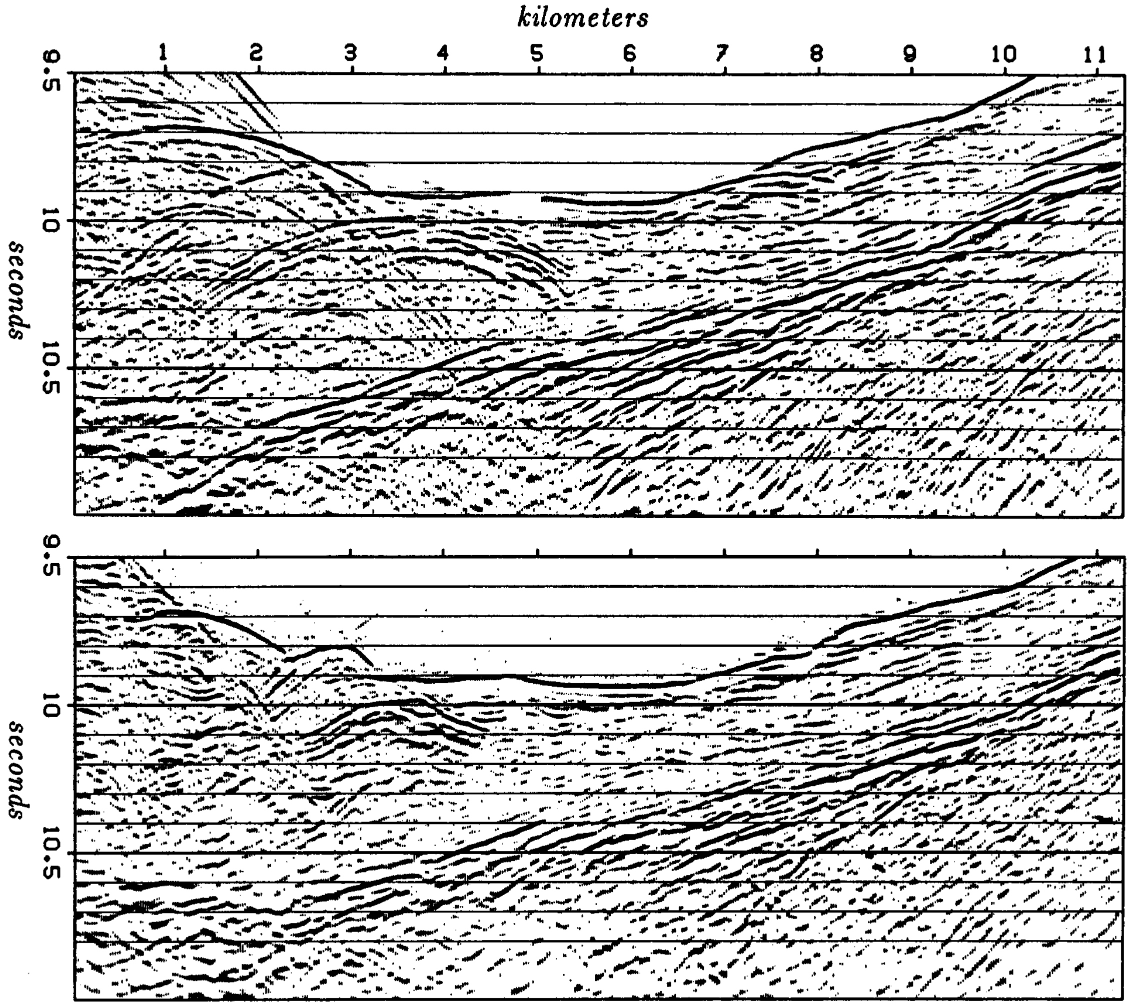

Figure 1.9: Top is 11 kilometers of reflection data from a survey line across the Japan trench (Tokyo University Oceanographic Research Institute). Bottom shows the result of migration processing. (Ottolini) xrf-japan [NR]

the left. This latter is presumably the top part of a plate that is beginning its descent into the earth. We can examine it for evidence of bending downward, such as tension fractures near the surface. (The topmost layer is soft recent marine sediment loosely attached to the plate). Notice that the top of the plot is not zero time. The time axis runs from 9.5 to 11.0 seconds. Before 9.5 sec there are no echoes—we are waiting for the waves to go between the ship and the ocean floor. Hyperbolic reflections around kilometers 1-3 are collapsed by migration to form interesting "blocky" shapes. Look at the sea floor topography near kilometer 8 and the difference between migrated and unmigrated data sections. After migration, the sea floor diffraction hyperbolas move away from the plate echo (kilometer 4). Fractures (especially the one at 6.2 km) are more sharply defined. Finally, if the plate bends downward, it is not apparent from the data given. The bending question really requires a more detailed analysis of lateral variation in seismic velocity.

For an example with petroleum interest, see Figure 1.10, data from offshore Texas. Sediments are dropped where coastal rivers enter the Gulf of Mexico. The added

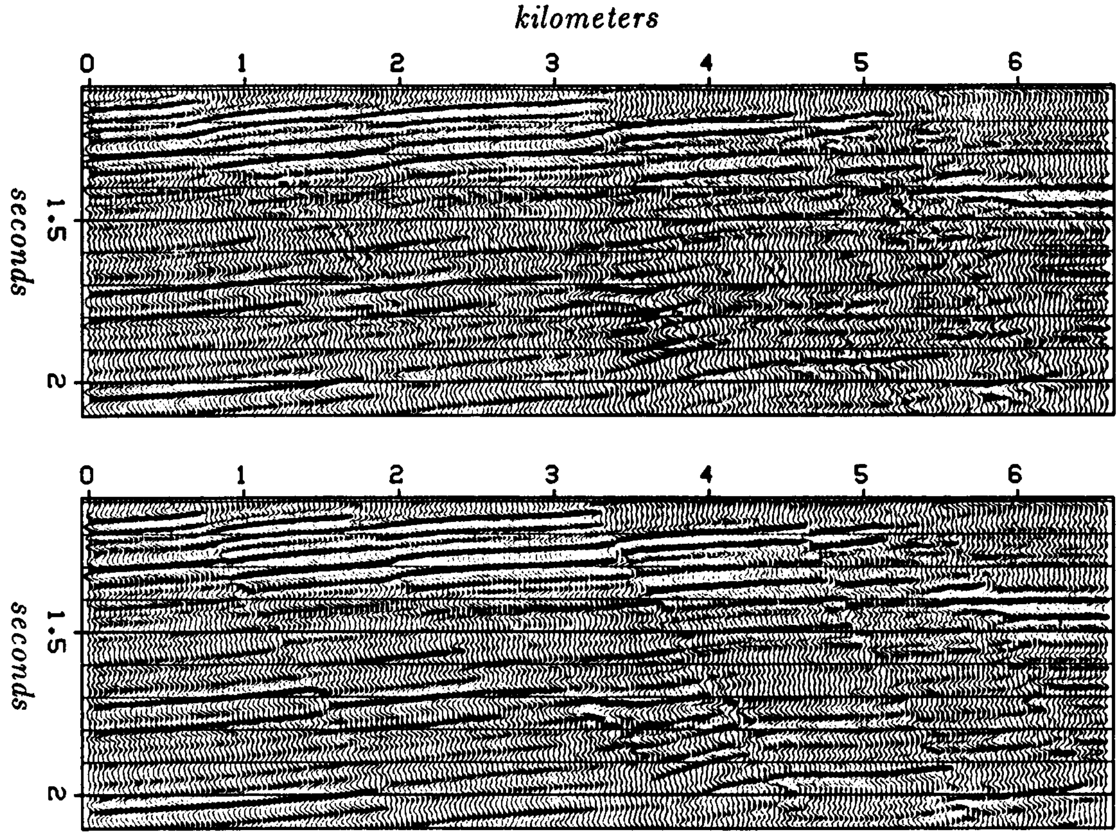

Figure 1.10: Top is 6.5 kilometers of reflection data from a survey line offshore from the Texas coast of the Gulf of Mexico. Bottom shows the result of migration processing. (Rothman). xrf-growth [NR]

weight causes slumping along steep faults. After a permeable sandstone layer has been identified by drilling, its reflection can be extrapolated up dip to the nearest fault on data like Figure 1.10. The fault is likely to break the continuity of the permeability trapping the upward flowing hydrocarbons. A sandstone at this depth can have a porosity of 25%. Assume a seismic velocity of 2.2 km/sec. Deduce the scale between physical volume and the data in Figure 1.10. Comparing the value of a volume of oil to the size of that same volume on Figure 1.10, you can see the importance of good images.

EXERCISES:

1 Prove the Pythagorean theorem, that is, the length of the hypotenuse vt of a

right triangle is determined by $x^2 + z^2 = v^2t^2$. Hint:

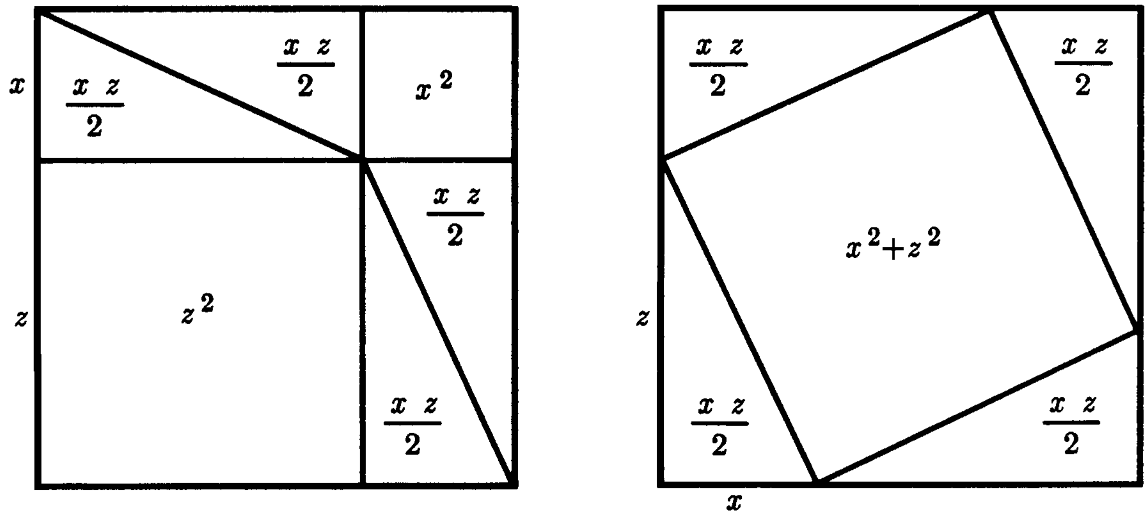

Figure 1.11: $\boxed{\text{xrf-pythag}}$ [NR]

2 Compute propagation angles for the hyperbola flanks in Figure 1.9.

3 Using the result of exercise 2, deduce the plunge angle of the plate.

4 How deep is the Japan trench (water velocity is 1.5 km/sec)?

5 On the Gulf Coast data, which direction is offshore? Why?

1.2 SURROUNDING DISCIPLINES

The previous section considered the *geometrical* aspects of wave propagation and how they relate to seismic imaging. Here we consider all the surrounding disciplines, how they relate, how they fail to relate to our central topic of earth imaging. Physics, the kind of physics one learns as an undergraduate, has a relevance to earth imaging, and also an irrelevance—the simple models don't always work. Likewise, mathematics, with its precise problem formulations is both relevant, and irrelevant to earth imaging. And geophysics itself, with its classic areas of theory and observation is both relevant and irrelevant to practical earth imaging. Finally geology brings insights as to why and why not basic physics is a realistic guide to practice. In all the surrounding disciplines of physics, mathematics, geophysics, and geology, I will give you some indication of the degree to which theory can be used as a guide by pointing out some inconsistencies between theory and current industrial practice.

1.2.1 Physics

Derivation of the acoustic wave equation

The acoustic wave equation describes sound waves in a liquid or gas. Another more complicated set of equations describes elastic waves in solids. Begin with the acoustic case. Define

ρ	$=$	mass per unit volume of the fluid
u	$=$	velocity flow of fluid in the x-direction
w	$=$	velocity flow of fluid in the z-direction
P	$=$	pressure in the fluid

Newton's law of momentum conservation says that a small volume within a gas will accelerate if there is an applied force. The force arises from pressure differences at opposite sides of the small volume. Newton's law says

$$\text{mass} \times \text{acceleration} \quad = \quad \text{force} \quad = \quad - \text{ pressure gradient}$$

$$\rho \frac{\partial u}{\partial t} \quad = \quad - \frac{\partial P}{\partial x} \tag{1.9}$$

$$\rho \frac{\partial w}{\partial t} \quad = \quad - \frac{\partial P}{\partial z} \tag{1.10}$$

The second physical process is energy storage by compression and volume change. If the velocity vector u at $x + \Delta x$ exceeds that at x, then the flow is said to be diverging. In other words, the small volume between x and $x + \Delta x$ is expanding. This expansion must lead to a pressure drop. The amount of the pressure drop is in proportion to a property of the fluid called its *incompressibility* K. In one dimension the equation is

$$\text{pressure drop} \quad = \quad (\text{incompressibility}) \times (\text{divergence of velocity}) \tag{1.11}$$

$$- \frac{\partial P}{\partial t} \quad = \quad K \frac{\partial u}{\partial x} \tag{1.12}$$

In two dimensions it is

$$- \frac{\partial P}{\partial t} \quad = \quad K \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \tag{1.13}$$

To arrive at the one-dimensional wave equation from (1.9) and (1.12), first divide (1.9) by ρ and take its x-derivative:

$$\frac{\partial}{\partial x} \frac{\partial}{\partial t} u \quad = \quad - \frac{\partial}{\partial x} \frac{1}{\rho} \frac{\partial P}{\partial x} \tag{1.14}$$

Second, take the time-derivatives of (1.12) and (1.13). In the solid-earth sciences we are fortunate that the material in question does not change during our experiments. This means that K is a constant function of time:

$$\frac{\partial^2 P}{\partial t^2} \quad = \quad -\, K\, \frac{\partial}{\partial t}\, \frac{\partial}{\partial x}\, u \tag{1.15}$$

Inserting (1.14) into (1.15), the one-dimensional scalar wave equation appears.

$$\frac{\partial^2 P}{\partial t^2} \quad = \quad K\, \frac{\partial}{\partial x}\, \frac{1}{\rho}\, \frac{\partial P}{\partial x} \tag{1.16}$$

In two space dimensions, the exact, acoustic scalar wave equation is

$$\frac{\partial^2 P}{\partial t^2} \quad = \quad K\, \left(\frac{\partial}{\partial x}\, \frac{1}{\rho}\, \frac{\partial}{\partial x} \;+\; \frac{\partial}{\partial z}\, \frac{1}{\rho}\, \frac{\partial}{\partial z} \right) P \tag{1.17}$$

You will often see the scalar wave equation in a simplified form, in which it is assumed that ρ is not a function of x and z. Two reasons are often given for this approximation. First, observations are generally unable to determine density, so density may as well be taken as constant. Second, we will soon see that Fourier methods of solution do not work for space variable coefficients. Before examining the validity of this approximation, its consequences will be examined. It immediately reduces (1.17) to the usual form of the scalar wave equation:

$$\boxed{\frac{\partial^2 P}{\partial t^2} \;=\; \frac{K}{\rho}\, \left(\frac{\partial^2}{\partial x^2} \;+\; \frac{\partial^2}{\partial z^2} \right)\; P} \tag{1.18}$$

To see that this equation is a restatement of the geometrical concepts of previous sections, insert the trial solution

$$P \quad = \quad \exp(-\, i\omega t \;+\; i\, k_x x \;+\; i\, k_z z) \tag{1.19}$$

What is obtained is the *dispersion relation of the two-dimensional scalar wave equation:*

$$\frac{\omega^2}{K/\rho} \quad = \quad k_x^2 \;+\; k_z^2 \tag{1.20}$$

Later an equation like (1.20) will be developed by considering only the geometrical behavior of waves. In that development the wave velocity squared is found where K/ρ stands in equation (1.20). Thus physics and geometry will be reconciled by the association

$$v^2 \quad = \quad \frac{K}{\rho} \tag{1.21}$$

Last, let us see why Fourier methods fail when the velocity is space variable. Assume that ω, k_x, and k_z are constant functions of space. Substitute (1.19) into (1.18) and you get the contradiction that ω, k_x, and k_z must be space variable if the velocity is space variable. Try again assuming space variability, and the resulting equation is still a differential equation, not an algebraic equation like (1.20).

Reflections and the high-frequency limit

It is well known that the contact between two different materials can cause acoustic reflections. A material contact is defined to be a place where either K or ρ changes by a spatial step function. In one dimension either $\partial K/\partial x$ or $\partial\rho/\partial x$ or both would be infinite at a point, and we know that either can cause a reflection. So it is perhaps a little surprising that while the density derivative is explicitly found in (1.17), the incompressibility derivative is not. This means that dropping the density gradients in (1.17) will not eliminate all possible reflections. Dropping the terms will slightly simplify further analysis, however, and since constant density is a reasonable case, the terms are often dropped.

There are also some well-known mathematical circumstances under which the first-order terms may be ignored. Fix your attention on a wave going in any particular direction. Then ω, k_x, and k_z have some prescribed ratio. In the limiting case that frequency goes to infinity, the P_{tt}, P_{xx}, and P_{zz} terms in (1.18) all tend to the *second* power of infinity. Suppose two media gradually blend into one another so that $\partial\rho/\partial x$ is less than infinity. The terms neglected in going from (1.17) to (1.18) are of the form $\rho_x\, P_x$ and $\rho_z\, P_z$. As frequency tends to infinity, these terms only tend to the *first* power of infinity. Thus, in that limit they can be neglected.

These terms are usually included in theoretical seismology where the goal is to calculate synthetic seismograms. But where the goal is to create earth models from seismic field data—as in this book—these terms are generally neglected. Earth imaging is more difficult than calculating synthetic seismograms. But often the reason for neglecting the terms is simply to reduce the clutter. These terms may be neglected for the same reason that equations are often written in two dimensions instead of three: the extension is usually possible but not often required. Further, these terms are often ignored to facilitate Fourier solution techniques. Practical situations might arise for which these terms need to be included. With the finite-difference method, they are not difficult to include. But any effort to include them in data processing should also take into account other factors of similar significance, such as the assumption that the acoustic equation approximates the elastic world.

EXERCISES:

1 Soil is typically saturated with water below a certain depth, which is known as the *water table.* Experience with hammer-seismograph systems shows that seismic velocity typically jumps up to water velocity ($V_{-2H_2O} = 1500\ m/s$) at the water table. Say that in a certain location, the ground roll is observed to be greater than the reflected waves, so a decision has been made to bury geophones. The troublesome ground roll is observed to travel at six-tenths the speed of a water wave. How deep must the geophones be buried below the water table to attenuate the ground roll by a factor of ten? Assume the data contains all frequencies from 10 to 100 Hz. (Hints: $\ln 10 \approx 2$, $2\pi \approx 6$, etc.)

2 Consider the function

$$P(z,t) \quad = \quad P_0 \, \frac{1}{\sqrt{Y(z)}} \, e^{i\,\omega\, t \;-\; i \int_0^z \; \omega \sqrt{\frac{\rho(\xi)}{K(\xi)}} \; d\xi} \tag{1.22}$$

where

$$\begin{aligned} P_0 \quad &= \quad \text{constant} \\ Y \quad &\equiv \quad \frac{1}{\sqrt{\rho(z)\, K(z)}} \end{aligned}$$

as a trial solution for the one-dimensional wave equation:

$$\left(\frac{\partial^2}{\partial t^2} \;-\; \frac{K(z)}{\rho(z)} \, \frac{\partial^2}{\partial z^2} \right) P \quad = \quad -\, \frac{K(z)}{\rho(z)^2} \, \frac{\partial \rho}{\partial z} \, \frac{\partial P}{\partial z} \tag{1.23}$$

Substitute the trial solution (1.22) into the wave equation (1.23). Discuss the trade-off between changes in material properties and the validity of your solution for different wavelengths.

Failure of Newtonian viscosity

Also remarkable is the failure of basic textbook seismology to explain the observed frequency-dependence of the dissipation parameter Q. The simplest theoretical approach to dissipation is to add a strain-rate term to Hooke's stress-strain law. This predicts stronger relative dissipation of high frequencies than of low frequencies. Experimentally, relative dissipation is observed to be roughly constant over many decades of frequency. Other simple Newtonian theories yield polynomial ratios in $-i\omega$ for the stress/strain ratio. These theories contain scale lengths and characteristic frequencies. They do not predict constant Q. The heterogeneity of the rock at all scales seems to be an essential attribute of a successful theory.

1.2.2 Geophysics

Curiously, our closest discipline, general geophysics, also seems to contradict, as well as support, some of the things we try to do in earth imaging.

Nonobservation of converted shear waves

In earthquake seismology and in laboratory measurement there are two clearly observed velocities. The faster velocity is a pressure wave (P-wave), and the slower velocity is a shear wave (S-wave). The shear wave can be polarized with ground motion in a horizontal plane (SH) or in a vertical plane (SV). Theory, field data, and laboratory measurement are in agreement. Successful experimental work with S-waves in the prospecting environment was done by Cherry and Waters [1968] and Erickson, Miller and Waters [1968].

It is remarkable that more than 99% of industrial petroleum prospecting ignores the existence of shear waves. Mathematically the earth is treated as if it were a liquid or a gas. The experimental work with shear waves used special equipment to generate and record vibration perpendicular to the survey line, i.e. SH-waves. The picture of the earth given by these transverse waves is often impaired by the soil layers, but sometimes the SH-wave picture is clear and consistent. Surprisingly, even good SH-wave data is often difficult to relate to the P-wave picture. These experimental studies show that the shear waves typically travel about half the speed of the pressure waves except in the soil layer, where the shear wave speed is often much slower and more variable. Observed shear waves usually have lower frequency than pressure waves. A shear wave with half the frequency and half the velocity of a pressure wave has just the same wavelength and hence the same resolving power as the pressure wave. Indeed, experimental work shows that shear waves do offer us about the same spatial resolution as pressure waves. Most land seismic data shows only the vertical component of motion, and all marine seismic data records the pressure. So in the conventional recording geometry, ideally we should never see SH-waves. More precisely, SH-waves should be small, arising only from the earth's departure from simple stratification.

The puzzling aspect of shear waves in reflection seismology is the failure of petroleum prospectors using the standard operating arrangement to routinely observe P-to-S conversions. Theory predicts that P-waves hitting an interface at an angle should be partially converted to SV-waves. Furthermore, for the 30°-60° angle reflections that are routinely encountered, these converted waves should have a size comparable to the P-wave.

The routine geometry of recording and processing discriminates somewhat against converted shear waves. But it discriminates against multiple reflections too (in much the same way) and we see multiples all the time. Furthermore the signature of converted waves should resemble that of multiples, but be distinctly different. Converted waves should show up routinely in velocity surveys (chapter 6). Figure 1.12 shows a zero-offset section containing some multiple reflections. The multiple reflection is recognizable as a replica of earlier topography. Converted waves would replicate the topography but the time scaling would be in the ratio of about 3/2 instead of exactly 4/2. With a sufficiently complex topography, as in Figure 1.12, the probability is low that the converted wave would be mistaken for another primary reflection.

Converted waves should have good diagnostic value in exploration. But the likelihood of seeing converted shear waves in conventional data seems to be so remote that most interpreters have given up looking. Why aren't converted waves seen in conventional data? Some reasons can be offered:

- In marine data there would have to be an additional conversion to P for the

Figure 1.12: A zero-offset section from east Africa with multiple reflections. (Teledyne) $\boxed{\text{xrf-africa}}$ [NR]

water path.

- In land data the soil is especially absorptive of shear.
- Vertical component recorders tend to see P better than S. This is especially so because rays bend toward the vertical in the near surface.

Of all the reasons why converted shear waves should be weaker than pressure waves, none is overwhelming. A wide range of amplitudes are recorded in a wide variety of environments. Data are often displayed with automatic gain control (AGC). Weakened amplitude appears to be insufficient cause for the failure of observation. We should keep looking. Converted waves are certainly more prevalent than our recognition of them. (I have never identified a converted shear wave on conventional recordings).

So although converted shear waves might some day play a significant role in reflection seismology, we now return to the mainstream—how to deal effectively with that which is routinely observable.

Reliability of reverberation modeling

The seismological literature contains an abundance of theory to describe seismic waves in layered media. A significant aspect of applied seismology is the general neglect of intrabed reverberation. When a wave reflects from an interface, the strength of the reflected wave is a small fraction, typically less than 10%, of the strength of the incident wave. This reflected wave is the one that is mainly dealt with in this book. However, the reflected wave itself reflects again and again, ad infinitum. For short path geometries, there can be very many of these rays. The question is whether these reverberations can ever amount to enough to make considering them worthwhile. The answer seems to be that although reverberation may be significant, seismologists are rarely able to improve interpretation of reflection survey data with the more complicated theory that is required to incorporate reverberation.

The situation is somewhat improved when well logs are available, but even then there are serious difficulties. The best possible lateral resolving power, say about 20-50 meters, is obtained after migration. The well log, however, is not a 20-50 meter lateral average of the earth. Next time you see a highway cut through sedimentary rocks, think of the difference between a point and a lateral average over 20-50 meters. In practice, people smooth the well log (vertical smoothing). Too little smoothing gives too much reverberation. Too much smoothing gives no reverberation. The amount of vertical smoothing is an empirically determined parameter, and results are significantly sensitive to it. Vertical averages of the well log may or may not be a satisfactory approximation to the required horizontal average.

What you can get from reflection seismology

Luckily, it has been discovered that certain functions of ρ and K can be reliably determined and mapped. The velocity v and the acoustic impedance R are given by the equations

$$v = \sqrt{K/\rho} \tag{1.24}$$

$$R = \sqrt{K\rho} \tag{1.25}$$

Mathematically, it is a simple job to back-solve equations (1.24) and (1.25), which gives

$$K = v\,R \tag{1.26}$$

$$\rho = \frac{R}{v} \tag{1.27}$$

In practice, the solutions (1.26) and (1.27) have little value because the two parameters v and R are seen through nonoverlapping spectral windows. The acoustic impedance R is seen through the typical 10 to 100 Hz spectral window of good quality reflection data. Since the low frequency part of the spectrum is missing, it is common to say that it is not the impedance which is seen, but the gradient reflectivity $=$ $c(x,z) = d\ln R/dz = R^{-1}dR/dz$ which is more familiar at a layer boundary where it has the form $(R_1 - R_2)/(R_1 + R_2)$.

The velocity v is seen through a much smaller window. Observation of it involves studying travel time as shot-to-geophone offset varies and will be described in chapter 6. With this second window it is hard to discern sixteen independent velocity measurements on a 4-second reflection time axis. So this window goes from zero to about 2 Hz, as depicted in Figure 1.13.

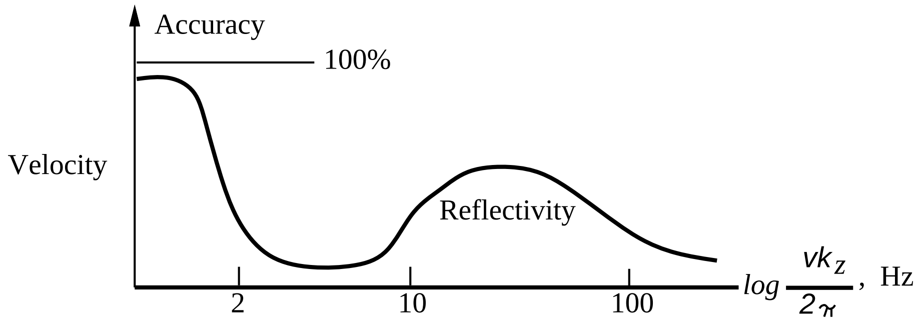

Figure 1.13: Reliability of information obtained from surface seismic measurements. $\boxed{\text{xrf-rely}}$ [NR]

Note that there is an information gap from 2-10 Hz. Even presuming that *rock physics* can supply us with a relationship between ρ and K, the gap seriously interferes with the ability of a seismologist to predict a well log before the well is drilled. What seismologists can do somewhat reliably is predict a filtered log.

The observational situation described above has led reflection seismologists to a specialized use of the word *velocity.* To a reflection seismologist, *velocity* means the low spatial frequency part of "real velocity." The high-frequency part of the "real velocity" isn't called *velocity:* it is called *reflectivity.* Density is generally disregarded as being almost unmeasurable by surface reflection seismology.

1.2.3 Mathematics

Philosophy of inverse problems

Physical processes are often simulated with computers in much the same way they occur in nature. The machine memory is used as a map of physical space, and time evolves in the calculation as it does in the simulated world. A nice thing about solving problems this way is that there is never any question about the uniqueness of the solution. Errors of initial data and model discretization do not tend to have a catastrophic effect. Exploration geophysicists, however, rarely solve problems of this type. Instead of having (x,z)-space in the computer memory and letting t evolve, we usually have (x,t)-space in memory and extrapolate in depth z. This is our business, taking information (data) at the earth's surface and attempting to extrapolate to information at depth. Stable time evolution in nature provides no "existence proof" that our extrapolation goals are reasonable, stable, or even possible.

The time-evolution problems are often called *forward problems* and the depth-extrapolation problems *inverse problems.* In a *forward problem,* such as one with acoustic waves, it is clear what you need and what you can get. You need the density $\rho(x,z)$ and the incompressibility $K(x,z)$, and you need to know the initial source of disturbance. You can get the wavefield everywhere at later times but you usually only want it at the earth's surface for comparison to some data. In the *inverse problem* you have the waves seen at the surface, the source specification, and you would like to determine the material properties $\rho(x,z)$ and $K(x,z)$. What has been learned from experience is that routine observations do not give reasonable estimates of images or maps of ρ and K.

Mathematical inverse problems

In mathematics the solution to an inverse problem has come to mean the "determination" of material properties from wavefields. Often this is achieved with a "convergent sequence." Geophysicists are less precise (or more inclusive) about what they mean by "determination." In some chapters of this book reflectors are "determined" by the exploding-reflection concept. In chapter 6 shot-to-geophone offset is incorporated, and reflectivity $c(x,z)$ and velocity $v(z)$ are "determined" with a buried-experiment concept. In chapter 11 the concept is developed of suppressing multiple reflections and finding the "true" amplitudes of reflections by having the upcoming wave vanish before the onset of the downgoing wave. Other imaging concepts seem likely to result from future processing schemes. It might be possible to show that some of our "determinations" coincide with those of mathematicians, but such coincidence is not our goal here. Such a goal is formulated more directly in my third book *Earth soundings analysis: processing versus inversion.*

1.2.4 Geology

Because of the complexity of sedimentary rocks, there is not universal agreement on an appropriate mathematical description.

The clastic section

Generally speaking, most petroleum reservoir rocks are sandstones. Sandstones are most often made by the sands that are deposited near the mouths of rivers where the water velocity is no longer sufficient to move them. The sands deposit along the terminus of the sand bars found at the river mouth, often along a slope of 25° or so, as depicted in Figure 1.14. Although the sands are not laid down in flat layers, the

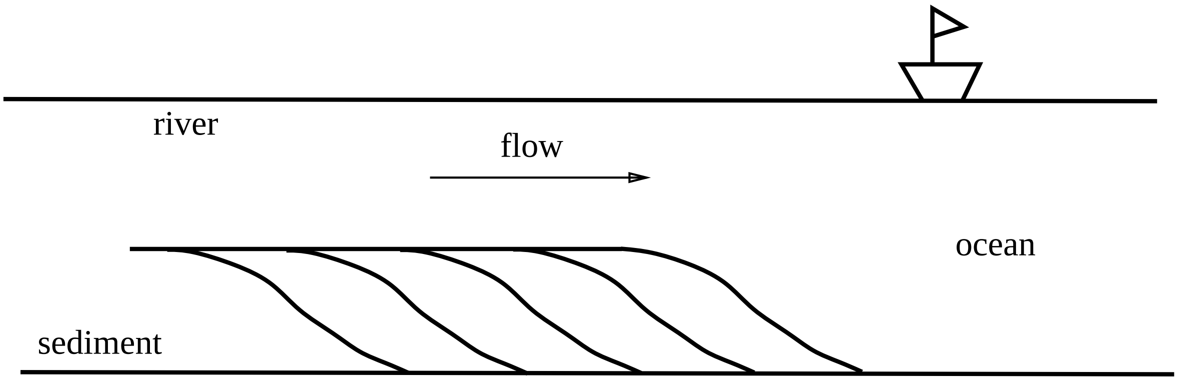

Figure 1.14: Sands (petroleum reservoir rocks) deposit on fairly steep slopes where rivers run into the ocean. xrf-river [NR]

process may build a horizontal layer.

Clays are more fine-grained materials (dirt) that are carried out to deeper water before they settle to make shale. Shale deposits tend to be layered somewhat more horizontally than sandstones. Specific locations of sand deposition change with the passing of storms and seasons, leaving a wood-grain-like appearance in the rock.

The river delta itself is a complicated, ever-changing arrangement of channels and bars, constantly moving along the coastline. At any one time the delta seems to be moving seaward as deposits are left, but later settling, compression, or elevation of sea level can cause it to move landward.

Sand is important because its porosity enables oil to accumulate and its permeability enables the oil to move to a well. Shale is important because it contains the products of former life on earth, and their hydrocarbons. These escape to nearby sands, but often not to the earth's surface, because of covering impermeable shales. The acoustic properties of sands and shales often overlap, though there is a slight tendency for shales to have a lower velocity than sands. Geophysicists on the surface see with seismic wavelengths ($\approx$ 30 meters) the final interbedded three-dimensional mixture of sands and shales.

Mixtures of sands and shales are called *clastic* rocks. The word *clast* means *break.* Clastic rocks are made from broken bits of crystalline rock. Most sedimentary rocks are clastic rocks. Most oil is found in clastic rocks. But much oil is also found in association with carbonates such as limestone. Carbonates are formed in shallow marine environments by marine organisms. Many carbonates (and clastics) contain oil that cannot be extracted because of lack of permeability. Permeability in carbonate rocks arises through several obscure processes. The seismologist knows carbonates as rocks with greater velocity than clastic rocks. Typically, a carbonate has a 20-50% greater velocity than a nearby clastic rock. Clastic rock sometimes contains limestone, in which case it is called *marl.*

Chrono-stratigraphy

Strange as it may seem, there is not universal agreement about the exact nature of seismic reflections. Physicists tend to think of the reflections as caused by the interface between rock types, as a sand-to-shale contact. The problem with this view is that sands and shales interlace in complex ways, both larger and smaller than the seismic wavelength. Many geologists, particularly a group known as *seismic stratigraphers,* have a different concept. (See *Seismic Stratigraphy—Applications to Hydrocarbon Exploration,* memoir 26 of the American Association of Petroleum Geologists). They have studied thousands of miles of reflection data along with well logs. They believe a reflection marks a constant geological time horizon. They assert that a long, continuous reflector could represent terrigenous deposition on one end and marine deposition on the other end with a variety of rock types in between. Data interpretation based on this assumption is called *chrono-stratigraphy.* The view of the seismic stratigrapher seems reasonable enough for areas that are wholly clastic, but when carbonates and other rocks are present, the physicist's view seems more appropriate. For further details, the book of Sheriff [1980] is recommended.