Anti-aliased Kirchhoff 3-D migration

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SUMMARY

A significant degradation in the quality of Kirchhoff migration images can arise when the migration operator summation trajectory is too steep for a given input seismic trace spacing and frequency content. This phenomenon is called "operator aliasing" and is strictly distinct from and independent of data aliasing. Unfortunately, the conditions which lead to significant operator aliasing are common in most 3-D Kirchhoff migration work, due to the sparse and irregular nature of 3-D acquisition geometries, and the need to minimize the size of 3-D migration input data volumes.

We anti-alias the migration operators "on the fly" with N-point triangle filters. Using a Z-transform representation, we reduce each N-point triangle filter operation to an efficient three-point filter operation, with the added minor overhead of a causal and anticausal integration of the input trace data prior to migration. Our anti-aliased migration is computationally efficient since it requires about the same number of floating point operations as a conventional Kirchhoff migration, and is conservative in its memory use since it does not require storage of multiple trace-data copies to implement the anti-aliasing. Our anti-aliased migration is accurate in that it applies local anti-alias filters separately for each of the many migration operators which pass through a given input data point.

We demonstrate our method with a 3-D implementation on a massively parallel Connection Machine CM-5, and compare our new anti-aliased Kirchhoff migration to a conventional aperture-weighted Kirchhoff migration in an application to a 3-D marine seismic data example. Our results indicate that our anti-aliasing method greatly enhances the 3-D resolution of steep salt-sediment interfaces and faults, and suppresses false reflections caused by conventional Kirchhoff-migration aliasing artifacts. Our method is applicable to other space-time Kirchhoff operations such as NMO velocity-stacking and DMO.

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INTRODUCTION

The process of seismic migration involves a mapping of the input seismic trace data by a migration operator to the output structural image. Spatial aliasing of the migration process can be problematic in all three domains: data, operator and image. These three types of spatial aliasing are distinct and independent, although they are often confused.

Aliasing defined

Data aliasing occurs when there are seismic frequencies f and dips θ along reflection and diffraction events in recorded seismic data which violate the following criterion: $f \leq f_d = v_r/(4 \sin\theta_r \Delta x)$. The term f_d is the maximum unaliased frequency in the data for a given trace spacing Δx, a velocity v_r at the receiver location, and a local plane-wave incident angle θ_r at the recording surface. Once data have been aliased during the recording process, it is difficult to suppress subsequent processing artifacts except to reshoot the survey at a finer group spacing, or try to interpolate the original data without interpolating the aliases, e.g., Spitz (1991) and Claerbout (1992). Yilmaz (1988) gives a good overview of data aliasing and its effects on all methods of migration.

Image aliasing occurs when the output sampling of the image space is too coarse to properly represent migrated dips. This is mostly a cosmetic problem and can be remedied by choosing an appropriate image mesh upon which to estimate the migrated structural image. Note that the Kirchhoff-migrated values at each mesh point in an aliased image space are uncontaminated and correct (assuming no data or operator aliasing). For example, a single migrated output trace at a well location may be entirely accurate without having to estimate any adjacent migrated traces. This "target-oriented" capability of Kirchhoff migration is not possible with spectral or finite-difference migration methods. Kirchhoff image aliasing is only a problem from an aesthetic point of view, and when the migrated image is needed as input to subsequent multi-channel processing.

Operator aliasing occurs when the operator dip along the migration summation trajectory is too steep for a given input seismic trace spacing and frequency content. Spatial aliasing of Kirchhoff integral operators is likely because the operators are designed in the discrete space-time domain rather than the spectral domain. Great care must be taken to ensure that the f-k spectrum of a given space-time operator contains no aliased energy wrapped beyond the Nyquist frequency and wavenumbers. This implies that space-time operators which are spatially undersampled and contain significant steep-dip and high-frequency energy are likely to suffer from severe operator aliasing. In contrast, migration operators designed explicitly in the f-k domain are unaliased by definition, and operators designed in the f-x domain are easily modified to suppress spatial aliasing.

Previous work

Conventionally, anti-aliasing of Kirchhoff operators can be partially accomplished by trace interpolation, aperture control, and operator dip filtering. Trace interpolation decreases the spatial sample interval and thus increases the spatial Nyquist frequency, but at the potentially prohibitive cost of increasing the total 3-D migration input load (e.g., Yilmaz, 1988). Migration aperture control and operator dip filtering have the effect of suppressing aliased steep-dip contributions to the output image. Unfortunately, these suppressed steep dips may be precisely the events of interest when imaging near-vertical structure such as salt flanks and faults.

Hale (1991) proposed a method to anti-alias Kirchhoff dip-moveout (DMO) operators by replicating time-shifted versions of the input seismic traces to approximate a summation along an equal-time-sampled version of the operator. Hale's subsequent nearest-neighbor spatial interpolation implies a boxcar averaging of the input trace values along the time direction, whereas we use triangle filtering to increase accuracy at steep migration dips. Hale's test of summing DMO impulse responses and matching the result to the original impulse ensures both operator aliasing and image aliasing are suppressed, which may invoke more lowpass-filtering and loss of resolution than is strictly necessary for the case of migration operator anti-aliasing alone. Hale's replication of time-shifted input traces and spatial interpolations are inefficient from both a floating-point and memory perspective for 3-D migration algorithms, such as ours, that process thousands of traces simultaneously.

Gray (1992) proposed an approximate but cost-effective method to anti-alias Kirchhoff migration operators by performing aperture-dependent lowpass temporal filtering of the input trace data. He suggested that approximately three copies of the input data be lowpass-filtered at different high-cut frequencies. During migration the wide-aperture portions of the migration operator would be drawn increasingly from the lowest-frequency-content filtered trace copies. This corresponds to the notion that the wide-aperture portions of the migration operator have the steepest dips and therefore only the lowest frequencies will be unaliased. We more accurately calculate the complete vector of high-cut frequencies separately for each diffraction trajectory, and design and apply our anti-alias filters on the fly accordingly. Our method is as cost-effective as Gray's approach, but does not require the memory-intensive storage of multiple data copies to implement the anti-aliasing.

We present an improved method for anti-aliasing Kirchhoff migrations. We first derive the relevant operator aliasing criteria and then present our efficient method of anti-aliasing the migration operator by local triangle filtering of the seismic trace. Finally, we compare applications of our anti-aliased Kirchhoff migration to conventional aperture-weighted Kirchhoff migration on a 3-D marine seismic data example.

ANTI-ALIASING THEORY

The operator anti-aliasing criterion defines the maximum unaliased temporal frequency in the seismic data at the location that the operator trajectory intercepts a seismic trace, and is a function of the local operator dip and the trace spacing. In order to remain unaliased, the sequence of seismic trace samples summed along the operator trajectory must satisfy the dip Nyquist sampling criterion:

$$f_{max} \leq \frac{1}{2\Delta T} = \frac{1}{2\left(\partial t_k / \partial \rho\right) \Delta \rho} \ . \tag{1}$$

The factor ΔT is the moveout time along the migration operator between adjacent traces. The term $\partial t_k/\partial\rho$ is the local spatial derivative of the migration operator at the point of intersection with the seismic trace, and $\Delta\rho$ is the seismic trace spacing along the recording surface. The anti-aliasing criterion (1) suggests local lowpass filtering of the seismic trace to ensure that no frequencies larger than f_{max} are incorrectly migrated into the output image. As discussed later, we implement the anti-aliasing lowpass filtering efficiently and robustly with local triangle filters.

Kirchhoff time migration

It is instructive to evaluate the anti-aliasing criterion for the special case of Kirchhoff *time* migration, which can be extended to the case of Kirchhoff *depth* migration. For time migration, we derive an expression for the anti-aliasing criterion (1) by analytically evaluating the operator derivative term $\partial t_k/\partial\rho$. We begin by considering the analytic expression of the Kirchhoff 3-D prestack time migration operator, which has the form

$$t_k \ = \ t_s + t_r \ \approx \ \sqrt{\left(\frac{\rho_s}{v}\right)^2 + \tau^2} + \sqrt{\left(\frac{\rho_r}{v}\right)^2 + \tau^2} \ . \tag{2}$$

The term t_k is the total two-way Kirchhoff migration reflection traveltime, τ is the one-way vertical traveltime or pseudo-depth to the reflection point, and v is the rms migration velocity that varies as a function of space and pseudo-depth (x, y, τ). The lengths ρ_s and ρ_r are the distances measured along the planar recording surface between the source or receiver, respectively, to the vertical projection of the image point on the recording surface, expressed as

$$\rho_\star \ = \ \sqrt{(x_i - x_\star)^2 + (y_i - y_\star)^2} \ , \tag{3}$$

Equation (3) assumes that the recording surface is a horizontal plane at a constant datum depth z_0. The subscript $\star$ is a wildcard for either the source or receiver coordinate subscripts s and r, and the subscript i refers to the surface location of the subsurface image point projected up onto the recording surface.

The spatial derivative of (2) is needed to determine the migration operator aliasing criterion, which can be derived as

$$\frac{\partial t_k}{\partial \rho} = +\sqrt{\left(\frac{\partial t_k}{\partial x_i}\right)^2 + \left(\frac{\partial t_k}{\partial y_i}\right)^2} \approx \frac{1}{v^2}\left(\frac{\rho_s}{t_s} + \frac{\rho_r}{t_r}\right) . \tag{4}$$

We evaluate the effective rms spatial sampling interval $\Delta\rho$ for the prestack migration operator as

$$\Delta\rho \approx \sqrt{\frac{1}{2}\left(dx_s^2 + dy_s^2 + dx_r^2 + dy_r^2\right)} \tag{5}$$

where we define $dx_\star$ and $dy_\star$ in the prestack case as

$$dx_\star = \frac{|x_i - x_\star|\Delta x}{\rho_\star} \tag{6}$$

and

$$dy_\star = \frac{|y_i - y_\star|\Delta y}{\rho_\star} . \tag{7}$$

The terms Δx and Δy are the true inline and crossline sampling intervals of the seismic trace data, and are assumed to be regular but possibly unequal in length. A more complicated analysis is required for irregular trace spacings, which this paper does not address.

Given the analytic expressions for the spatial derivative of the Kirchhoff time migration operator $\partial t_k/\partial\rho$, and the definition of the areal trace spacing interval $\Delta\rho$, we derive an analytic expression for the time migration operator anti-aliasing criterion. For the prestack case, the anti-aliasing criterion is

$$f_{max} \leq \frac{v^2}{2\Delta\rho}\left(\frac{\rho_s}{t_s} + \frac{\rho_r}{t_r}\right)^{-1} , \tag{8}$$

where the prestack definition of $\Delta\rho$ given by (5) – (7) must be used.

Kirchhoff depth migration

In contrast to the time migration method above, the depth migration anti-aliasing criterion does not have a general analytic solution. Kirchhoff depth migration operators in $v(x, y, z)$ media do not have simple hyperbolic forms, and must be evaluated numerically through heterogeneous migration velocity models. However, we propose three methods to provide satisfactory depth migration anti-aliasing results.

The first and most straightforward method is simply to use the rms velocity field v and the time migration anti-aliasing criterion (8), of the previous section, to design the anti-aliasing lowpass filters along the non-hyperbolic depth migration operators. This approach is robust because the dip along the time migration operator is a reasonable approximation to the *average* dip $\partial t_k/\partial\rho$ along the true depth migration trajectory.

The second depth migration anti-aliasing technique applies to Kirchhoff depth migration driven by traveltime tables. In this case, the operator derivative $\partial t_k/\partial \rho$ in the general anti-aliasing criterion (1) can be evaluated by differencing the source and receiver traveltimes at the trace location with respect to an adjacent source and receiver location. This approach is more accurate than the time migration approximation, but requires extra memory and I/O overhead to load the adjacent traveltime tables, or extra computation time if the adjacent traveltimes are interpolated from a sparse set of traveltime tables already loaded into memory.

The third depth migration anti-aliasing technique applies to Kirchhoff depth migration in which the traveltime values are computed by raytracing on the fly. In this case, an extra set of rays can be traced from adjacent source and receiver locations down to all subsurface points, or interpolated from a single set of dynamic rays by paraxial interpolation. Then the difference in ray arrival times can be used to approximate the operator derivative $\partial t_k/\partial \rho$ in (1). This method is more computationally intensive than the traveltime table approach, but has significantly smaller memory requirements.

In the sections that follow, we assume that the operator derivative $\partial t_k/\partial \rho$ of equation (1) is a known quantity, whether for time or depth migration, since it can be calculated by any of the three methods described above.

ANTI-ALIAS FILTER DESIGN

Given that we know the maximum unaliased frequency f_{max} at each point along the migration operator, we need to design a method to locally lowpass filter the seismic data in order to satisfy the anti-aliasing criterion (1). We implement the local lowpass filters as triangular smoothing. The Z-transform representation of an arbitrary N-point triangle filter can be expressed as

$$g(z) = \frac{-z^{-k-1} + 2 - z^{k+1}}{\alpha(1-z)(1-z^{-1})} \; , \tag{9}$$

where the triangle length $N = 2k+1$, and the filter scaling coefficient $\alpha = (k+1)^2$. The amplitude spectrum of the triangle filter can be derived as

$$A(\omega) = \frac{\sin^2(\omega \Delta t (k+1)/2)}{\sin^2(\omega \Delta t/2)} \; , \tag{10}$$

which is the discrete version of a continuous sinc^2 function, and has spectral notches at the frequencies

$$f_n = \frac{\omega_n}{2\pi} = \frac{n}{(k+1)\Delta t} \;\; ; \;\; n = 1, 2, 3, \dots \; . \tag{11}$$

We design the triangle filter length N such that the maximum unaliased frequency f_{max} is equal to the first notch in the triangle spectrum at frequency f_1. Equating the anti-aliasing criterion (1) to the first triangle filter notch frequency of (11), we derive the appropriate anti-aliasing triangle filter length N to be

$$N \geq \max(\; 4 \left(\frac{\partial t_k}{\partial \rho} \right) \frac{\Delta \rho}{\Delta t} - 1 \, , \;\; 1 \;\;) \, . \tag{12}$$

The value of the local migration operator dip $\partial t_k / \partial \rho$ is given by equation (4) in the case of time migration, or by the methods discussed in the previous section on depth migration.

Furthermore, we note that the denominator of $g(z)$ represents a causal integration of the trace with $1/(1-z)$, followed by an anticausal trace integration with $1/(1-z^{-1})$. The numerator of $g(z)$ is a gapped three-point Laplacian operator, where the length of the gap $(k+1)$ determines the triangle length N and high-cut frequency f_{max}. Therefore, if the input trace data are integrated twice before migration, an arbitrary N-point anti-alias triangle filter can be designed and applied on the fly for the cost of a three-point operator. This observation offers a large savings in computational effort when applying the triangle filters.

DATA EXAMPLES

We compare our anti-aliased Kirchhoff migration to a conventional aperture-weighted Kirchhoff migration, both implemented on a Connection Machine CM-5. Preliminary 3-D results were shown by Lumley and Claerbout (1993). Figure 1 shows how our anti-aliasing method enhances a 3-D Kirchhoff migration image of a salt intrusion, given a sparse 50m-binned input dataset. We will discuss 2-D and 3-D data examples in more detail at the oral presentation.

CONCLUSIONS

We present an anti-aliasing method for Kirchhoff migration operators, based on local triangle filtering. The N-point triangles are efficiently applied as 3-point filters after causal and anticausal integration of the seismic trace data. This method is well suited for parallel processing algorithms, where storage of multiple copies of various high-cut filtered versions of the input data is not often practical. We compare the anti-aliasing method to a standard Kirchhoff 3-D migration, using a salt intrusion data set from the Gulf of Mexico.

Our results indicate that careful operator anti-aliasing can enhance steep-dip reflections from faults and salt-sediment interfaces during the process of 3-D seismic migration imaging. Our method is applicable to other Kirchhoff space-time operators such as NMO velocity stacking and DMO.

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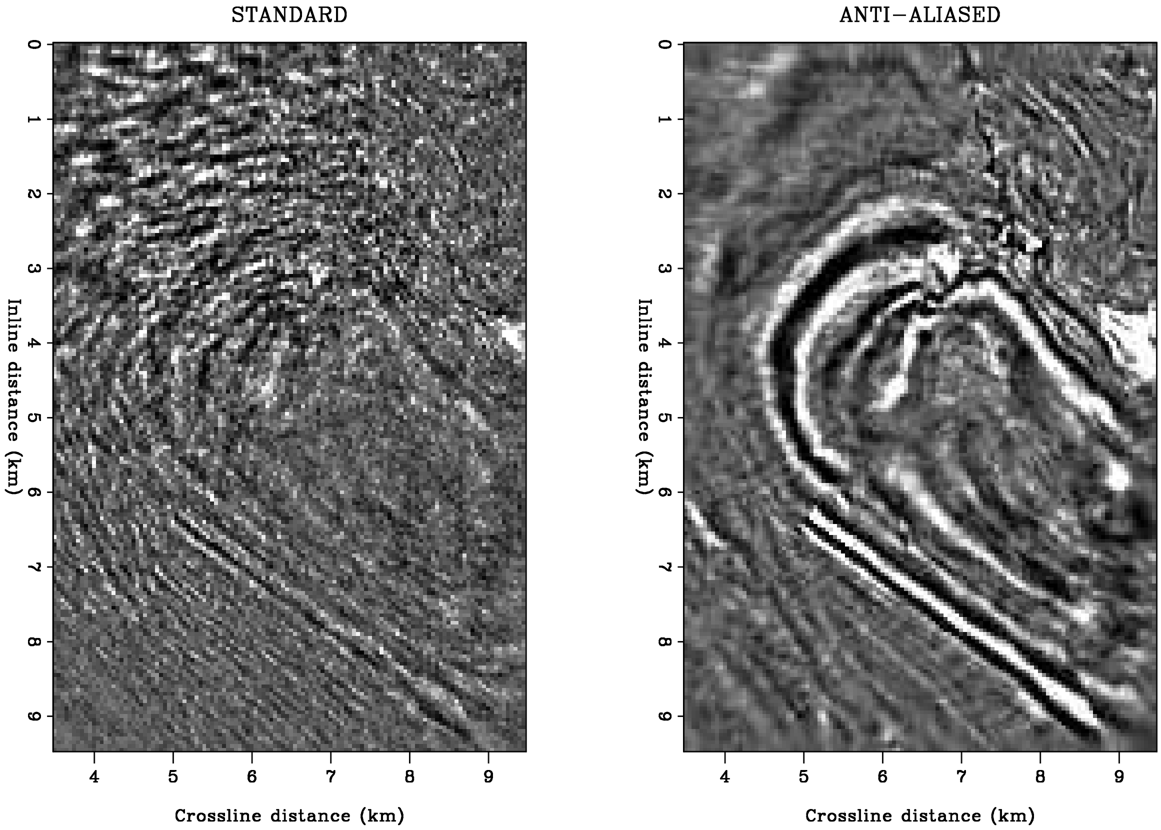

Figure 1: Kirchhoff 3-D migrated depth slices. The left panel is a standard Kirchhoff 3-D migration with a 60-degree migration aperture and $\cos^2\theta$ operator dip filter, the right panel is our new anti-aliased migration method.