Lecture Notes on

Nonlinear Inversion and Tomography:

I. Borehole Seismic Tomography

From a Series of Lectures by

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Glossary

acoustic – pertaining to sound waves in a gas or fluid (such as air or water), generally limited to compressional waves.

backprojection – a one-step, approximate reconstruction method.

block – an element of a three-dimensional region whose properties are to be reconstructed. Usually, the properties are assumed to constant within the block.

cell – an element of a two-dimensional or three-dimensional region whose properties are to be reconstructed. Usually, the properties are assumed to be constant within the cell.

consistent – a system of equations with at least one solution satisfying all the physical constraints on a model.

determined – a linear system with as many equations as unknowns (assuming that the equations are linearly independent). If the equations are consistent, there is generally a unique physical solution to such a systeme.

elastic – pertaining to sound waves in a solid, and explicitly including both compressional and shear waves.

feasible – pertaining to a part of a set (especially the set of all possible models) that satisfies all known physical constraints, such as positivity. Any model that is not feasible is infeasible.

homogeneous – constant, that is a physical property constant on the scale of investigation.

image – a picture showing qualitative differences in a physical property of some region.

imaging – the process of producing an image.

inconsistent – a linear system with no physical (or feasible) solution. For example, the system

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

has the unique solution

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix},$$

but this solution is unphysical because it fails to satisfy the positivity constraint. The cause of inconsistency is usually a major error in data collection, but more subtle interactions between forward modeling and the data can also produce inconsistency.

infeasible – pertaining to a part of a set (especially the set of all possible models) that fails to satisfy any of the physical constraints, such as positivity. This set is complementary to the feasible set.

inhomogeneous – not constant, that is a physical property varying on the scale of investigation.

inverse – the opposite rule. For example, subtraction is the opposite of addition, while division is the opposite of multiplication.

inversion – the process of reconstructing a two-dimensional image or three-dimensional map of some physical property in a selected region.

konoscope – a device for reconstructing the properties of a three-dimensional region using tomography or inversion.

map – a picture or volume representation often showing quantitative differences in a physical property of some region. A map is generally quantitative whereas an image is qualitative. A map is two- or three-dimensional whereas an image is two-dimensional.

migration – the process of reconstructing an image of earth reflectivity from seismic reflection data. Also known as **wave equation migration**.

nonfeasible – same as infeasible.

overdetermined – any linear system with more equations than unknowns. (Caveat: if many of the equations are linearly dependent, then the reduced system may actually be either determined or underdetermined; however, it generally requires much computation to decide if this is so.) Generally no exact solution to such a system exists, so approximate methods of solution such as least-squares are used to find "best" approximate solutions. Inverting for local averages of physical properties may produce an overdetermined mathematical inversion problem.

pixel – a picture element, or two-dimensional cell.

reconstruction – the act of constructing again from pieces that have been disassembled, as in a puzzle.

seismic – pertaining to sound waves in the earth, and explicitly including both compressional and shear waves.

seismogram – a record of seismic signals. Seismogram is to seismograph as photograph is to camera.

seismograph – a device for measuring seismograms.

seismography – the study or observation of seismic signals.

tomogram – the reconstructed image of some physical property produced by tomography. Tomogram is to konoscope as photograph is to camera, or as micrograph is to microscope.

tomograph – same as tomogram.

tomography – the study of cross sections; the process of reconstructing a two-dimensional image of some physical property of a selected plane region.

underdetermined – any linear system with fewer equations than unknowns. There are generally many solutions (often an infinite number) to such a system. Inhomogeneous physical systems whose properties may be described as essentially continuous functions of position may be considerd to have an infinite number of unknowns; therefore, any attempt to reconstruct the continuous system from finite data leads to an underdetermined physical inversion problem.

voxel – a volume element, or three-dimensional cell.