Lecture Notes on

Nonlinear Inversion and Tomography:

I. Borehole Seismic Tomography

From a Series of Lectures by

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