Permeability and Relative Permeability in Rocks

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Abstract

Important features of the topology of the pore space of rocks can be usefully quantified by analyzing digitized images of rock cross sections. One approach computes statistical correlation functions using modern image processing techniques. These correlation functions contain information about porosity, specific surface area, tortuosity, formation factor, and elastic constants, as well as the fluid permeability and relative permeability. The physical basis of this approach is discussed and examples of the results for various sandstones are presented. The analysis shows that Kozeny-Carman relations and Archie's empirical laws must be modified to account for finite percolation thresholds in order to avoid unphysical behavior in the calculated relative permeabilities.

1. Introduction

Most rocks are porous to some degree. Methods of characterizing that porosity have been studied for many years in order to understand the physical properties of rocks and to gain insight into possible scenarios for their formation. Two such properties of great practical interest are the fluid permeability and the electrical conducitivity of rocks containing conducting fluids. In order for an electron, an ion, or a fluid particle to travel from one part of a rock to another distant part, it is necessary for the particle to follow a tortuous path through the rock, avoiding the solid/insulating grains that compose the rock. Thus, it may be expected on very general physical grounds that there should be a close quantitative relationship between the fluid permeability and the electrical conductivity. No such rigorous relationship has yet been found however.

Brace (1977) and later Paterson (1983) and Walsh and Brace (1984) have shown that a type of Kozeny-Carman relation may be used to relate the fluid permeability to the electrical formation factor. Brace (1977) found that such a formula could predict the fluid permeability k of a rock within about a factor of 2 over 9 orders of magnitude in the variation of k if the hydraulic radius and formation factor of the rock are known. Bernabé, Brace, and Evans (1982) studied the dependence of permeability on porosity and pore geometry in hot-pressed calcite and found significant deviations from Kozeny-Carman in the vicinity of the percolation threshold. Walsh and Brace (1984) rederived the Kozeny-Carman relation and wrote it in terms of porosity, specific surface area, and formation factor. Berryman and Blair (1986) proposed using image processing methods as a means of estimating the parameters in this Kozeny-Carman relation and tested this approach on porous glass and various sandstones. The results from image processing were similar to those of Brace (1977) in that the predicted and measured permeabilities generally agreed well within a factor of 2. In fact, the authors considered these results to be suspiciously good, and subsequently showed (Berryman and Blair, 1987) that the success of the method was due in part to the fortuitous cancellation of two compensating errors: (a) the Kozeny-Carman relation is an approximation based on an implicit assumption of smooth-walled pores and (b) the image processing method has inherent resolution limitations that force the measured specific surface area to be consistent with the assumptions in the derivation of the Kozeny-Carman relation.

If we can predict the permeability for single-phase flow through a porous material from digitized information obtained from cross sections of rocks, then we may also be able to obtain estimates of traditional relative permeability as well when two fluid phases are present. With traditional relative permeability (Corey, 1977; Bear, 1979; Dullien, 1979), interactions between the two fluid phases are neglected and each fluid phase is treated as a single phase flowing through a porous matrix composed of the solid phase and the other fluid phase. The traditional approach to relative permeability is only an approximation to the real situation in two-phase flow, and many researchers have challenged the assumption that the interactions between the fluid phases are negligible (Raats and Klute, 1968; deGennes, 1983; Thigpen and Berryman, 1985; Whitaker, 1986; Berryman, Thigpen, and Chin, 1988; Rose, 1988; Rose, 1989; Kalaydjian, 1990; Rothman, 1990). If the interactions are significant, then the generalization of the image processing method for estimating relative permeability coefficients is not automatic. Nevertheless, the interactions must depend on the fluid/fluid interface area, which is a quantity that can be obtained from images of cross sections.

In Section 2, we summarize the analysis of the permeability for single-phase flow and of the relative permeability for two-phase flow. We include a discussion of the cross terms that arise due to the fluid/fluid interaction. In Section 3, we present a simple but rigorous result relating the fluid/fluid interface to other measured surface areas. In Section 4, we analyze Kozeny-Carman relations for single-phase flow, for two-phase flow without fluid/fluid interaction, and for two-phase flow with such interaction. We also consider the possibility that the two fluids have mixed wettability characteristics. In Section 5, we describe methods used to obtain and quantify cross sections of sandstones that display the two distinct fluid phases occupying the pore space of a rock. Section 6 summarizes our results for Berea sandstone. A discussion of the results to date and the possibilities for extensions in the future is presented in the final section.

2. Permeability and relative permeability

We will first consider a single-phase fluid and then generalize to the two-phase problem. Throughout this discussion, we assume that the porous solid frame is rigid.

2.1 Single-phase flow

Consider a porous medium of total volume Ω filled with a fluid occupying the subvolume Ω_f. The points of contact between the saturating fluid and the solid form the interior surface Γ_I. The exterior fluid surface through which the fluid may enter or exit the volume Ω is Γ_E. We assume the applied stress tensor Π_{ij} is known on Γ_E. The applied stress takes the form

$$\Pi_{ij} = -p\delta_{ij} + \sigma_{ij} \tag{1}$$

where the viscosity tensor σ is related to the fluid velocity field $\vec{v}$

$$\sigma_{ij} = \mu(v_{i,j} + v_{j,i}), \tag{2}$$

for $i,j = 1,2,3$. In this notation, i and j index the directions in a cartesian coordinate system $(x = x_1, y = x_2, z = x_3)$ and the subscript appearing after a comma refers to a partial derivative; thus, $v_{2,3} = \partial v_2/\partial z$. The fluid pressure is p and the fluid viscosity is μ. Then, the energy dissipation in the fluid is given by (Landau and Lifshitz, 1959)

$$\mathcal{D} = \frac{1}{2\mu\Omega}\int_{\Omega_f} \sigma_{ij}\sigma_{ij}\, d^3x \geq 0 \tag{3}$$

where the summation convention is assumed for repeated indices in (3). If we neglect body forces (*e.g.*, gravity) and suppose that the macroscopic applied pressure gradient arises due the pressure difference ΔP across a distance ΔZ in the direction $\hat{z}$, then the relationship between the microscopic stresses and the macroscopic forces is easily seen to be given by (Berryman and Milton, 1985)

$$\mathcal{D} = \frac{k}{\mu}(\frac{\Delta P}{\Delta Z})^2 \tag{4}$$

or equivalently

$$\mathcal{D} = -\vec{J}\cdot\hat{z}\frac{\Delta P}{\Delta Z} \tag{5}$$

where $\vec{J}$ is the volumetric flow rate per unit area and

$$\vec{J} = -\frac{k}{\mu}\frac{\Delta P}{\Delta Z}\hat{z}. \tag{6}$$

Equation (6) is Darcy's law in the absence of body forces and the new constant appearing in (4) and (6) is the fluid permeability or Darcy's constant k.

In the presence of body forces, we generalize (6) to

$$\vec{J} = \frac{k}{\mu}\vec{X} \tag{7}$$

where

$$\vec{X} = -\nabla p + \rho\vec{g}. \tag{8}$$

The density of the fluid is ρ and the gravitational acceleration vector is $\vec{g}$.

2.2 Two-phase flow

Now suppose that the pore space is completely filled with two immiscible fluids $1, 2$. These fluids could be water and oil or water and air or any other pair of Newtonian fluids. Then the total energy dissipation in the two fluids will quite generally be given by the sum of the energy dissipation in the individual fluids

$$\mathcal{D} = \frac{1}{2\mu_1\Omega}\int_{\Omega_1} \sigma_{ij}^{(1)}\sigma_{ij}^{(1)}\, d^3x + \frac{1}{2\mu_2\Omega}\int_{\Omega_2} \sigma_{ij}^{(2)}\sigma_{ij}^{(2)}\, d^3x \geq 0 \tag{9}$$

where μ_n and Ω_n are the viscosity and volume of the nth fluid.

Furthermore, by analogy with the single-phase results, we must have in general that

$$\frac{1}{2\mu_n \Omega} \int_{\Omega_n} \sigma_{ij}^{(n)} \sigma_{ij}^{(n)}\, d^3x = \vec{J}^{(n)} \cdot \vec{X}^{(n)} \tag{10}$$

where $\vec{J}^{(n)}$ is the volumetric flow rate of the nth fluid and $\vec{X}^{(n)}$ is the force on the nth fluid. However, since there is more than one force present, Eq. (7) should be generalized to take into account the possibility that a force applied to one fluid may cause the other fluid to move as well. In particular, we expect the motion of one fluid to tend to drag the other along with it wherever the two fluids are in contact. Following the notation of Rothman (1990), we can write this resulting relation as

$$\vec{J}^{(1)} = L_{11}(\theta)\vec{X}^{(1)} + L_{12}(\theta)\vec{X}^{(2)} \tag{11}$$

$$\vec{J}^{(2)} = L_{21}(\theta)\vec{X}^{(1)} + L_{22}(\theta)\vec{X}^{(2)} \tag{12}$$

where $\theta = \Omega_1/(\Omega_1 + \Omega_2)$ is the saturation of (fraction of the pore space filled by) phase one. The Ls are linear coefficients that we show later are proportional to permeability and inversely proportional to viscosity.

Now define the force vector

$$f = \begin{pmatrix} \vec{X}^{(1)} \\ \vec{X}^{(2)} \end{pmatrix} \tag{13}$$

and the coefficient matrix

$$\mathcal{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}. \tag{14}$$

Then we see that the total dissipation can be written in the form

$$\mathcal{D} = f^T \mathcal{L} f \geq 0. \tag{15}$$

Since the force vector is arbitrary, we see that the coefficient matrix must be positive definite, which implies that

$$L_{11}L_{22} - L_{12}L_{21} > 0. \tag{16}$$

In closely related work, Thigpen and Berryman (1985) have shown that the average fluid velocities $\vec{V}^{(n)}$ obey the momentum equations

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \vec{V}^{(1)} \\ \vec{V}^{(2)} \end{pmatrix} = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \begin{pmatrix} \vec{X}^{(1)} \\ \vec{X}^{(2)} \end{pmatrix} \tag{17}$$

where $\phi_1 = \theta\phi$, $\phi_2 = (1-\theta)\phi$, and the Cs are drag coefficients that must satisfy the relations

$$C_{11} + C_{21} \geq 0, \qquad C_{22} + C_{12} \geq 0, \qquad C_{12} = C_{21} \leq 0, \tag{18}$$

to assure that the drag always opposes the motion and to guarantee the symmetry of the action-reaction (*i.e.*, momentum conservation). Since the fluid velocities are related to the volumetric flow rates by

$$\begin{pmatrix} \vec{J}^{(1)} \\ \vec{J}^{(2)} \end{pmatrix} = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \begin{pmatrix} \vec{V}^{(1)} \\ \vec{V}^{(2)} \end{pmatrix}, \tag{19}$$

the permeability matrix is then directly related to the drag coefficients. Substituting (19) into (11)-(12) and comparing to (17), we find that the coefficient matrix $\mathcal{L}$ is given in terms of the Cs as

$$\begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} \phi_1^2 C_{22} & -\phi_1\phi_2 C_{12} \\ -\phi_1\phi_2 C_{12} & \phi_2^2 C_{11} \end{pmatrix} \tag{20}$$

where $\Delta = C_{11}C_{22} - C_{12}^2$. Thus, it follows directly from momentum conservation for linear flow of two immiscible fluids that the coefficients for the cross terms are equal

$$L_{12} = L_{21}. \tag{21}$$

This equality has been postulated by Kalaydjian (1990) and by Rothman (1990) based on Onsager reciprocity (Fitts, 1962; Landau and Lifshitz, 1969). A thorough analysis of the possible pitfalls contained in such a postulate has been given by Fitts (1962). Rothman's numerical experiments have also substantially confirmed the result (21), while Kalaydjian's analytical results – obtained assuming that (21) holds – are in good agreement with those of Rothman when the fluid viscosities are equal. Furthermore, it follows directly from (18) and (20) that

$$L_{12} \leq (1-\theta)L_{11}/\theta, \qquad L_{12} \leq \theta L_{22}/(1-\theta). \tag{22}$$

Both Kalaydjian's analytical and experimental results are in good agreement with the combined bounds implied by (22). Rothman's numerical studies also provide experimental confirmation of (22). The details of these results will be reported elsewhere.

3. Specific surface area of the fluid/fluid interface

There is one rigorous formula (a simple sumrule) that can be easily derived relating the various internal surface areas s_W, s_{NW}, s_T, and s_I (where the subscripts refer respectively to wetting, nonwetting, total pore, and fluid/fluid interface areas). Figure 1 shows the simplest geometrical arrangement when one of the two fluids completely wets the internal surface area of the rock. We will call this condition "normal wettability." Then, the nonwetting fluid introduces a fluid/fluid interface. From Figure 2 it is clear that the total surface area of the wetting fluid is now $s_W = s_T + s_I$, while the surface area of the nonwetting fluid is the same as that of the fluid/fluid interface $s_{NW} = s_I$. Thus, the sum of the two gives the general result

$$s_W + s_{NW} = s_T + 2s_I. \tag{23}$$

Indeed, in the hypothetical situation where the two fluids occupy separate pore spaces, we see easily that $s_W + s_{NW} = s_T$ since then there is no fluid/fluid interface. The result for segregated fluids is therefore a special case of (23) with $s_I = 0$. Furthermore, if the two fluids have "mixed wettability" properties so that the pictures in Figures 1 and 2 are over simplified, then (23) is still valid. For "mixed wettability" conditions, both fluids wet different parts of the internal surface area. Quite generally the surface area of one fluid will be of the form $s_W = \omega s_T + s_I$ while the other will be of the form $s_{NW} = (1-\omega)s_T + s_I$, where ω is some fraction $0 \le \omega \le 1$. Summing these two expressions again gives (23).

We stress that (23) is not an approximation, but a rigorous result when the pore space is filled with two fluid phases. If more than two fluid phases are present, then (23) must be generalized to account for all the interfaces between pairs of fluids. The utility of a rigorous result such as (23) becomes apparent when we consider that measurements of the various specific surface areas may proceed almost independently, yet we may need to know if the results are consistent for a given rock sample. One simple check on the results that follows immediately from (23) is the condition

$$s_W + s_{NW} - s_T \ge 0. \tag{24}$$

If (24) is violated to a significant degree by the data for these three specific surface areas, then we may be sure that our experimental method has some inconsistency, bias, or error that must be eliminated. The second important practical use for (23) comes from the fact that it may be used to compute the fluid/fluid interface area per unit volume, since it implies that

$$s_I = \frac{1}{2}(s_W + s_{NW} - s_T). \tag{25}$$

Equation (25) will be useful whenever we need quantitative information about the interface area.

4. Kozeny-Carman relations

Since Darcy's constant is difficult to model, many approximate methods have been developed to estimate fluid permeability. Among these techniques are the effective medium based methods (Koplik, Lin, and Vermette, 1984; Wong, Koplik, and Tomanic, 1984; Doyen, 1988), percolation theory (Katz and Thompson, 1986; Balberg, 1986; Katz and Thompson, 1987; Thompson, Katz, and Krohn, 1987), the approaches using rigorous variational bounds (Berryman and Milton, 1985; Rubinstein and Torquato, 1989), and a great variety of methods based on Kozeny-Carman empirical relations. Our experience has been that intelligent use of the Kozeny-Carman relations can give excellent results, so we will concentrate on that approach here.

4.1 Single-phase flow

The Kozeny-Carman relation for single-phase flow derived by Walsh and Brace (1984) and used extensively by Berryman and Blair (1986; 1987) is given by

$$k = \frac{\phi^2}{2s^2F} = \frac{\phi^3}{2s^2\tau^2}, \tag{26}$$

where ϕ is the porosity, s is the specific surface area, F is the electrical formation factor, τ is the electrical tortuosity, and the relationship

$$\tau^2 = \phi F \tag{27}$$

has been used in the second part of Eq. (26).

4.2 Two-phase flow: traditional relative permeability

When two fluid phases are present in the pore space, we will distinguish two types of wetting behavior. Normal wettability will refer to the situation where one phase tends to wet the pore surface area, while the second phase tends not to wet it as long as the other phase is present in sufficient quantity. Mixed wettability will refer to the situation where both phases tend to wet distinct parts of the pore space regardless of their relative concentrations. We will now consider both of these situations in turn, while neglecting the possibility of cross terms in the relative permeabilities.

4.2.1 Tortuosity

We first examine the effect of tortuosity on two-phase flow. If we can predict the permeability for single-phase flow through a porous material from digitized information obtained from cross sections of rocks, we should also be able to obtain reasonable estimates of the traditional relative permeabilities. When both fluids percolate (form a continuous connected phase) throughout the porous sample, we will assume that to a first approximation the effective tortuosity for each fluid phase is the same as that for the porous solid

$$\phi F = \tau^2 = \tau_W^2 = \tau_{NW}^2. \tag{28}$$

For both phases to percolate, the saturation level must be moderate (say in the range 20% to 80%, depending on the porosity) and the permeability/porosity must not be very small. Then, the assumption (28) is both warranted and necessary in the present context, since it is unlikely that the relevant electrical formation factor measurements can generally be made for both phases.

Equation (28) is clearly an approximation that must be modified when the saturation level of either phase approaches the percolation threshold. In this case, we can introduce into (28) additional factors of

$$(\phi_W - \phi_W^c)^\alpha, \qquad (\phi_{NW} - \phi_{NW}^c)^\beta, \tag{29}$$

where the critical volume fractions or percolation thresholds for the wetting and nonwetting phases are given respectively by ϕ_W^c and ϕ_{NW}^c and the exponents α and β are to be determined through experiment (Bernabé, Brace, and Evans, 1982). Typical values of these exponents have been shown (Archie, 1942; Madden, 1976; Corey, 1977; Sen, Scala, and Cohen, 1981; Sen, 1984) to be $\alpha \simeq \beta \simeq 2$ for water saturations down to about 0.15-0.20 ($\simeq \phi_W^c/\phi$). Equation (28) is then modified to

$$\tau^2 = \tau_W^2[(\phi_W - \phi_W^c)/\phi]^\alpha = \tau_{NW}^2[(\phi_{NW} - \phi_{NW}^c)/\phi]^\beta. \tag{30}$$

In general, we expect the percolation thresholds to be small fractions $\phi_W^c \simeq \phi_{NW}^c \simeq 0.02$. With these volume fractions, we can also associate critical saturation levels $\theta_W^c = \phi_W^c/\phi$ and $\theta_{NW}^c = 1 - \phi_{NW}^c/\phi$. (Note the difference in the definitions of the ϕs and the θs: The ϕs are volume fractions, so ϕ_W is the volume fraction of the wetting phase and ϕ_{NW} is the same for the nonwetting phase, and we have the sumrule $\phi_W + \phi_{NW} = \phi$, since we assume the wetting and nonwetting phases together saturate the pore volume. In contrast, θ_W^c is the critical saturation value of the wetting fluid at wetting percolation, whereas θ_{NW}^c is the critical saturation value of the *wetting* phase at *nonwetting* percolation. This definition is made to simplify the formulas later.) There is no simple relationship between the critical saturations θ_W^c and θ_{NW}^c, since the shape of the effective pore space for each phase is different. However, it is reasonable to suppose that $\theta_W^c \simeq 1 - \theta_{NW}^c$. For example, if the porosity ϕ is about 20% and critical volume fraction for percolation in the rock pores is about 2%, then the critical saturation θ for the wetting phase is on the order of 10%, while for the nonwetting phase the critical value of θ is about 90%. In the absence of other information about the percolation thresholds, we may assume they take their limiting values of $\phi_W^c = 0$ and $\phi_{NW}^c = 0$.

4.2.2 Normal wettability

Applying the Kozeny-Carman relation (26) and the approximation (28) to the separate fluid phases, we have

$$k_W = \frac{\phi_W^3}{2s_W^2\tau_W^2} \tag{31}$$

and

$$k_{NW} = \frac{\phi_{NW}^3}{2s_{NW}^2\tau_{NW}^2}. \tag{32}$$

If the picture in Figure 1 is typical of the fluid arrangement in the pores, then we expect to find that $s_W = s_T + s_I$ and $s_{NW} = s_I$. The relative permeabilities are given by

$$k_{rW} = \frac{k_W}{k} = \frac{\theta^3}{(s_W/s_T)^2} = \frac{\theta^3}{(1+s_I/s_T)^2} \tag{33}$$

and

$$k_{rNW} = \frac{k_{NW}}{k} = \frac{(1-\theta)^3}{(s_{NW}/s_T)^2} = \frac{(1-\theta)^3}{(s_I/s_T)^2}, \tag{34}$$

so the tortuosity τ has completely cancelled out of these expressions.

Again, if the picture in Figure 1 is representative of the distribution of nonwetting fluid to wetting fluid, then to a reasonable approximation we should expect that

$$s_I \simeq (1-\theta)s_T. \tag{35}$$

Equation (35) will be valid when the grain surface is fairly smooth, or when our *estimate* of the grain surface area is obtained using a low resolution method such as image processing. Otherwise, we generally expect (35) to be replaced by $s_I << (1-\theta)s_T$, since the fluid/fluid interface will be smooth regardless of the roughness of the grain surface. So, continuing to neglect the effects of the changing tortuosity close to a percolation threshold and using (35), we should find that

$$k_{rW} \simeq \frac{\theta^3}{(2-\theta)^2} \tag{36}$$

and

$$k_{rNW} \simeq (1-\theta). \tag{37}$$

The results (36) and (37) show we should expect a strong asymmetry between the relative permeabilities for the wetting and the nonwetting phases. These first approximations to the relative permeabilities show that k_{rW} for the wetting phase is a strongly nonlinear function of the wetting phase saturation θ. Physically, this makes sense because the wetting phase is trapped between the solid and the nonwetting fluid, so the typical open throat for the wetting phase is significantly smaller in the presence of the nonwetting phase. By contrast, the nonwetting phase sees a porous medium that is very similar to that of the solid matrix – the effective pores have just been reduced in size by the coating action of the wetting phase. Until percolation threshold effects become important, the relative permeability for this phase should be approximately proportional to $(1-\theta)$ as shown in (37).

4.2.3 Mixed wettability

If we can measure the various specific surface areas using either image processing methods (as will be described later in this paper) or some other means, then it is straightforward to determine whether the condition of normal wettability or that of mixed wettability applies. We check for mixed wettability by comparing the two specific surface areas s_{NW} and s_I. For normal wettability, we have $s_{NW} = s_I$, while for mixed wettability we will find that $s_{NW} > s_I$. The interface area itself may still be computed unambiguously from the rigorous formula (25). Strictly speaking our notation should change now, since the phase labelled NW is also a wetting phase under these conditions. However, we will keep the notation and simply be aware that the NW-phase is assumed to wet a smaller proportion of the pore wall than does the W-phase.

Now recalling that for mixed wettability $s_W = \omega s_T + s_I$ and $s_{NW} = (1-\omega)s_T + s_I$ where ω is the relative wetting fraction, the formulas (33) and (34) must be modified so the relative permeabilities are given by

$$k_{rW} = \frac{k_W}{k} = \frac{\theta^3}{(s_W/s_T)^2} = \frac{\theta^3}{(\omega + s_I/s_T)^2} \tag{38}$$

and

$$k_{rNW} = \frac{k_{NW}}{k} = \frac{(1-\theta)^3}{(s_{NW}/s_T)^2} = \frac{(1-\theta)^3}{(1-\omega+s_I/s_T)^2}. \tag{39}$$

Since we now have a wetting fraction ω as well as the saturation parameter θ, it becomes more difficult to do any sort of general analysis of the relative permeabilities in such complex circumstances. The image processing methods for obtaining these parameters should nevertheless provide reasonable estimates of all these parameters and therefore of the relative permeabilities as well.

4.3 Two-phase flow: relative permeability including cross terms

The theory of the relative permeability in the presence of fluid/fluid interactions has still to be developed. We will have to be satisfied here with preliminary results. Let the subscripts $1, 2$ refer respectively to wetting and nonwetting fluid phases. Then, a reasonable approximation to the coefficient matrix $\mathcal{L}$ that is consistent with what we know at present is

$$\begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} = \begin{pmatrix} k_{rW}k/\mu_W & \chi \\ \chi & k_{rNW}k/\mu_{NW} \end{pmatrix} \tag{40}$$

where we have used the empirically derived expression for the cross terms

$$\chi \simeq min[(1-\theta)L_{11}/\theta, \quad \theta L_{22}/(1-\theta)], \tag{41}$$

which has been found to agree well with the available data of Kalaydjian (1990) and Rothman (1990). The motivation for considering a relation of the form (41) comes from the rigorous bounds (22), based on momentum conservation. Equation (41) tends to be a good estimator of the cross terms everywhere except in the vicinity of $\theta \simeq 0.5$, where

(41) overestimates the cross terms by nearly a factor of two for the real data example of Kalaydjian (1990).

5. Sample preparation and image processing methodology

In this section we describe the methodology used to produce and analyze images of Berea sandstone in which the pore space was segregated into regions occupied by simulated wetting and nonwetting phases. First, a low melting point bismuth metal alloy was imbibed into a dry evacuated core sample of Berea sandstone, at a temperature of approximately 100° C. After imbibition, the sample was cooled to room temperature, solidifying the metal alloy. The bulk saturation with the metal alloy was determined to be 57% based on weight. The core sample was then cut into slabs in the direction perpendicular to its axis, and examination of the slabs showed that the saturation decreased along the length of the core from bottom to top.

One slab, taken from the lower section of the sample was chosen for our analysis. This slab was vacuum impregnated with epoxy and a standard petrographic thin section was prepared. The thin section was examined using a scanning electron microscope (SEM) in backscatter mode. In this mode the metal (nonwetting) phase appeared bright, while the epoxy (wetting) phase appeared dark and the mineral phase (grains) had intermediate brightness. Hardcopy images of the pore structure were produced as photographic prints and negatives. The negatives were then digitized using an Eikonix scanning digitizer. Once they were in digital form the image scale was determined and the images were cropped using standard techniques.

Images of the total porosity and segments of the pore space occupied by the wetting and nonwetting phases were produced as follows. The thresholding technique used to identify pore space filled by the nonwetting metal phase was a slightly modified version of the method described by Berryman and Blair (1986). Unfortunately, this technique did not work as well for the epoxy phase, especially in areas along the metal/epoxy interface. This problem was overcome by applying thresholding to sections of the image to delineate the epoxy/grain and metal/grain boundaries in each section. A mosaic was then constructed from the sections to produce an image of the total pore space. Finally, an image of the wetted (epoxy filled) pore space was produced by subtracting the nonwetted (metal filled) pore space from the total pore space. Comparison of the digital images with the original prints was very useful in segregating the pore space.

While we are aware that this simulation produced by first imbibing a nonwetting phase and subsequently injecting a wetting phase into the pore space may not produce two-phase spatial distributions representative of simultaneous two-phase flow, we believe that the insight gained from this approach is valuable nevertheless.

6. Results

Total porosity and the amount of pore space occupied by the nonwetting and wetting phases was determined from four images with magnifications of 54× and 100×. A representative section is shown in Figure 3a. The associated image displaying the segregated pore space is shown in Figure 3b. These Figures show that as expected the nonwetting phase filled most of the large pores. The fraction of pore space occupied by each phase was simply calculated and results are presented in Table 1. The value of total porosity was determined to be 0.23 and this agrees well with values determined by laboratory measurements. Laboratory data indicate that the bulk saturation of the rock with the nonwetting phase was approximately 0.57. However, saturation varied widely over the length of the core and our value of 0.77 at the location of the slab provides precise data on the fine scale distribution of the nonwetting phase.

Two-point autocorrelation functions were calculated for images of the total pore space and for pore space occupied by each phase, following the technique developed by Berryman (1985), and Berryman and Blair (1986). From these data for the total pore space the value of the two-point correlation function S_2 at each length increment was calculated as the radial average of values for that length. S_2^{NW} and S_2^{W} were calculated in a similar way from the respective autocorrelation functions. The two-point correlation functions for the total pore space and the nonwetted and wetted pore space shown in Figure 3b are presented in Figure 3c. This figure shows that the correlation function for the nonwetting, imbibed phase closely resembles the correlation function for the total pore space. This is typical for the images we analyzed and is consistent with our previous observation that the nonwetting phase sees a porous medium that is similar to the solid matrix. Mean grain size can also be estimated from the correlation function, and our results indicate a value of 80 microns for this section.

We also analyzed four images with magnifications in the range 220 to 1100x. Images at this magnification were used to estimate the specific surface areas, following the work of Debye, Anderson, and Brumberger (1957), who showed that the derivative of the two-point correlation function near the origin is proportional to the specific surface area, and Berryman and Blair (1987) who related the resolution appropriate for measuring s_T to the correlation length computed for the material. A representative image is shown in Figure 4a, and the segregated pore space is shown in Figure 4b. The two-point autocorrelation functions S_2, S_2^{NW}, S_2^{W} for the total pore space and the respective sections of pore space occupied by each phase are shown are shown in Figure 4c. This figure shows that at this scale the correlation function for the total pore space is not dominated by either pore phase. To estimate the specific surface area s_T for the total pore space, we fit a line to the first few points of the function S_2 for each image of the total pore space [see Berryman and Blair (1986) for a description of the method]. The slopes of these lines were then averaged to produce a value of S_2' for the total pore space of the cross section; this value is given in Table 1. Specific surface area (s_T) was then calculated as $s_T = -4S_2'(0)$ following Berryman and Blair (1986). Values of specific surface area for both the wetting (s_W) and nonwetting phases (s_{NW}) were calculated in a similar fashion. These measured values of specific surface area were then used to calculate specific area of the interface (s_I) following Eq. (25). Results are given in Table 1. The value of s_T of 0.1072 for the total pore space agrees well with the values of 0.1109 and 0.1231 we reported previously for a different sample of Berea sandstone [see Berryman and Blair (1986)]. In addition, these results also show that the pore space occupied by the wetting phase has the larger contribution to the total specific surface area.

Permeability for the cross section was estimated using the computed values of porosity and specific surface area in conjunction with equation (26). Assuming a value of 10.0–15.0 for the formation factor, we estimate the intrinsic permeability of the rock to be 220 md–150 md. This is within a factor of two or three of the value of 350 md measured in the laboratory for the specimen. The estimated value of the wetting fraction was $\omega = 0.74$ and the ratio of the interface surface area to the total surface area was found to be $s_I/s_T = 0.087$, both of which are consistent with observations of the images. Thus, the wetting phase does in fact cover most of the surface area and the wetting and nonwetting phases do not actually have much interfacial contact throughout the sample.

We also estimated relative permeability for the wetting and nonwetting phases, first using (38) and (39). Estimated values of relative permeability for both phases are given in Table 1. These results show qualitatively that the wetting phase dominates the specific surface area, while the nonwetting phase dominates the permeability due to its higher partial saturation. This also indicates that for the imaged cross section, the wetting phase is filling the dead end pores, and that much of the specific surface area is contained in those dead end pores.

Although the results for the wetting phase were quantitatively sensible ($k_W \simeq .02$), the results for the nonwetting phase were not, since the computed relative permeability for the nonwetting phase was found to be greater than unity ($k_{rNW} \simeq 3.7$). It is possible that the relative permeability found in an actual two-phase flow measurement might be greater than unity if the wetting phase acts as a lubricant for the nonwetting phase (by relaxing the no-slip boundary condition), thus permitting it to flow through the pores more easily than in the absence of the wetting phase. But, the relative permeability cannot be greater than unity in our procedure because such lubrication effects are neglected. Furthermore, Berryman and Blair (1987) present a theorem stating that the permeability of a less porous material is *always* less than or equal to that of a more porous material when the pore space of the less porous material is contained in the pore space of the more porous one. Since the nonwetting phase has intruded into the rock's pore space and not created new pore space in the process, this problem satisfies the conditions of the theorem and therefore we may conclude that it is physically unacceptable to ignore the effects of percolation thresholds and the increase in tortuosity for the nonwetting phase as we have done so far.

Thus, to improve our estimates of relative permeability, we *must* incorporate equation (30) into equations (31) and (32). Assuming $\alpha = \beta = 2$, equations (38) and (39) can be written as

$$k_{rW} = \frac{\theta^3(\theta - \theta_W^c)^2}{(s_W/s_T)^2} \tag{42}$$

and

$$k_{rNW} = \frac{(1-\theta)^3(\theta_{NW}^c - \theta)^2}{(s_{NW}/s_T)^2}, \tag{43}$$

where the relative permeabilities are no longer expressed as functions of saturation and the surface areas alone, but also depend on the percolation thresholds. We do not have an independent measure of the value of the percolation threshold for the nonwetting phase θ_{NW}^c, but it is interesting to note that we can use (43) together with the theorem mentioned previously (showing that $k_{rNW} \le 1$ is required physically) to obtain an estimate of the value of the percolation threshold. This computation shows that $(1-\theta_{NW}^c)\phi \ge .05$ and that $\theta_{NW}^c \le 0.75$. These numbers are completely consistent with the fact that the measured nonwetting saturation $1-\theta \simeq 0.77 \ge 0.25 \simeq 1-\theta_{NW}^c$. This computation (by construction) yields a physically consistent set of numbers for the various permeabilities and the threshold values.

Another nonrigorous relation we can use to gain some information is the approximate relation $\theta_W^c \simeq 1-\theta_{NW}^c$ between the percolation thresholds for the wetting and the nonwetting phases. Thus, we find that $\theta_W^c \simeq 0.25$, with some (possibly significant) experimental error. The measured value of the wetting saturation was $\theta = 0.23$, so the wetting phase was imbibed at close to its percolation threshold. The value of the relative permeability obtained from (33) is already so low that any corrections from the presence of the percolation threshold value in (42) will be neglected. Clearly, the wetting phase permeability is very low in this example, and we would be pushing the image processing method well beyond its limitations to try to give a more precise estimate.

Accurate estimates of the percolation threshold values also should not be expected from this approach since the exponents α and β (assumed here to follow Archie's law) must be obtained simultaneously along with the threshold values.

Specific surface area for each phase was also used with the fractional pore volume of each phase to compute the effective hydraulic radius $r = \phi/s$, for the total and segregated pore space. These values are also shown in Table 1 and indicate that the effective hydraulic radius of the pore space occupied by the nonwetting phase is larger than that of the total pore space.

7. Discussion

The analysis in this paper has stressed two-phase flow through porous media, but it should be clear that generalization to multi-phase flow is possible. For example, Oak, Baker, and Thomas (1990) summarize the current understanding of relative permeability for three-phase flow. In the presence of water, gas, and oil, our analysis should be generalized from the 2×2 system in (11)-(12) to a 3×3 system with six independent coefficients instead of three. The result (23) for the relationship among the surface areas is also easily generalized to

$$s_{water} + s_{gas} + s_{oil} = s_{pore} + 2s_{w/g} + 2s_{w/o} + 2s_{g/o}. \tag{44}$$

The rigorous result for two-phase flow (23) has been shown to be a very valuable tool for checking the consistency of computed surface areas, and (44) may also prove useful for three-phase flow analysis.

Our analysis of relative permeability has shown that Kozeny-Carman relations and Archie's empirical laws must be modified to account for a finite percolation threshold. Our continuing effort to estimate physical properties of rocks from images has therefore led us to an unexpected result. A simplified theory that ignores the percolation thresholds leads to unphysical values (> 1) for the relative permeability of the nonwetting phase. Inclusion of the percolation thresholds in the calculation leads to a set of physically reasonable numbers for the various permeabilities, and threshold values themselves. This approach cannot be used to find accurate estimates of the percolation threshold values since the exponents α and β must be determined simultaneously, but with assumed values for the exponents a consistent picture of the physical parameters can be obtained.

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FIGURE CAPTIONS

Fig. 1. Geometrical arrangement of wetting and nonwetting fluids in a typical pore.

Fig. 2. Illustration of the sumrule presented in Eq. (23).

Fig. 3a. Before image of Berea sandstone at $\times 54$ magnification.

Fig. 3b. Image of Berea sandstone after segmentation into solid grains (black), nonwetting (gray) and wetting (white) phases.

Fig. 3c. Two-point correlation functions for Berea sandstone: S_2 for total pore space, S_2^W for the wetted pore space, and S_2^{NW} for the nonwetted pore space.

Fig. 4a. Before image of Berea sandstone at $\times 540$ magnification.

Fig. 4b. Image of Berea sandstone after segmentation into solid grains (black), nonwetting (gray) and wetting (white) phases.

Fig. 4c. Two-point correlation functions for Berea sandstone: S_2 for total pore space, S_2^W for the wetted pore space, and S_2^{NW} for the nonwetted pore space.