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The Elastic Coefficients of Double-Porosity Models for Fluid Transport in Jointed Rock

James G. Berryman
Lawrence Livermore National Laboratory
P. O. Box 808 L-202
Livermore, CA 94551-9900
Telephone: 510-423-2905
FAX: 510-422-1002
and
*Herbert F. Wang**
Department of Geology and Geophysics
University of Wisconsin
1215 West Dayton Street
Madison, WI 53706
Telephone: 608-262-5932
FAX: 608-262-0693

*Address for correspondence.

Abstract

Phenomenological equations (with coefficients to be determined by specified experiments) for the poroelastic behavior of a dual porosity medium are formulated and the coefficients in these linear equations are identified. The generalization from the single porosity case increases the number of independent coefficients for volume deformation from three to six for an isotropic applied stress. The physical interpretations are based upon considerations of different temporal and spatial scales. For very short times, both matrix and fractures behave in an undrained fashion. For very long times, the double porosity medium behaves like an equivalent single porosity medium. At the macroscopic spatial level, the pertinent parameters (such as the total compressibility) may be determined by appropriate field tests. At an intermediate or mesoscopic scale pertinent parameters of the rock matrix can be determined directly through laboratory measurements on core, and the compressibility can be measured for a single fracture. All six coefficients are determined from the three poroelastic matrix coefficients and the fracture compressibility from the single assumption that the solid grain modulus of the composite is approximately the same as that of the matrix for a small fracture porosity. Under this assumption, the total compressibility and three-dimensional storage coefficient of the composite are the volume averages of the matrix and fracture contributions.

1. Introduction

Analysis of the quasistatic behavior of porous fluid-saturated mechanical systems is generally based on "poroelastic theory." The first detailed studies of the coupling between the pore-fluid pressure and solid stress fields were described using a linear elastic theory by Biot [1941]. The quasistatic constitutive equations relate the strain tensor linearly to both the stress tensor and the fluid pressure. Time dependent, quasistatic fluid flow is incorporated by combining the continuity requirement with Darcy's law. As originally formulated, Biot's theory applies to a homogeneous, porous medium. However, porosity and permeability often occur in rock masses on several distinct spatial scales. Thus, the need arises for more general models incorporating qualitatively different types of porosity (*e.g.*, matrix versus fracture) as well as different types of rock mass for code calculations at the reservoir or aquifer scale. Biot's theory nevertheless continues to play an important role in these more complex models: the mechanical behavior of the matrix materials from which they are usually constructed is often described by Biot's equations supplemented with fluid-flow coupling terms between the blocks.

In problems of fluid flow in hydrocarbon reservoirs and aquifers, the simplest and most frequent idealization is the dual-porosity/dual-permeability medium in which a porous matrix is partitioned into blocks by a fracture network [Barenblatt and Zheltov, 1960; Warren and Root, 1963]. It is generally assumed that the fracture permeability is much greater than the matrix permeability, while the fracture porosity is much smaller than the matrix porosity. Therefore, fluid flow occurs primarily through the fracture network, but fluid storage occurs mostly in the porous matrix.

The typical pictorial representation of the double porosity medium shows disaggregated matrix blocks surrounded entirely by fluid in fractures. In this scenario, apparently the blocks must be supported entirely by fluid pressure. The poroelastic model for a double porosity medium to be presented here accounts more realistically for some of the external stress to be supported partially by asperities bridging fracture surfaces.

Since the pore-pressure field is not in general decoupled from the stress field, it is necessary to incorporate Biot's concepts of poroelasticity into the dual porosity model. First the static constitutive equations must be formulated. Then the equations of fluid transport can be included via the continuity requirement. The independent variables for volumetric change are chosen to be the external confining pressure, p_c, and the fluid pressures in the matrix $p_f^{(1)}$ and in the fracture $p_f^{(2)}$, respectively. The dependent variables are chosen to be the volumetric strain, and Biot's increment of fluid content (fluid volume accumulation per unit bulk volume) in the matrix $\zeta^{(1)}$ and fracture $\zeta^{(2)}$. The phenomenological approach then relates each dependent variable linearly to the independent variables, with coefficients to be determined by experiments to be specified. This choice of variables leads to a symmetric coefficient matrix because the scalar product of the dependent and independent variables is an energy density.

The governing equations for fluid transport in a dual-porosity/dual-permeability medium are written as a pressure diffusion equation in the fractured medium with a matrix-to-fracture source term that, in its simplest form, is proportional to the difference in pressure between fluid in the matrix and fluid in the adjacent fracture [Wilson and Aifantis, 1982; Khaled et al., 1984; Beskos and Aifantis, 1986; Elsworth and Bai, 1990; Elsworth and Bai, 1992; Bai et al., 1993a].

Dual porosity, including dual permeability, models [Barenblatt and Zheltov, 1960; Warren and Root, 1963] have been applied in numerous situations to simulate fluid flow in fractured reservoirs and aquifers (e.g., Streltsova [1988]). Similarly, Biot's [1941] linear poroelastic theory has been applied in numerous situations in which the stress field is coupled to fluid flow, *e.g.*, tidal and barometric effects on well levels [Van der Kamp and Gale, 1983; Rojstaczer and Agnew, 1989], subsidence and seismicity due to fluid withdrawal from gas reservoirs [Segall *et al.*, 1994], seismicity due to water level changes in reservoir lakes [Roeloffs, 1978], fluid pressure changes due to a moving dislocation [Roeloffs and Rudnicki, 1984; 1985].

Because most near surface rock masses are fractured to some degree, it is natural to examine coupling a dual porosity, fluid flow model with deformation, that is, to extend Biot's theory to the dual porosity situation of separate matrix and fracture phases. Although this model has been presented in the literature (e.g., Wilson and Aifantis [1982], Cho *et al.* [1991], Bai *et al.* [1993b]), our re-examination of the problem (1) provides improved understanding of the physical meaning of the poroelastic constants, (2) justifies the intuitive result that the instantaneous matrix and fracture pore pressure buildups during undrained compression should be the same as that expected in each phase individually as if each were independent of the other, (3) shows how to obtain the parameters in the dual porosity situation from parameters for each individual phase, and (4) corrects the approximate formulation from previous workers for fully-coupled problems.

The focus of this paper is on the rigorous identification of the coefficients in a linear formulation of the phenomenological equations. Although some of the parameters identified are not readily measurable, the attending analysis nevertheless clarifies the significance of all the coefficients. The procedure is analogous to that of Biot and Willis [1957] in which they establish physical interpretations of the coefficients in Biot's equations as well as relations between macroscopic parameters and properties of the more microscopic constituents. The analysis is performed by considering both different temporal and different spatial scales. Three temporal scales are considered: very short times (both matrix and fracture phases are undrained), intermediate (only one phase is drained), and long times (matrix and fracture pressures are equilibrated). Similarly, three levels of spatial scale are considered: macroscopic (reservoir and fracture scale), intermediate or mesoscopic (core sample scale), and microscopic (individual grain and crack scale). At the macroscopic level, the pertinent parameters (such as the fracture compressibility) may in principle be determined by appropriate field tests or large block tests. At the scale of core samples, pertinent parameters of the rock matrix may be determined directly through laboratory measurements. Two special cases are identified: (1) When the fracture fluid pressure is set equal to that of the confining pressure, the matrix material is effectively isolated from the fracture behavior and the analysis may be conveniently simplified. (2) When a special choice of the matrix fluid pressure is made, the matrix can be constrained to behave rigidly on the average, and so the fracture behavior is effectively isolated from that of the matrix. These two cases allow a natural decoupling of the double-porosity system so the coefficients in the equations may be determined in easily described laboratory experiments.

2. Overview of Deformation Dependent Flow

In this section, we give a brief overview of the general ideas to be pursued in the remainder of the paper. Precise definitions of quantities introduced are given in the following section.

The equations for deformation-coupled flow in a single porosity medium are easily derived from Biot's equations of poroelasticity by taking the low frequency limit, assuming only that inertial effects and second derivatives with respect to time are negligible. If the solid displacement is u_i and the fluid pressure is p (positive in compression), the resulting equations are

$$\frac{3K}{2(1+\nu)}e_{,i} + \frac{3(1-2\nu)K}{2(1+\nu)}u_{i,jj} = \alpha p_{,i}, \tag{1}$$

where the solid dilatation is $e = u_{j,j}$, K and ν are the drained bulk modulus and Poisson's ratio, respectively, and

$$\frac{k}{\mu}p_{,ii} = \frac{\alpha}{BK_u}\dot{p} + \alpha\dot{e}, \tag{2}$$

where K_u is the undrained bulk modulus and B is Skempton's coefficient [Skempton, 1954]. The dots indicate time derivatives. The remaining constants are the Biot-Willis parameter $\alpha = 1 - K/K_s = (1 - K/K_u)/B$, the permeability k, and the fluid viscosity μ. [Another constant we will need, but that does not appear explicitly here, is the undrained Poisson's ratio ν_u.] All these constants are given their precise definitions in the next section. These equations are in any case well known and equivalent forms may be found in Rice and Cleary [1976], Cleary [1977], etc.

The generalization of (1) and (2) to double porosity media is straightforward. First, we assume that there are two types of porosity and corresponding pressures $p^{(1)}$ and $p^{(2)}$ within the fluids contained in each pore type. (See Figure 1.) Then, to generalize the equation for the solid displacement, we merely change the forcing term on the right hand side to allow for more components, so (1) becomes

$$\frac{3K}{2(1+\nu)}e_{,i} + \frac{3(1-2\nu)K}{2(1+\nu)}u_{i,jj} = \alpha^{(1)}p^{(1)}_{,i} + \alpha^{(2)}p^{(2)}_{,i}, \tag{3}$$

where we have introduced phenomenological constants $\alpha^{(1)}$ and $\alpha^{(2)}$ whose precise physical significance must be determined. An important characteristic of this equation is that each point in space now has two fluid pressures associated with it, and therefore these pressures are (not the true microscopic pressures in the fluid, but actually) averages over some representative volume element. Similarly, Khaled *et al.* [1984] write two equations in place of (2) having the forms

$$\frac{k^{(1)}}{\mu}p^{(1)}_{,ii} = \frac{\alpha^{(1)}}{B^{(1)}K^{(1)}_u}\dot{p}^{(1)} + \alpha^{(1)}\dot{e} - \gamma(p^{(2)} - p^{(1)}), \tag{4}$$

and

$$\frac{k^{(2)}}{\mu}p^{(2)}_{,ii} = \frac{\alpha^{(2)}}{B^{(2)}K^{(2)}_u}\dot{p}^{(2)} + \alpha^{(2)}\dot{e} + \gamma(p^{(2)} - p^{(1)}), \tag{5}$$

and we will show how to interpret their coefficients in terms of a more complete derivation.

The coefficients $k^{(1)}$ and $k^{(2)}$ are permeabilities associated with matrix and fracture porosity, respectively. The remaining factors are straightforward generalizations of the constants appearing in (2). Long time analysis to be presented later shows that $k^{(1)} + k^{(2)} = k$ and also provides other relations among the various constants. The terms proportional to pressure differences have been introduced to drive the pressures in the two types of porosity towards a single equilibrium pressure that will be approached at long times. (See Zimmerman *et al.* [1992] for a discussion of the physical significance of the coefficient γ.) More detailed derivations of these equations may be found in the papers by Wilson and Aifantis [1982], Khaled et al. [1984], and Beskos and Aifantis [1986]. In particular, note that, by setting $\dot{e} = 0$, the flow equations are no longer coupled to the strain equation (3), so we recover the classical rigid double-porosity model of Barenblatt and Zheltov [1960]. The main point to be emphasized here is that these equations are just linear relations between stresses and strains. Except for questions about the neglect of possible cross-coupling terms (which we show later is a real issue but fortunately a small one), the form of the equations is not seriously in doubt. However, we show later that (4) and (5) neglect time-dependent cross-coupling terms that are not likely to be negligible [cf. equations (52) and (53)]. Furthermore, the meanings and values of various coefficients may be nontrivial to deduce from the physics and mechanics of the underlying microscopic problem.

The remainder of the paper is devoted to a careful analysis of the precise physical significance of the various parameters appearing in equations (3)–(5), or the related equations we derive to replace them.

3. Single Porosity Models and Long Time Behavior

In the absence of driving forces that can maintain pressure differentials over long time periods, double porosity models must reduce to single porosity models in the long time limit when the matrix pore pressure and crack/fracture pore pressure become equal. It is therefore necessary to remind ourselves of the basic results for single porosity models in poroelasticity. One important role these results play is to provide constraints for the long time behavior in the problems of interest. A second significant use of these results (which we address later in this paper) arises when we make laboratory measurements on core samples having properties characteristic of the matrix material. Then the results presented in this section apply specifically to the matrix stiffnesses, porosity, etc.

For isotropic materials and hydrostatic pressure variations, the two independent variables in linear mechanics of porous media are the confining (external) pressure p_c and the fluid (pore) pressure p_f. The differential pressure $p_d \equiv p_c - p_f$ is often used to eliminate the confining pressure. The equations of the fundamental dilatations are then

$$-\frac{\delta V}{V} = \frac{\delta p_d}{K} + \frac{\delta p_f}{K_s} \tag{6}$$

for the total volume V,

$$-\frac{\delta V_\phi}{V_\phi} = \frac{\delta p_d}{K_p} + \frac{\delta p_f}{K_\phi} \tag{7}$$

for the pore volume $V_\phi = \phi V$, and

$$-\frac{\delta V_f}{V_f} = \frac{\delta p_f}{K_f} \tag{8}$$

for the fluid volume V_f. These equations provide a shorthand notation for defining the constants that appear in them, which may be defined precisely by expressions such as

$$\frac{1}{K} = -\frac{1}{V}\left(\frac{\partial V}{\partial p_d}\right)_{p_f}, \quad \frac{1}{K_s} = -\frac{1}{V}\left(\frac{\partial V}{\partial p_f}\right)_{p_d}, \quad \text{etc.} \tag{9}$$

(Also, see the Appendix and Table A.) Thus, equation (6) serves to define the various constants of the porous solid, such as the drained frame bulk modulus K and the unjacketed bulk modulus K_s for the composite frame. Equation (7) defines the jacketed pore modulus K_p and the unjacketed pore modulus K_ϕ. Similarly, (8) defines the bulk modulus K_f of the pore fluid.

Treating δp_c and δp_f as the independent variables in our poroelastic theory, we define the dependent variables $\delta e \equiv \delta V/V$ and $\delta\zeta \equiv (\delta V_\phi - \delta V_f)/V$, both of which are positive on expansion, and which are respectively the total volume dilatation and the increment of fluid content. Then, it follows directly from the definitions and from (6), (7), and (8) that

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} 1/K & 1/K_s - 1/K \\ -\phi/K_p & \phi(1/K_p + 1/K_f - 1/K_\phi) \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}. \tag{10}$$

Now we consider two well-known *gedanken* experiments: the drained test and the undrained test [Gassmann, 1951; Biot and Willis, 1957; Geertsma, 1957]. (For a single porosity system, these two experiments are sometimes considered equivalent to the "slow loading" and "fast loading" limits respectively. However, these terms are relative since, for example, the fast loading — equivalent to undrained — limit is still assumed to be slow enough that the average fluid and confining pressures are assumed to have reached equilibrium.) The drained test assumes that the porous material is surrounded by an impermeable jacket and the fluid is allowed to escape through a tube that penetrates the jacket. Then, in a long duration experiment, the fluid pressure remains in equilibrium with the external fluid pressure (*e.g.*, atmospheric) and so $\delta p_f = 0$ and hence $\delta p_c = \delta p_d$; so the changes of total volume and pore volume are given exactly by the drained constants $1/K$ and $1/K_p$ as defined in (6) and (7). On the other hand, the undrained test assumes that the jacketed sample has no tubes to the outside world, so pore pressure responds only to the confining pressure changes. With no means of escape, the increment of fluid content cannot change, so $\delta\zeta = 0$. Then, the second equation in (10) shows that

$$0 = -\phi/K_p(\delta p_c - \delta p_f/B), \tag{11}$$

where Skempton's pore-pressure buildup coefficient B [Skempton, 1954] is defined by

$$B \equiv \left.\frac{\partial p_f}{\partial p_c}\right|_{\delta\zeta=0} \tag{12}$$

and is therefore given by

$$B = \frac{1}{1 + K_p(1/K_f - 1/K_\phi)}. \tag{13}$$

It follows immediately from this definition that the undrained modulus K_u is determined by (also see Carroll [1980])

$$\frac{1}{K_u} \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial p_c}\right)_{\delta\zeta=0} = \frac{1-\alpha B}{K}, \tag{14}$$

where we introduced the combination of moduli known as the Biot-Willis parameter $\alpha = 1 - K/K_s$. This result was apparently first obtained by Gassmann [1951] (though not in this form) for the case of microhomogeneous porous media (*i.e.*, $K_s = K_\phi = K_m$, the bulk modulus of the single mineral present) and by Brown and Korringa [1975] and Rice [1975] for general porous media with multiple minerals as constituents.

Finally, we condense the general relations from (10) together with the reciprocity relations [Brown and Korringa, 1975] into symmetric form as

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \frac{1}{K}\begin{pmatrix} 1 & -\alpha \\ -\alpha & \alpha/B \end{pmatrix}\begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}. \tag{15}$$

Specifically, the reciprocity relation results in the identity $K_p = \alpha/\phi K$, which has been used to arrive at (15).

A storage coefficient, which is a central concept in describing poroelastic aquifer behavior in hydrogeology, related inversely to one defined in Biot's original 1941 paper is

$$S \equiv \left.\frac{\partial\zeta}{\partial p_f}\right|_{\delta p_c=0} = \frac{\alpha}{BK}. \tag{16}$$

This storage coefficient is the change in increment of fluid content per unit change in the fluid pressure, *defined for a condition of no change in external pressure.* It has also been called the three-dimensional storage coefficient by Kümpel [1991].

We may equivalently eliminate the Biot-Willis parameter α and write (15) in terms of the undrained modulus so that

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \frac{1}{K}\begin{pmatrix} 1 & -(1-K/K_u)/B \\ -(1-K/K_u)/B & (1-K/K_u)/B^2 \end{pmatrix}\begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}. \tag{17}$$

Equation (17) has the advantage that all the parameters have very well defined physical interpretations, and are also fairly easily generalized for a double porosity model.

The total strain energy functional (including shear) for this problem may be written in the form

$$2E = \delta\tau_{ij}\delta e_{ij} + \delta p_f \delta\zeta, \tag{18}$$

where δe_{ij} is the change in the average strain with $\delta e_{ii} \equiv \delta e$ being the dilatation, $\delta\tau_{ij}$ being the change in the average stress tensor for the saturated porous medium with $\frac{1}{3}\delta\tau_{ii} = -\delta p_c$. It follows that

$$\delta p_c = -\frac{\partial E}{\partial e} \tag{19}$$

and

$$\delta p_f = \frac{\partial E}{\partial \zeta}, \tag{20}$$

both of which are also consistent with Betti's reciprocal theorem [Love, 1927] since the matrices in (15) and (17) are symmetric. The shear modulus G is related to the bulk modulus and Poisson's ratio by $G = 3(1-2\nu)K/2(1+\nu)$. Then, it follows that the stress equilibrium equation is

$$\tau_{ij,j} = (K_u + \tfrac{1}{3}G)e_{,i} + Gu_{i,jj} - BK_u\zeta_{,i} = 0 \tag{21}$$

and Darcy's law takes the form

$$\frac{k}{\mu}p_{,ii} = \dot{\zeta}. \tag{22}$$

From these equations, (1) and (2) may be easily derived using the identity

$$\frac{\zeta}{K_u - K} = \frac{p}{(BK_u)^2} + \frac{e}{BK_u}, \tag{23}$$

which follows from (17). Equation (23) is used to eliminate the confining pressure p_c from the equations.

4. Double Porosity Models: Identifying Phenomenological Coefficients

The theory to be described implicitly assumes the existence of a macroscopic length scale at which the rock/pore/fracture/fluid mixture can be treated in some sense as both homogeneous and isotropic. The homogeneity assumption is an absolute requirement. The isotropy assumption is merely a convenience — providing a desirable simplification of the analysis of the more general problem in which fractures are likely not to be randomly oriented, and thus produce a preferred axis for fluid flow as well as anisotropy in the elastic coefficients. These questions of anisotropy can be treated in a straighforward extension of the present work. On the other hand, the homogeneity requirement imposes a maximum length scale for practical application of the theory to be described. For reservoirs that are sufficiently inhomogeneous, we would expect that the present analysis would apply only to localized regions of the reservoir, and that the entire reservoir could then be modeled by a type of finite element analysis wherein the present theory would provide the means of assigning constants to those finite elements.

We begin by assuming two distinct phases at the macroscopic level: a porous matrix phase with the effective properties $K^{(1)}$, $G^{(1)}$, $K_m^{(1)}$, $\phi^{(1)}$ occupying volume fraction $V^{(1)}/V = v^{(1)}$ of the total volume and a macroscopic crack or joint phase occupying the remaining fraction of the volume $V^{(2)}/V = v^{(2)} = 1 - v^{(1)}$. We emphasize that the crack/joint/fracture phase is a pure void phase. Since our approach is phenomenological, we need to make no assumption of microhomogeneity in the solid composing the porous frame. The key feature distinguishing the pores in the two phases — and therefore requiring this analysis — is the very high fluid permeability $k^{(22)}$ of the crack or joint phase and the relatively lower permeability $k^{(11)}$ of the matrix phase. We could also introduce a third independent permeability $k^{(12)} = k^{(21)}$ for fluid flow at the interface between the matrix and crack phases, but for simplicity we assume here that this third permeability is essentially the same as that of the matrix phase, so $k^{(12)} = k^{(11)}$.

We have three distinct pressures: confining pressure δp_c, pore-fluid pressure $\delta p_f^{(1)}$, and joint-fluid pressure $\delta p_f^{(2)}$. Treating $\delta p_c, \delta p_f^{(1)}$, and $\delta p_f^{(2)}$ as the independent variables in our double porosity theory, we define the dependent variables $\delta e \equiv \delta V/V$ (as before), $\delta\zeta^{(1)} = (\delta V_\phi^{(1)} - \delta V_f^{(1)})/V$, and $\delta\zeta^{(2)} = (\delta V_\phi^{(2)} - \delta V_f^{(2)})/V$, which are respectively the total volume dilatation, the increment of fluid content in the matrix phase, and the increment of fluid content in the joints. We assume that the fluid in the matrix is the same kind of fluid as that in the cracks or joints, but that the two fluid regions may be in different states of average stress and therefore need to be distinguished by their respective superscripts.

Linear relations among strain, fluid content, and pressure then take the general form

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}. \tag{24}$$

By analogy with (15) and (17), it is easy to see that $a_{12} = a_{21}$ and $a_{13} = a_{31}$. The symmetry of the new off-diagonal coefficients may be demonstrated by using Betti's reciprocal theorem in the form

$$\begin{pmatrix} \delta e & -\delta\zeta^{(1)} & -\delta\zeta^{(2)} \end{pmatrix} \begin{pmatrix} 0 \\ -\delta\overline{p}_f^{(1)} \\ 0 \end{pmatrix} = \begin{pmatrix} \delta\overline{e} & -\delta\overline{\zeta}^{(1)} & -\delta\overline{\zeta}^{(2)} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -\delta p_f^{(2)} \end{pmatrix}, \tag{25}$$

where unbarred quantities refer to one experiment and barred to another experiment to show that

$$\delta\zeta^{(1)}\delta\overline{p}_f^{(1)} = a_{23}\delta p_f^{(2)}\delta\overline{p}_f^{(1)} = a_{32}\delta\overline{p}_f^{(1)}\delta p_f^{(2)} = \delta\overline{\zeta}^{(2)}\delta p_f^{(2)}. \tag{26}$$

Hence, $a_{23} = a_{32}$. Similar arguments have often been used to establish the symmetry of the other off-diagonal components. Thus, we have established that the matrix in (24) is completely symmetric, so we need to determine only six independent coefficients. To do so, we consider a series of *gedanken* experiments, including tests in both the short time and long time limits. The key idea here is that at long times the two pore pressures must come to equilibrium ($p_f^{(1)} = p_f^{(2)} = p_f$ as $t \to \infty$) as long as the cross permeability $k^{(12)}$ is finite. However, at very short times, we may assume that the process of pressure equilibration has not yet begun. We nevertheless assume that the pressure in each of the two components have individually equilibrated on the average, even at short times.

We should emphasize that in the present section we describe *thought experiments*, and as such they may *not* necessarily be expected to be realizable in the laboratory or field in all cases. Practical methods of evaluating the coefficients are treated in the two following sections.

Also, note that the existence of a second pore pressure and increment of fluid content leads to the possibility of several distinct generalizations of the Skempton and/or Biot-Willis coefficients. The somewhat complicated notation we introduce will attempt to emphasize the defining boundary conditions for the various cases. We will clarify these differences in the Discussion Section.

4.1 Undrained joints, undrained matrix, short time

There are several different, but equally valid, choices of time scale on which to define Skempton-like coefficients for the matrix/fracture system under consideration. Elsworth and Bai [1992] use a definition based on the idea that for very short time both fluid systems will independently act undrained after the addition of a sudden change of confining pressure. This idea implies that $\delta\zeta^{(1)} = 0 = \delta\zeta^{(2)}$ which, when substituted into (24), gives

$$\begin{aligned} -\delta e &= a_{11}\delta p_c + a_{12}\delta p_f^{(1)} + a_{13}\delta p_f^{(2)} \\ 0 &= a_{12}\delta p_c + a_{22}\delta p_f^{(1)} + a_{23}\delta p_f^{(2)} \\ 0 &= a_{13}\delta p_c + a_{23}\delta p_f^{(1)} + a_{33}\delta p_f^{(2)}. \end{aligned} \tag{27}$$

Defining

$$B_{EB}^{(1)} \equiv \left.\frac{\partial p_f^{(1)}}{\partial p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} \quad \text{and} \quad B_{EB}^{(2)} \equiv \left.\frac{\partial p_f^{(2)}}{\partial p_c}\right|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} \tag{28}$$

we can solve (27) for the two Skempton's coefficients and find the results

$$B_{EB}^{(1)} = \frac{a_{23}a_{13} - a_{12}a_{33}}{a_{22}a_{33} - a_{23}^2} \tag{29}$$

and

$$B_{EB}^{(2)} = \frac{a_{23}a_{12} - a_{13}a_{22}}{a_{22}a_{33} - a_{23}^2}. \tag{30}$$

The effective undrained modulus is found to be given by

$$\frac{1}{K_{u^{EB}}} \equiv -\frac{\partial e}{\partial p_c}|_{\delta\zeta^{(1)}=\delta\zeta^{(2)}=0} = a_{11} + a_{12}B_{EB}^{(1)} + a_{13}B_{EB}^{(2)}. \tag{31}$$

These definitions will be compared to others as our analysis progresses.

4.2 Drained joints, undrained matrix, intermediate time

Again consider a sudden change of confining pressure on a jacketed sample, but this time with tubes inserted in the joint (fracture) porosity so $\delta p_f^{(2)} = 0$, while $\delta\zeta^{(1)} = 0$. We will call this the drained joint, undrained matrix limit. The resulting equations are

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - a_{12}\delta p_f^{(1)} \\ 0 &= -a_{21}\delta p_c - a_{22}\delta p_f^{(1)} \\ -\delta\zeta^{(2)} &= -a_{31}\delta p_c - a_{32}\delta p_f^{(1)}, \end{aligned} \tag{32}$$

showing that the pore-pressure buildup in the matrix is

$$B[u^{(1)}] \equiv \left.\frac{\partial p_f^{(1)}}{\partial p_c}\right|_{\delta\zeta^{(1)}=\delta p_f^{(2)}=0} = -\frac{a_{21}}{a_{22}}. \tag{33}$$

Similarly, the effective undrained modulus for the matrix phase is found from (32) to be determined by

$$\frac{1}{K[u^{(1)}]} \equiv -\left.\frac{\partial e}{\partial p_c}\right|_{\delta\zeta^{(1)}=\delta p_f^{(2)}=0} = a_{11} + a_{12}B[u^{(1)}]. \tag{34}$$

The generalized storage coefficient a_{23} is a measure of cross-coupling between the matrix and fracture fluids. The discussion section examines the significance of this coefficient more fully, but for now it is useful to notice that, if $a_{23} = 0$, then (29) and (33) are the same.

4.3 Drained matrix, undrained joints, intermediate time

The next thought experiment might be difficult or impossible to realize in either the laboratory or the field. Nevertheless, consider another sudden change of confining pressure on a jacketed sample, but this time with the tubes inserted in the matrix porosity so $\delta p_f^{(1)} = 0$, while $\delta\zeta^{(2)} = 0$. We will call this the drained matrix, undrained joint limit. The equations are

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - a_{13}\delta p_f^{(2)} \\ -\delta\zeta^{(1)} &= -a_{21}\delta p_c - a_{23}\delta p_f^{(2)} \\ 0 &= -a_{31}\delta p_c - a_{33}\delta p_f^{(2)}, \end{aligned} \tag{35}$$

showing that the pore-pressure buildup in the cracks is

$$B[u^{(2)}] \equiv \left.\frac{\partial p_f^{(2)}}{\partial p_c}\right|_{\delta\zeta^{(2)}=\delta p_f^{(1)}=0} = -\frac{a_{31}}{a_{33}}. \tag{36}$$

Similarly, the effective undrained modulus for the joint phase is found from (35) to be determined by

$$\frac{1}{K[u^{(2)}]} \equiv -\left.\frac{\partial e}{\partial p_c}\right|_{\delta\zeta^{(2)}=\delta p_f^{(1)}=0} = a_{11} + a_{13}B[u^{(2)}]. \tag{37}$$

We may properly view Eqs. (33), (34), (36), and (37) as "defining" relations among these parameters.

Notice that, if $a_{23} = 0$, then (30) and (36) are the same.

4.4 Drained test, long time

The long duration drained (or "jacketed") test for a double porosity system should reduce to the same results as in the single porosity limit. The conditions on the pore pressures are that the background fluid pressure is uniform and that the changes in the pressures satisfy $\delta p_f^{(1)} = \delta p_f^{(2)} = 0$ in the long time limit. The total volume then obeys $\delta e = -a_{11}\delta p_c$, and therefore it follows that

$$a_{11} \equiv \frac{1}{K}, \tag{38}$$

where K is the overall drained bulk modulus of the system including the fractures.

4.5 Undrained test, long time

The long duration undrained test for a double porosity system should also produce the same physical results as a single porosity system (assuming only that it makes sense at some appropriate larger scale to view the medium as homogeneous). Again the background fluid pressures have equilibrated, so the basic equations for the changes in the pressures and fluid response are

$$\begin{aligned} \delta p_f^{(1)} = \delta p_f^{(2)} &= \delta p_f, \\ \delta\zeta \equiv \delta\zeta^{(1)} + \delta\zeta^{(2)} &= 0, \end{aligned} \tag{39}$$

assuming the total mass of fluid is confined. Then, using the reciprocity condition $a_{23} = a_{32}$, it follows that

$$\begin{aligned} \delta e &= -a_{11}\delta p_c - (a_{12} + a_{13})\delta p_f, \\ 0 &= -(a_{21} + a_{31})\delta p_c - (a_{22} + 2a_{23} + a_{33})\delta p_f, \end{aligned} \tag{40}$$

showing that the overall pore-pressure buildup coefficient is given by

$$B \equiv \left.\frac{\partial p_f}{\partial p_c}\right|_{\delta\zeta=0} = -\frac{a_{21} + a_{31}}{a_{22} + 2a_{23} + a_{33}}. \tag{41}$$

Similarly, the undrained bulk modulus is found to be given by

$$\frac{1}{K_u} \equiv -\left.\frac{\partial e}{\partial p_c}\right|_{\delta\zeta=0} = a_{11} + (a_{12} + a_{13})B. \tag{42}$$

4.6 Fluid injection test, long time

The conditions on the pore pressures for the long duration, single porosity limit for the three-dimensional storage coefficient S are $\delta p_f^{(1)} = \delta p_f^{(2)} = \delta p_f$, while the confining pressure remains constant. It follows therefore that

$$S \equiv \left.\frac{\partial \zeta}{\partial p_f}\right|_{\delta p_c=0} = a_{22} + 2a_{23} + a_{33}. \tag{43}$$

4.7 Generalized Biot-Willis Parameters

Equation (38) has already determined the coefficient a_{11}. Then, (42) shows that

$$a_{12} + a_{13} = -\frac{1/K - 1/K_u}{B} = -\alpha/K, \tag{44}$$

while (34) and (38) show that

$$a_{12} = -\frac{1/K - 1/K[u^{(1)}]}{B[u^{(1)}]} \equiv -\overline{\alpha}^{(1)}/K \tag{45}$$

and similarly (37) and (38) show that

$$a_{13} = -\frac{1/K - 1/K[u^{(2)}]}{B[u^{(2)}]} \equiv -\overline{\alpha}^{(2)}/K. \tag{46}$$

In (45) and (46), we have introduced generalized Biot-Willis parameters $\overline{\alpha}^{(1)}$ and $\overline{\alpha}^{(2)}$ for the matrix and joint phases, defined by these equations. These two parameters are defined this way for notational convenience, but for example $\overline{\alpha}^{(1)}$ should not be confused with the true Biot-Willis parameter $\alpha^{(1)}$ of the matrix material without fractures, which we anticipate will generally differ in value from $\overline{\alpha}^{(1)}$.

Combining (45) and (46) with (44) shows that

$$\alpha = \overline{\alpha}^{(1)} + \overline{\alpha}^{(2)}. \tag{47}$$

Relations such as (47) showing dependencies among the various constants are useful because they show that constants potentially difficult to measure (such as the undrained joint modulus $K[u^{(2)}]$) can actually be determined from other more easily accessible data.

These results show that all the constants in the first row of the matrix in (24) have now been determined in terms of quantities that could in principle be measured. Similar manipulations give the remaining constants as we will show.

4.8 Summary of results

Combining these results, we obtain the following general relations

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \frac{1}{K} \begin{pmatrix} 1 & -\overline{\alpha}^{(1)} & -\overline{\alpha}^{(2)} \\ -\overline{\alpha}^{(1)} & \overline{\alpha}^{(1)}/B[u^{(1)}] & Ka_{23} \\ -\overline{\alpha}^{(2)} & Ka_{23} & \overline{\alpha}^{(2)}/B[u^{(2)}] \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}, \tag{48}$$

where the expression for the remaining off-diagonal term

$$a_{23} \equiv (\alpha/B - \overline{\alpha}^{(1)}/B[u^{(1)}] - \overline{\alpha}^{(2)}/B[u^{(2)}])/2K \tag{49}$$

follows from (41) after substituting for other coefficients using (33), (36), and (44)–(46). Comparing (15) with the long time behavior ($\delta p_f^{(2)} \rightarrow \delta p_f^{(1)}$) of (48) provides a simple means of checking the validity of (48).

The comparative simplicity of (48) may be sufficient reason to consider this set of definitions as the most "natural" one. However, since not all of the parameters are easily measured, we will need to continue our analysis in the next two sections. For now, we make use of these results to derive equations for fluid flow.

In analogy with (23), we need to write equations for $\zeta^{(1)}$ and $\zeta^{(2)}$ eliminating the confining pressure. Using (45) and (46), the results of these calculations are:

$$\zeta^{(1)} = \overline{\alpha}^{(1)} e + \frac{\overline{\alpha}^{(1)}}{B[u^{(1)}]K[u^{(1)}]} p_f^{(1)} + (a_{23} - \frac{\overline{\alpha}^{(1)}\overline{\alpha}^{(2)}}{K}) p_f^{(2)}, \tag{50}$$

and

$$\zeta^{(2)} = \overline{\alpha}^{(2)} e + \frac{\overline{\alpha}^{(2)}}{B[u^{(2)}]K[u^{(2)}]} p_f^{(2)} + (a_{23} - \frac{\overline{\alpha}^{(1)}\overline{\alpha}^{(2)}}{K}) p_f^{(1)}. \tag{51}$$

It is important to note that the presence of the nonzero coefficient $[a_{23} - (\overline{\alpha}^{(1)}\overline{\alpha}^{(2)})/K)]$ of the cross-coupling terms is a departure from the approach of Khaled *et al.* [1984] and Elsworth and Bai [1992]. These authors have simply assumed that this coefficient vanishes identically. This is an approximation that needs further justification. We elaborate this point in the Discussion Section.

Since confining pressure has been eliminated, we can drop the subscripts (but not the superscripts) on the two fluid pressures. We assume a Darcy-like behavior of the system. For single porosity, the appropriate equation is given by (22). A straightforward generalization of this result for two fluids and two pressures is then given by the following equations:

$$\frac{k^{(11)}}{\mu}p^{(1)}_{,ii} + \frac{k^{(12)}}{\mu}p^{(2)}_{,ii} = \dot{\zeta}^{(1)} = \frac{\overline{\alpha}^{(1)}}{B[u^{(1)}]K[u^{(1)}]}\dot{p}^{(1)} + \overline{\alpha}^{(1)}\dot{e} + \left(a_{23} - \frac{\overline{\alpha}^{(1)}\overline{\alpha}^{(2)}}{K}\right)\dot{p}^{(2)} \quad (52)$$

and

$$\frac{k^{(22)}}{\mu}p^{(2)}_{,ii} + \frac{k^{(21)}}{\mu}p^{(1)}_{,ii} = \dot{\zeta}^{(2)} = \frac{\overline{\alpha}^{(2)}}{B[u^{(2)}]K[u^{(2)}]}\dot{p}^{(2)} + \overline{\alpha}^{(2)}\dot{e} + \left(a_{23} - \frac{\overline{\alpha}^{(1)}\overline{\alpha}^{(2)}}{K}\right)\dot{p}^{(1)}. \quad (53)$$

We include possible cross-coupling terms on the left-hand side as an appropriate generalization of Darcy's law when two fluids and two fluid pressures are present (see Thigpen and Berryman [1985] for a discussion and derivation). These equations should be compared to (4) and (5). A detailed comparison of differences in numerical predictions is beyond the scope of the present paper, but will be pursued in future work.

5. Laboratory Measurements on Core Samples

Although the preceding section gives rigorous definitions of the various constants needed to generalize from single-porosity to double-porosity poroelasticity, nevertheless some of these constants are not easily measured directly either in the laboratory or in the field. Therefore, in this section and the next, we show how to relate all the coefficients to more easily measured and interpreted parameters. Thus, our emphasis changes now to practical means of identifying the coefficients.

It follows immediately from (15) and the definitions of the constants that the pertinent equations for pure matrix material must be given by

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)}/v^{(1)} \end{pmatrix} = \frac{1}{K^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \end{pmatrix}. \quad (54)$$

These constants can all be estimated [Rice and Cleary, 1976; Detournay and Cheng, 1993] or found in the laboratory by analysis of core samples of matrix material [Hart and Wang, 1995]. This fact is important because it suggests another way of identifying certain combinations of the general double-porosity coefficients in (24).

To obtain the connecting equations, first note that

$$\delta e \equiv \frac{\delta V}{V} = v^{(1)}\frac{\delta V^{(1)}}{V^{(1)}} + \frac{\delta V^{(2)}}{V} = v^{(1)}\delta e^{(1)} + v^{(2)}\delta e^{(2)}, \quad (55)$$

$$\delta e^{(i)} \equiv \frac{\delta V^{(i)}}{V^{(i)}} \quad \text{for} \quad i = 1, 2, \quad (56)$$

and

$$\delta\zeta^{(2)} \equiv \frac{\delta V_\phi^{(2)} - \delta V_f^{(2)}}{V} = \frac{\delta V^{(2)}}{V} - \frac{\delta V_f^{(2)}}{V}. \quad (57)$$

The final relation follows from the identity $\delta V^{(2)} = \delta V_\phi^{(2)}$, since the fracture volume is all void space.

Thus, to obtain the desired relations in a form analogous to (54), we must rewrite (24) as

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -v^{(2)}\delta e^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & \overline{a}_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}, \tag{58}$$

where the third diagonal element has been modified to eliminate the fluid contribution such that

$$\overline{a}_{33} = a_{33} - \frac{v^{(2)}}{K_f}. \tag{59}$$

Now we consider another thought experiment: Suppose the confining pressure is equal to the fracture pressure so $\delta p_c = \delta p_f^{(2)}$. This situation mimics that of the matrix core sample in a laboratory experiment by completely surrounding the matrix material with a uniform pressure field. (See Figure 2.) Then, by combining the appropriate rows and columns, we can telescope the 3×3 system down to a 2×2 system of the form

$$v^{(1)} \begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)}/v^{(1)} \end{pmatrix} = \begin{pmatrix} a_{11} + 2a_{13} + \overline{a}_{33} & a_{12} + a_{23} \\ a_{12} + a_{23} & a_{22} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \end{pmatrix}. \tag{60}$$

So, except for an overall factor of $v^{(1)} = V^{(1)}/V$ (which is generally very close to unity in the cases of interest), equations (54) and (60) are of the same form. We are therefore led to the identifications

$$\frac{v^{(1)}}{K^{(1)}} = a_{11} + 2a_{13} + \overline{a}_{33}, \tag{61}$$

$$-\frac{v^{(1)}\alpha^{(1)}}{K^{(1)}} = a_{12} + a_{23}, \tag{62}$$

and, using (12) and $K_p = \phi K/\alpha$ together with the definition of $\phi^{(1)}$ as the porosity of the matrix material, we find

$$\frac{v^{(1)}\alpha^{(1)}}{K^{(1)}} + v^{(1)}\phi^{(1)}(1/K_f - 1/K_\phi^{(1)}) = \frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K^{(1)}} = a_{22}. \tag{63}$$

The stress-strain relations may now be expressed in the form

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} 1/K & a_{12} & -\alpha/K - a_{12} \\ a_{12} & v^{(1)}\alpha^{(1)}/B^{(1)}K^{(1)} & -v^{(1)}\alpha^{(1)}/K^{(1)} - a_{12} \\ -\alpha/K - a_{12} & -v^{(1)}\alpha^{(1)}/K^{(1)} - a_{12} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}, \tag{64}$$

where the fracture self-storage coefficient is given by

$$a_{33} = v^{(2)}/K_f + v^{(1)}/K^{(1)} - (1 - 2\alpha)/K + 2a_{12}, \tag{65}$$

and from (33)

$$a_{12} = -v^{(1)}\alpha^{(1)}B[u^{(1)}]/B^{(1)}K^{(1)}. \tag{66}$$

Combining (61) and (62) then shows that

$$v^{(1)}/K_s^{(1)} = a_{11} + a_{12} + 2a_{13} + a_{23} + \overline{a}_{33}, \tag{67}$$

while combining (62) and (63) shows that

$$v^{(1)}\phi^{(1)}(1/K_f - 1/K_\phi^{(1)}) = a_{12} + a_{22} + a_{23}. \tag{68}$$

6. Fractures in a Rigidly Constrained Matrix

In the preceding section, setting $\delta p_f^{(2)} = \delta p_c$ created a situation where the matrix was completely surrounded by an effective confining pressure $\delta p_c = \delta p_f^{(2)}$. This trick essentially isolated the matrix material from the fracture behavior and permitted a simple identification of certain combinations of the double-porosity parameters in terms of the matrix constants.

Now we would like to isolate the the fractures just as we isolated the matrix in the last section. Unfortunately, the same trick does not apply. Setting $\delta p_f^{(1)} = \delta p_c$ does not generally isolate the fracture (although there might be particular geometries where it does), because neither the confining pressure nor the matrix fluid pressure are applied directly to the surface of the fractures. So instead we must consider some alternative thought experiment to achieve the desired decoupling. By considering an experiment on a double porosity medium so that the matrix material remains rigid (*i.e.*, $\delta e^{(1)} = 0$) while the fractures deform, this condition specifies a particular choice of the matrix fluid pressure $\delta p_f^{(1)} = \delta p^*$ (depending on both δp_c and δp_f, but normally $\delta p^* \neq \delta p_c$) and shows that the desired separation is then accomplished.

Consider equation (58). To determine whether a particular set of stresses results in the matrix remaining rigid or not, we need an equation isolating the matrix strain $\delta e^{(1)}$ from the overall strain δe. Since $\delta e = v^{(1)}\delta e^{(1)} + v^{(2)}\delta e^{(2)}$, it follows directly from (58) that we have zero matrix strain when

$$-v^{(1)}\delta e^{(1)} = (a_{11} + a_{31})\delta p_c + (a_{13} + \overline{a}_{33})\delta p_f^{(2)} + (a_{12} + a_{32})\delta p_f^{(1)} \equiv 0. \tag{69}$$

This final equality (setting the right hand side equal to zero) provides the needed rigidity condition. Solving the resulting equation for $\delta p_f^{(1)}$ in terms of δp_c and $\delta p_f^{(2)}$ provides the relation required to determine the value of matrix fluid pressure $\delta p_f^{(1)} = \delta p^*$ needed to guarantee overall matrix rigidity (see Fig. 3). Then, we obtain the desired equation for $v^{(2)}\delta e^{(2)}$ as a function of δp_c and $\delta p_f^{(2)}$:

$$v^{(2)}\delta e^{(2)} = \frac{a_{11}a_{32} - a_{31}a_{12}}{a_{12} + a_{32}}(-\delta p_c) + \frac{a_{13}a_{32} - \overline{a}_{33}a_{12}}{a_{12} + a_{32}}(-\delta p_f^{(2)}). \tag{70}$$

The definitions of the constants for the fracture system in a rigid matrix ($\delta e^{(1)} = 0$) are given by

$$v^{(2)}\begin{pmatrix} \delta e^{(2)} \\ -\delta\zeta^{(2)}/v^{(2)} \end{pmatrix} = \frac{v^{(2)}}{K^{(2)}}\begin{pmatrix} 1 & -\alpha^{(2)} \\ -\alpha^{(2)} & \alpha^{(2)}/B^{(2)} \end{pmatrix}\begin{pmatrix} -\delta p_c \\ -\delta p_f^{(2)} \end{pmatrix}. \tag{71}$$

It is important to recognize that, whereas $B^{(2)}$ and $K^{(2)}$ have their well defined (standard) interpretations for the fracture phase as experimental observables, $\alpha^{(2)}$ is now a parameter that strictly speaking does not have the usual Biot-Willis [1957] micromechanical interpretation in terms of frame and grain modulus since there is no grain modulus associated with the fracture phase. We can nevertheless define an effective grain modulus according to $K_s^{(2)} \equiv K^{(2)}/(1-\alpha^{(2)})$, but care should be taken not to overinterpret this parameter. With these definitions, we are finally led to the identifications:

$$\frac{v^{(2)}}{K^{(2)}} = \frac{a_{11}a_{32} - a_{13}a_{12}}{a_{12} + a_{23}} \tag{72}$$

and

$$\frac{v^{(2)}\alpha^{(2)}}{K^{(2)}} = \frac{\overline{a}_{33}a_{12} - a_{13}a_{23}}{a_{12} + a_{23}}. \tag{73}$$

Note that, if a_{23} vanishes, the right hand sides of these equations reduce to $-a_{13}$ and $\overline{a}_{33}$, respectively.

The analogous calculation of the fracture fluid increment $\delta\zeta^{(2)}$ shows that

$$-\delta\zeta^{(2)} = \frac{a_{13}a_{12} - a_{23}a_{11}}{a_{12} + a_{23}}(-\delta p_c) + \frac{a_{33}(a_{12} + a_{23}) - a_{23}(a_{13} + \overline{a}_{33})}{a_{12} + a_{23}}(-\delta p_f^{(2)}). \tag{74}$$

Since reciprocity shows that the off-diagonal terms of (71) are equal, we obtain a condition on the coefficients by equating the the second numerator of (70) with the first numerator of (74), showing that

$$a_{13}a_{12} - a_{11}a_{23} = a_{13}a_{23} - \overline{a}_{33}a_{12}. \tag{75}$$

Substituting from (64) and solving for a_{12}, we obtain the result

$$a_{12} = -\frac{\alpha^{(1)}}{K^{(1)}}\frac{K_s^{(1)}}{K_s}, \tag{76}$$

which should be compared to (66).

Note that in the special case when $a_{23} = 0$, (75) implies that $\overline{a}_{33} = -a_{13}$, while (72) and (73) then imply that $\alpha^{(2)} = 1$. Furthermore, under the same assumption, (71), (72), and (74) imply that $B^{(2)} = -a_{13}/a_{33}$, while (66) and (76) imply that $K_s^{(1)} = v^{(1)}K_s$. These results will be discussed further in the next section.

7. Discussion

7.1 Comparison with single porosity theory

The preceding analysis of the constitutive equations for a fractured or jointed, porous medium is one logical extension of Biot's linear theory for a double porosity material. The purpose of this section is to provide additional interpretation of the basic coefficient matrix in

(24). Biot's original formulation for an isotropic, poroelastic material requires three independent moduli for an isotropic state of stress (15). Biot's constant, α/K, is analogous to the thermal expansion coefficient in thermoelasticity, and hence will be referred to as the poroelastic expansion coefficient. This coefficient gives the amount of bulk expansion of the stress-free material for a uniform pore pressure increase. The other constant, $S \equiv \alpha/BK$, is the so-called three-dimensional storage coefficient as used in hydrogeology [Van der Kamp and Gale, 1983; Kümpel, 1991; Wang, 1993]. This constant quantifies the volume of fluid that must flow into a control volume due to an increase in pore pressure *at constant confining pressure.*

The double porosity theory (24) with six independent coefficients, a_{ij}, is a straightforward generalization of Biot's original equations. The six coefficients occur in three categories that correspond to the three original Biot coefficients. The coefficient $a_{11} = 1/K$ is an effective compressibility of the combined fracture-matrix system. The coefficients, $a_{12} = -\overline{\alpha}^{(1)}/K$ and $a_{13} = -\overline{\alpha}^{(2)}/K$, are generalized poroelastic expansion coefficients. The overbar notation is used to emphasize that the bulk dilatation due to a pore pressure increase in phase 1 while the confining pressure and $p_f^{(2)}$ are kept constant is *not* the same as the poroelastic expansion of the matrix itself, in general, although the analysis shows that it is a good approximation for a fractured, porous medium.

The terms a_{22}, a_{23} , $a_{32} = a_{23}$, and a_{33} are generalized storage coefficients, *i.e.*, a_{ij} is the volume of fluid that flows into a control volume (normalized by the control volume) of phase $i-1$ due to a unit increase in fluid pressure in phase $j-1$. They can be thought of as forming a tensor of storage coefficients, linking the vector composed of the increments of fluid content $\zeta^{(1)}$ and $\zeta^{(2)}$ to the two pore pressures $p_f^{(1)}$ and $p_f^{(2)}$. In Biot's single porosity theory, the storage coefficient $S = \alpha/BK$. The diagonal components a_{22} and a_{33} for the double porosity theory are $\overline{\alpha}^{(i)}/B[u^{(i)}]K$ for $i = 1, 2$, where again the notation emphasizes the similarity to the single porosity case, but also emphasizes that they are not necessarily the values of phase i as a single porosity material. The double porosity material is a composite, and hence has effective moduli that depend on the interactions between the two phases.

7.2 Stress formulation

In the following two paragraphs, we show that the six coefficients that completely describe volumetric deformations of the double porosity material can be obtained from three constants for the matrix and the fracture compressibility with just one additional assumption about the coupling between the matrix and fracture phases at constant stress. In our stress-based formulation, the coefficient a_{23} is a cross-storage coefficient for conditions of constant confining pressure. The assumption $a_{23} = 0$ is equivalent to $K_s^{(1)} = v^{(1)}K_s$, which obtains for a matrix of a single constituent and $v^{(1)} \simeq 1$. The assumption $a_{23} = 0$ immediately leads to equality between the various Skempton-like coefficients: $B^{(1)} = B_{EB}^{(1)} = B[u^{(1)}] = -a_{12}/a_{22}$ and $B^{(2)} = B_{EB}^{(2)} = B[u^{(2)}] = -a_{13}/a_{33}$. [For example, the result for $B^{(1)}$ follows from (62) and (66).] This assumption also leads from (61), (72), and (73) to the result that the overall compressibility a_{11} is the volume average of the matrix compressibility $1/K^{(1)}$ and the fracture compressibility $1/K^{(2)} = 1/(v^{(2)}k_n s)$, where k_n (GPa/m) is the fracture stiffness and s (m) is the fracture spacing (cf. Elsworth and Bai [1992]).

Numerical values for the matrix and fracture properties are given for Berea sandstone and Westerly granite in Table 1. Expressions and numerical values for all the a_{ij}s are given in

Table 2. Note that if $v^{(1)} \simeq 1$, then the results for a_{22} and a_{33} are the same as the normal three-dimensional storage coefficient for the matrix and fracture phases separately. Also, the long-time three-dimensional storage coefficient is the sum of the storage coefficients of the individual phases. These results are intuitively reasonable, and provide a means of predicting the deformation and fluid storage behavior of the fractured, porous medium from knowledge of the individual phases. In particular, note that all the coefficients in Table 2 were computed from the values of $K^{(1)}$, $K_s^{(1)}$, K_f, $\phi^{(1)}$, $v^{(2)}$, and $K^{(2)}$ quoted in Table 1. Thus, these six measurements (together with Poisson's ratio) are sufficient to determine completely the behavior of the double-porosity model.

In contrast to the preceding example, Table 3 presents data for Chelmsford granite and Weber sandstone taken from laboratory measurements by Coyner [1984]. The data available in these experiments differ somewhat from the preceding case, since Coyner's experiments included a series of tests on several types of laboratory scale rock samples at different confining pressures. The values quoted for K and K_s are those for a moderate confining pressure of 10 MPa (values at lower confining pressures were also measured but we avoid using these values because the rocks generally exhibit nonlinear behavior in that region of the parameter space), while the values quoted for $K^{(1)}$ and $K_s^{(1)}$ are at 25 MPa. Thus, based on the idea that the pressure behavior is associated with two kinds of porosity in the laboratory samples — a crack porosity, which is being closed between 10 and 25 MPa, and a residual matrix porosity above 25 MPa, we assume the available data are K, K_s, $K^{(1)}$, $K_s^{(1)}$, K_f, $\phi^{(1)}$, and $v^{(2)}$. We find that these data are sufficient to compute all the coefficients, and therefore no assumption need be made about the value of a_{23}. In Table 4, we find for both types of rock that this coefficient is positive but small — about an order of magnitude smaller than the other matrix elements. The only other unusual feature of the results computed using this laboratory data is the occurrence of values larger than unity for $B[u^{(1)}]$ in Chelmsford granite and for $B[u^{(2)}]$ and $B_{EB}^{(2)}$ in Weber sandstone. Also note that $\alpha^{(2)}$ for both rocks is very close to unity. In this example, seven measurements (together with Poisson's ratio) are sufficient to determine completely the volumetric changes of the double-porosity model. The main difference between this example and the preceding one is that having a direct measurement of K eliminates the necessity of assuming $a_{23} = 0$.

7.3 Strain formulation

Elsworth and Bai [1992], Khaled *et al.* [1984], and Wilson and Aifantis [1982] formulated the constitutive equations for $\delta\zeta^{(1)}$ and $\delta\zeta^{(2)}$ in terms of bulk strain, δe [cf. (50) and (51), respectively] in place of p_c. We will show that the assumption $a_{23} = 0$ leads to far more reasonable results than their analogous assumption $A_{23} \equiv a_{23} - a_{12}a_{13}/a_{11} = 0$, where the upper case coefficients A_{ij} signify corresponding coefficients for a strain-based formulation. Solving for a_{23} in the case of Berea sandstone leads to a value of 0.085 GPa^{-1}, which is comparable in magnitude with the poroelastic expansion coefficient a_{12}. Furthermore, using this value for a_{23} leads to a value of $B_{EB}^{(1)} = 0.11$ for the short-term Skempton's coefficient in the matrix phase. The problem stems from the fact that A_{23} is not likely to be negligible. The coefficient may be defined by

$$A_{23} \equiv \left. \frac{\partial \zeta^{(1)}}{\partial p_f^{(2)}} \right|_{\delta e = \delta p_f^{(1)} = 0} . \qquad (77)$$

The fractures will expand within the constraint of zero total strain because an increase in the fluid pressure in the fractures will cause compression of the matrix. Therefore, fluid must be withdrawn from the matrix in order to maintain $\delta p_f^{(1)} = 0$. Thus, the constant A_{23} is expected to be negative (for example, see values computed in Tables 2 and 4), while in fact the identity $A_{23} = -a_{12}a_{13}/a_{11} < 0$ follows from the assumption that $a_{23} = 0$. See Figures 4 and 5 for a comparison of these two coefficients.

The diagonal storage coefficient in the strain formulation is given by $A_{33} = a_{33} - a_{13}^2/a_{11}$. As in the discussion of A_{23}, this coefficient is for the case of constant *total* volume, not constant *fracture* volume. The fractures are able to expand in response to an increase in $p_f^{(2)}$ because the matrix contracts. Therefore, the fracture storage is not so small, as it would be for a rigid fracture. Elsworth and Bai [1992] calculated an unreasonably small value of $B_{EB}^{(1)} = 2.3\times10^{-4}$ for Berea sandstone, because they used the rigid fracture value for A_{33}.

The strain formulation is as valid a formulation as the stress formulation we have used. However, the *ad hoc* assumption that the cross storage coefficient $A_{23} = 0$ is not justified; significant coupling occurs between the fracture and matrix for conditions of constant total strain. The assumption that $A_{23} = 0$ or $A_{33} \simeq 0$ leads to significant underestimation of the early pressure buildup in the matrix at short times. On the other hand, the assumption that the cross-storage coefficient $a_{23} = 0$ is justified on the grounds that the overall solid grain modulus is likely to be close to that of the matrix grains (also compare Table 4). Furthermore, it could be argued that small values of this cross-coupling coefficient a_{23} are also needed to help justify the use of a double-porosity model. If the cross-coupling a_{23} were found to be about as large as the other storage coefficients, such a result would be a strong indication that a single porosity analysis was more appropriate for the data.

7.4 Temporal and spatial scales

The local mechanical and fluid pressure response of a double porosity material is time and scale dependent, because fluid may be exchanged between the two phases and because the two fluid pressures can be maintained independently (at least in a *gedanken* experiment). As demonstrated in our preceding detailed analysis and that of Wilson and Aifantis [1982], consideration of short, intermediate, and long time scales leads to theoretical relationships between measurements and phenomenological coefficients. The coefficients are "phenomenological" because they are determined from data indirectly by the rules specified here, and they are not determined at present by a microscopic theory. By making separate analyses at the mesoscopic scale of a typical core sample and of a typical fracture, we have shown that all six constants for volumetric deformations are determined from the matrix poroelastic coefficients and the fracture compressibility based on the single assumption that $a_{23} = 0$. Then all the constants defined for different time scales can be determined. At the long-time scale, the double-porosity medium behaves as an equivalent single porosity medium with a single macroscopic compressibility, Skempton's coefficient, and storage coefficient. In general, a_{23} does not vanish identically, but is likely to be significantly smaller in magnitude than the other matrix elements.

8. Conclusions

- The six volumetric deformation coefficients in the double porosity theory can be divided into three categories corresponding to the three volumetric deformation coefficients in the single porosity theory. The effective medium bulk modulus replaces the ordinary bulk modulus of a single porosity medium. Two poroelastic expansion coefficients, one for the pore pressure in each phase, replace the single poroelastic expansion coefficient in a single porosity medium. A symmetric two-by-two poroelastic storage tensor consisting of three coefficients, two diagonal and one off-diagonal, replaces the single storage coefficient in a single porosity medium.

- Under the circumstances studied in this paper, the magnitude of the off-diagonal storage coefficient a_{23} in a stress-based formulation can be assumed to be quite small (or zero in some cases) for a fractured, porous medium. (The assumption that the strain-based cross-coefficient $A_{23} = 0$ is not justified.) Since a_{23} is a measure of the cross-coupling between the matrix and the fracture fluids, small values of this coefficient are indicative that the division of the fluid into two well distinguished parts is actually justified. If this coupling term were large (or at least of the same order of magnitude as the other storage coefficients), it could be argued that the two fluids act in concert and, therefore, should not be distinguished in the manner we have done here. Vanishing of this coefficient is not required, but we fully expect small values to be the rule in applications of interest, *i.e.*, reservoir pumping analysis and wave propagation.

- Consideration of very short, intermediate, and long time scales yields definitions of a number of poroelastic moduli, many of which are physically realizable in laboratory or field experiments, and the interrelationships among these poroelastic moduli have been specified.

Finally, an important direction for future work is to deal with those situations where it might be either difficult or impossible to make the required measurements of the parameters (but predictive capability is vital, or at least highly desirable). Then, we will want to introduce a new microscopic point of view in order to relate the phenomenological coefficients to quantities such as grain modulus and porosity at the microscale. The standard model of the microscale (and the one used in the examples — but not in the theory — in the present paper) is that used by Gassmann [1951], which carries the restrictive assumption that the solid frame is composed of only one type of elastic constituent. A more general point of view has been introduced recently by Berryman and Milton [1991, 1992], Norris [1992], and Berryman [1992], who show how to relate macroscopic coefficients in poroelasticity to quantities at the microscale when two types of solid constituents are present. Making use of this approach will permit us to make the microscopic identification of coefficients in more complex and therefore more realistic geologic media.

It should also be noted that the present work has important implications for wave propagation through double porosity media, as well as for the analysis of reservoir pumping scenarios that has been emphasized here. Many of the coefficients required for analysis of wave propagation are the same as those in the present study, the main difference being that inertial effects have played no role in the preceding analysis. Inertial effects will clearly play a more prominent role in subsequent wave studies.

Appendix. Poroelasticity Notation

Kümpel [1991] has given a detailed review and comparison of various notations used in the literature on poroelasticity. We do not need to reproduce this extensive comparison here. But, to aid the reader who may be more familiar with some other choice of notation, we include Table A, giving comparisons to some of the more commonly used symbols. Green and Wang [1986] provide comparisons to earlier work of Biot [1941]. Also, see Wang [1993].

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List of Symbols		
k	=	overall fluid permeability
$k^{(1)}, k^{(2)}$	=	matrix and fracture permeabilities
p_c	=	confining pressure
p_d	=	$p_c - p_f$, the differential pressure
p_f	=	fluid pressure
$p_f^{(1)}, p_f^{(2)}$	=	matrix and fracture fluid pressures
$p_f^{(d)}, p_f^{(u)}$	=	drained and undrained fluid pressures
$v^{(1)}, v^{(2)}$	=	volume fractions occupied by matrix and fractures with $v^{(1)} + v^{(2)} = 1$
B	=	Skempton's coefficient
G	=	shear modulus
K	=	bulk modulus of drained porous frame (jacketed)
K_f	=	fluid bulk modulus
K_m	=	material (or grain) bulk modulus
K_p	=	$\phi K/\alpha$, an effective pore bulk modulus (jacketed)
K_s	=	an effective solid bulk modulus (unjacketed)
K_u	=	bulk modulus of undrained (confined) porous frame
K_ϕ	=	an effective pore bulk modulus (unjacketed)
S	=	α/BK, the storage coefficient
V	=	total volume
$V^{(1)}, V^{(2)}$	=	total matrix and fracture volumes
V_s	=	$(1-\phi)V$, the solid volume
V_ϕ	=	ϕV, the pore volume
α	=	Biot-Willis parameter
β	=	volume thermal expansion coefficient
γ	=	pressure difference coefficient
ζ	=	increment of total fluid content
$\zeta^{(1)}, \zeta^{(2)}$	=	increments of matrix and fracture fluid content
θ	=	temperature
μ	=	fluid viscosity
ν	=	Poisson's ratio
ϕ	=	total porosity $= v^{(1)}\phi^{(1)} + v^{(2)}$
$\phi^{(1)}$	=	matrix porosity (fracture porosity is unity)

TABLE 1. Material Properties

Parameter	Berea Sandstone	Westerly Granite
$K^{(1)}$ (GPa)	8.0^a	25.0^a
$\nu^{(1)}$	0.20^a	0.25^a
$K_s^{(1)}$ (GPa)	36.0^a	45.4^a
$\alpha^{(1)}$	0.78	0.45
K_f (GPa)	3.3^a	3.3^a
$\phi^{(1)}$	0.064^b	0.00106^b
$B^{(1)}$	0.847	0.984
$S^{(1)}$ (GPa^{-1})	0.115	0.0183
$v^{(2)}$	0.0064^c	0.0106^c
$K^{(2)}$(GPa)	0.00775^c	0.0876^c

[a]From Rice and Cleary [1976]

[b]From Touloukian *et al.* [1989]

[c]From Elsworth and Bai [1992]

Table 2. Double porosity parameters computed from material properties in Table 1.

Parameter	Formula	Berea Sandstone	Westerly Granite
K (GPa)	$[v^{(1)}/K^{(1)} + v^{(2)}/K^{(2)}]^{-1}$	1.0520	6.25
K_s (GPa)	$K_s^{(1)}/v^{(1)}$	36.2	45.9
α	$1 - K/K_s$	0.971	0.864
a_{11} (GPa^{-1})	$1/K$	0.951	0.1601
a_{12} (GPa^{-1})	$-v^{(1)}\alpha^{(1)}/K^{(1)}$	-0.0969	-0.0178
a_{13} (GPa^{-1})	$-v^{(2)}/K^{(2)}$	-0.826	-0.121
a_{22} (GPa^{-1})	$v^{(1)}\alpha^{(1)}/B^{(1)}K^{(1)}$	0.1144	0.01817
a_{23} (GPa^{-1})	0.0	0.0	0.0
a_{33} (GPa^{-1})	$v^{(2)}(1/K_f + 1/K^{(2)})$	0.828	0.124
A_{23} (GPa^{-1})	$a_{23} - a_{12}a_{13}/a_{11}$	-0.0842	-0.0135
$\alpha^{(2)}$	1.0	1.0	1.0
B	$-(a_{12} + a_{13})/(a_{22} + a_{33})$	0.979	0.977
$B^{(1)}, B[u^{(1)}], B_{EB}^{(1)}$	$-a_{12}/a_{22}$	0.847	0.984
$B^{(2)}, B[u^{(2)}], B_{EB}^{(2)}$	$-a_{13}/a_{33}$	0.998	0.974
K_u (GPa)	$[a_{11} - (a_{12} + a_{13})^2/(a_{22} + a_{33})]^{-1}$	21.5	39.6
$K[u^{(1)}]$ (GPa)	$[a_{11} - a_{12}^2/a_{22}]^{-1}$	1.151	7.01
$K[u^{(2)}]$ (GPa)	$[a_{11} - a_{13}^2/a_{33}]^{-1}$	7.92	23.4
S (GPa^{-1})	α/BK	0.943	0.1415
$S^{(2)}$ (GPa^{-1})	$\alpha^{(2)}/B^{(2)}K^{(2)}$	129.5	11.67

TABLE 3. Material Properties

Parameter	Chelmsford Granite	Weber Sandstone
K (GPa)	8.0[a]	4.0[a]
K_s (GPa)	54.5[a]	37.0[a]
α	0.85[a]	0.89[a]
$K^{(1)}$ (GPa)	17.0[a]	10.0[a]
$\nu^{(1)}$	0.25	0.15
$K_s^{(1)}$ (GPa)	55.5[a]	38.0[a]
$\alpha^{(1)}$	0.69[a]	0.74[a]
K_f (GPa)	3.3	3.3
$\phi^{(1)}$	0.0011	0.095[a]
$B^{(1)}$	0.992	0.355
$S^{(1)}$ (GPa^{-1})	0.0409	0.208
$v^{(2)}$	0.011[a]	0.0095

[a]From Coyner [1984]

TABLE 4. Double porosity parameters computed from material properties in Table 3.

Parameter	Formula	Chelmsford Granite	Weber Sandstone
a_{11} (GPa^{-1})	$1/K$	0.125	0.250
a_{12} (GPa^{-1})	$-\alpha^{(1)}K_s^{(1)}/K^{(1)}K_s$	-0.0413	-0.076
a_{13} (GPa^{-1})	$-\alpha/K - a_{12}$	-0.0649	-0.147
a_{22} (GPa^{-1})	$v^{(1)}\alpha^{(1)}/B^{(1)}K^{(1)}$	0.0405	0.206
a_{23} (GPa^{-1})	$-v^{(1)}\alpha^{(1)}/K^{(1)} - a_{12}$	0.00119	0.00270
a_{33} (GPa^{-1})	$v^{(2)}/K_f + v^{(1)}/K^{(1)} - (1-2\alpha)/K + 2a_{12}$	0.0664	0.145
$\overline{a}_{33}$ (GPa^{-1})	$a_{33} - v^{(2)}/K_f$	0.0630	0.142
A_{23} (GPa^{-1})	$a_{23} - a_{12}a_{13}/a_{11}$	-0.0203	-0.042
$\alpha^{(2)}$	$(\overline{a}_{33}a_{12} - a_{13}a_{23})/(a_{11}a_{23} - a_{13}a_{12})$	0.997	0.994
B	$-(a_{12}+a_{13})/(a_{22}+2a_{23}+a_{33})$	0.973	0.624
$B^{(1)}$	$-(a_{12}+a_{23})/a_{22}$	0.992	0.355
$B[u^{(1)}]$	$-a_{12}/a_{22}$	1.022	0.368
$B_{EB}^{(1)}$	$(a_{23}a_{13} - a_{12}a_{33})/(a_{22}a_{33} - a_{23}^2)$	0.993	0.355
$B^{(2)}$	$(\overline{a}_{33}a_{12} - a_{13}a_{23})/[a_{33}(a_{12}+a_{23}) - a_{23}(a_{13} - \overline{a}_{33})]$	0.950	0.980
$B[u^{(2)}]$	$-a_{13}/a_{33}$	0.978	1.011
$B_{EB}^{(2)}$	$(a_{23}a_{12} - a_{13}a_{22})/(a_{22}a_{33} - a_{23}^2)$	0.961	1.004
K_u (GPa)	$[a_{11} - (a_{12}+a_{13})^2/(a_{22}+2a_{23}+a_{33})]^{-1}$	46.3	8.99
$K^{(2)}$ (GPa)	$v^{(2)}(a_{12}+a_{13})/(a_{11}a_{23} - a_{13}a_{12})$	0.179	0.0666
S (GPa^{-1})	α/BK	0.1092	0.357
$S^{(2)}$ (GPa^{-1})	$\alpha^{(2)}/B^{(2)}K^{(2)}$	5.87	15.24

Table A. Comparison of the notation used in this paper with that of several other authors.

	Notation				
Quantity Defined	This Work	Brown and Korringa [1975]	Green and Wang [1986]	Rice and Cleary [1976]	Zimmerman *et al.* [1986]
Porosity	ϕ	ϕ	ϕ	v_0	ϕ
$-\frac{1}{V}\left(\frac{\partial V}{\partial p_d}\right)_{p_f}$	$1/K$	κ_A	β	$1/K$	C_{bc}
$-\frac{1}{V}\left(\frac{\partial V}{\partial p_f}\right)_{p_d}$	$1/K_s$	κ_M	β_s	$1/K_s^{'}$	$C_{bc} - C_{bp}$
$-\frac{1}{V_\phi}\left(\frac{\partial V_\phi}{\partial p_d}\right)_{p_f}$	$1/K_p$	$\kappa^{'}/\phi$	$(\beta - \beta_s)/\phi$	$1/v_0 H$	C_{pc}
$-\frac{1}{V_\phi}\left(\frac{\partial V_\phi}{\partial p_f}\right)_{p_d}$	$1/K_\phi$	κ_ϕ	β_ϕ	$1/K_s^{''}$	$C_{pc} - C_{pp}$

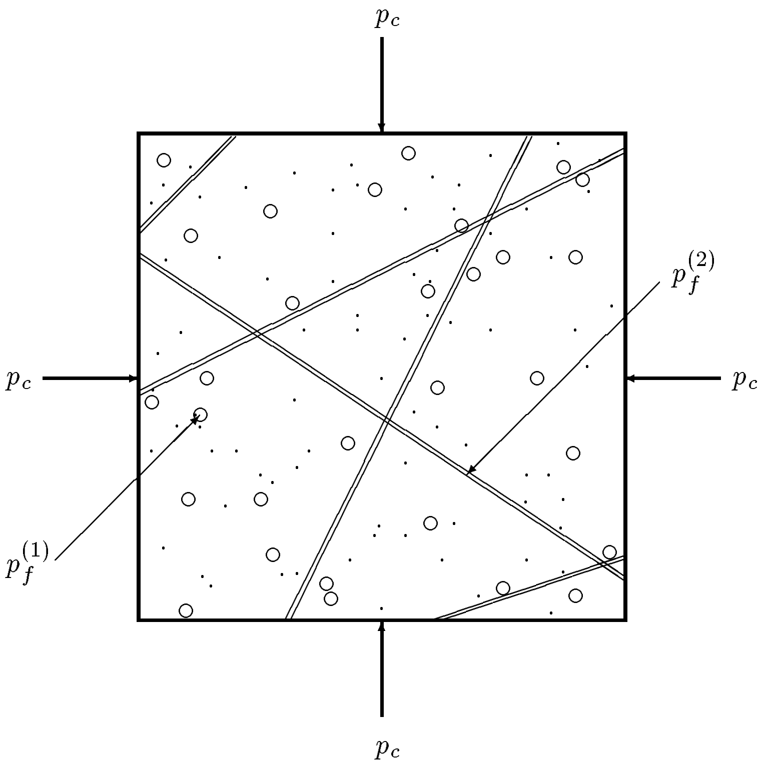

Figure 1: The elements of a double porosity model are: porous rock matrix intersected by fractures. Three types of macroscopic pressure are pertinent in such a model: external confining pressure p_c, internal pressure of the matrix pore fluid $p_f^{(1)}$, and internal pressure of the fracture pore fluid $p_f^{(2)}$.

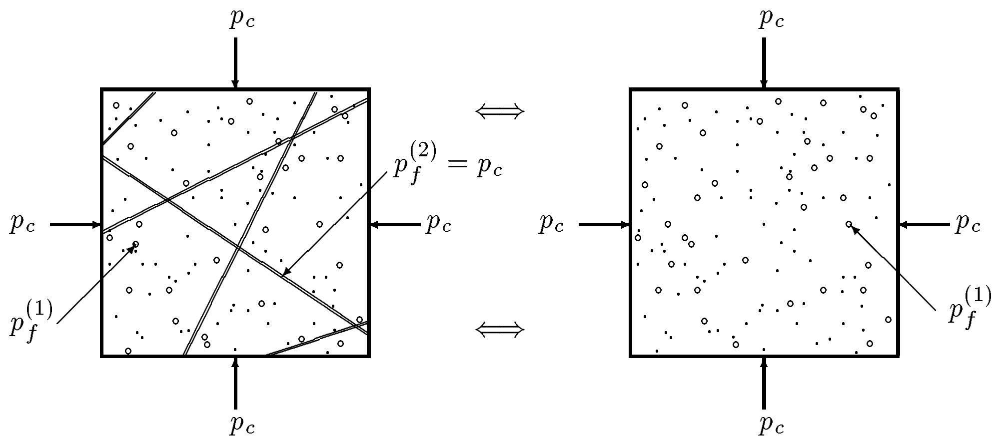

Figure 2: Setting $p_f^{(2)} = p_c$ establishes a constant pressure around the matrix material which is then equivalent to the situation experienced by a core sample of matrix material in a laboratory test.

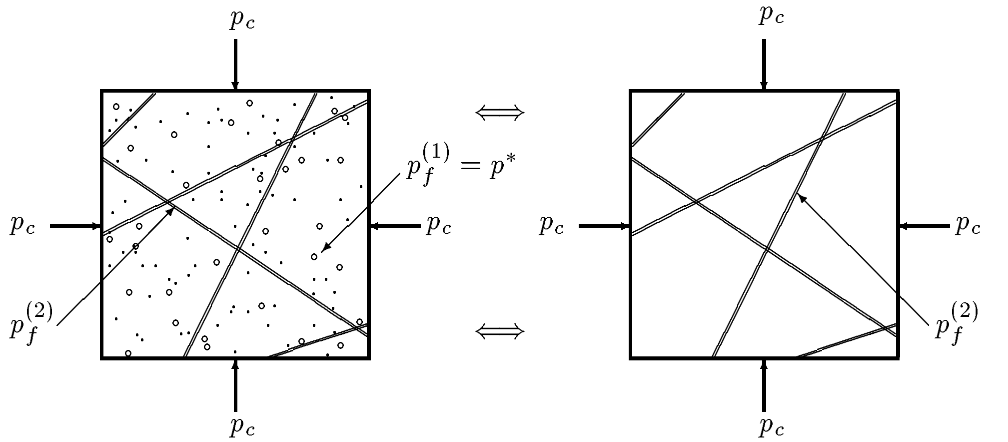

Figure 3: For any given value of confining pressure p_c and fracture fluid pressure $p_f^{(2)}$, there is a special choice of the matrix fluid pressure $p_f^{(1)} = p^*$ that guarantees no macroscopic strain ($\delta e^{(1)} = 0$) in the matrix. This situation allows the behavior of the fracture to be isolated from that of the matrix essentially as if the matrix material were macroscopically rigid.

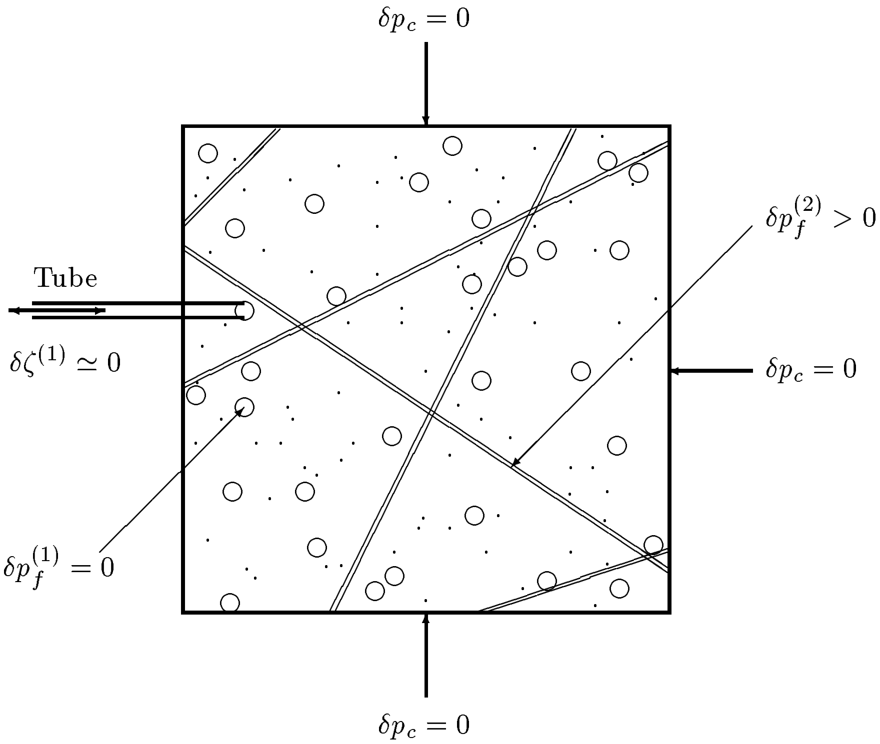

Figure 4: The coefficient a_{23} determines the change in matrix fluid storage when confining pressure is constant, matrix pore pressure is constant, and the fracture fluid pressure changes. The analysis shows that this coefficient is very small, but can be either positive or negative. A correspondingly small amount of fluid flows either into or out of the matrix with an increase in fracture pressure, accompanied by an increase in the total solid volume.

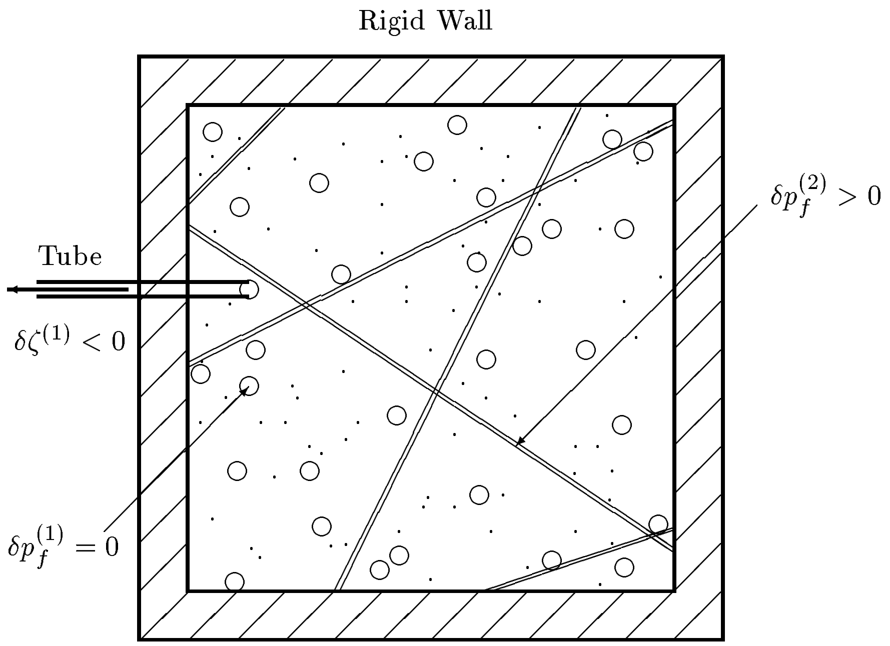

Figure 5: The coefficient A_{23} determines the change in matrix fluid storage when total strain is constant, matrix pore pressure is constant, and the fracture fluid pressure changes. The analysis shows that this coefficient is negative, but not small, so that fluid flows out of the matrix with an increase in fracture pressure.