Inverting Ultrasonic Data on Solid/Fluid Mixtures for Biot-Gassmann Parameters

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Abstract

Although the Biot-Gassmann equations are known to predict acoustic data for water-saturated glass bead packings very well, these equations often seem to give anomalous results for naturally occurring rocks. The theory shows that, as the compressibility of the pore fluid increases, the inverse of the coefficients C and M in Biot's equations should increase linearly, and the theory also gives definite predictions about the values of the slope and intercept of the resulting lines. Although the anticipated linear behavior is observed in real data, the values of the slopes are smaller than expected for some rocks. Therefore, the process of inverting this data for the desired coefficients must incorporate other physical constraints and then produces, not a single value but rather, a range of possible values for each coefficient.

1 Introduction

All soils and most rocks are porous. The pores are often filled with air or water, but in situations of most economical or environmental interest they are filled with other fluids such as hydrocarbons or fluid-born chemical contaminants. Sound waves readily propagate through such solid/fluid mixtures and are therefore commonly used by geophysicists to locate economically desirable fluids or to attempt to map the underground migration of environmentally undesirable fluids. Images of underground velocity distributions may be obtained either by surface seismic reflection surveying or by cross-borehole transmission tomography. These sound velocity maps can often be obtained with little concern for their ultimate interpretation. But accurate interpretation of fluid content requires careful analysis of the dependence of wave velocity and attenuation on the pore-fluid distribution.

In the context of elasticity theory, measurements of shear and compressional velocities are commonly used to invert for shear and bulk moduli. However, at frequencies typical of seismic exploration, sound waves propagating through solid/fluid mixtures are governed by Biot's equations of poroelasticity [8]. Since the fluid constituents are not just incidental contaminants but rather the feature of primary interest in these problems, it is essential for the success of this effort that both the forward and inverse problems of poroelasticity be understood to a much higher degree than has been previously possible. Since fluid-filled porous media contain multiple constituents, poroelasticity theory

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contains more parameters than elasticity theory and therefore inversion for constituent moduli requires more measurements. Unrealistic simplifications must be avoided or those physical effects resulting from the presence of the fluid and from inhomogeneities in the solid may be misinterpreted — resulting in lost opportunities to discover resources or incorrect assessments of hazards.

The Biot-Gassmann theory of poroelasticity has been highly successful in explaining ultrasonic data on fluid-saturated porous packings of sintered glass beads [22,1,19]. However, it is also known that systematic discrepancies exist for ultrasonic data on fluid-saturated rocks [27,20]. The cause of these discrepancies is still unknown, but two of the more likely sources are: (1) inhomogeneities in the rocks and (2) high-frequency dispersion. Gassmann's equation [14] — one of the key results in the theory — is strictly valid only for homogeneous porous media and for static deformations; thus, improvements in the analysis of ultrasonic data may arise from a careful reexamination of the limitations, implications, and possibility of extensions of Gassmann's result.

Brown and Korringa [13] have shown that Gassmann's equation for the saturated bulk modulus can be generalized for an inhomogeneous porous medium by introducing two more bulk moduli — one for the solid deformation (K_s) and one for the pore deformation (K_ϕ). For the forward problem, Berryman and Milton [5] have shown that these constants can be determined exactly from bulk moduli of the constituents for a porous frame composed of just two minerals in welded contact. Berryman and Milton [6] have shown further that exact results for these constants can also be obtained for two minerals together with cracks if the thermal expansion coefficients of the constituents and the porous frame are also known.

In Section 2, we present the Biot-Gassmann equations of poroelasticity, including a discussion of differences between Gassmann's specialized result for the coefficients and the general results of Brown and Korringa. In Section 3, we discuss the inverse problem for the Biot-Gassmann parameters.

2 Biot-Gassmann Equations

Consider a porous medium whose connected pore space is saturated with a single-phase viscous fluid. The fraction of total volume occupied by fluid is the porosity ϕ, which is assumed to be uniform on some appropriate length scale. Bulk modulus and density of the fluid are K_f and ρ_f, respectively. Bulk and shear moduli of the drained porous frame are K and μ. For simplicity, assume the frame is composed of a single granular constituent whose bulk and shear moduli and density are K_m, μ_m, and ρ_m. Frame moduli may be measured on drained samples, or may be estimated using one of a variety of methods from the theory of composites [2-4].

For long wavelength acoustic pulses ($\lambda >> h$ where h is a typical pore size) propagating through such a porous medium, we define average values of local displacements in the solid and also in the saturating fluid. The average displacement vector in the solid frame is $\mathbf{u}$, while that in the pore fluid is $\mathbf{u}_f$. A more useful way of quantifying fluid displacement is to introduce average displacement of fluid relative to frame which is $\mathbf{w} = \phi(\mathbf{u}_f - \mathbf{u})$. For small strains, frame dilatation is

$$e = \nabla \cdot \mathbf{u}. \tag{1}$$

Similarly, average fluid dilatation is

$$e_f = \nabla \cdot \mathbf{u}_f, \tag{2}$$

which includes fluid flow terms as well as dilatation. The increment of fluid content is defined by

$$\zeta = -\nabla \cdot \mathbf{w} = \phi(e - e_f). \tag{3}$$

With these definitions, Biot [8-10] introduces a quadratic strain-energy functional of the independent variables e and ζ for an isotropic, linear porous medium

$$2E = He^2 - 2Ce\zeta + M\zeta^2 - 4\mu I_2, \tag{4}$$

where, if $e_{xy} = \frac{1}{2}(u_{x,y} + u_{y,x}), \ldots$, then $I_2 = e_{xx}e_{yy} + e_{yy}e_{zz} + e_{zz}e_{xx} - e_{xy}^2 - e_{yz}^2 - e_{zx}^2$ is the second strain invariant [7]. Elementary bounds on coefficients in the equations of poroelasticity are presented by Thigpen and Berryman [26]. Thermodynamic and mechanical stability require non-negativity of E, which implies that $H \geq 0$, $M \geq 0$, $HM - C^2 \geq 0$, and $\mu \geq 0$. Then, components of the average stress tensor τ for the saturated porous medium are

$$\tau_{ij} = [(H - 2\mu)e - C\zeta]\delta_{ij} + 2\mu e_{ij}, \tag{5}$$

and fluid pressure p_f is

$$p_f = M\zeta - Ce. \tag{6}$$

Two coupled equations of motion for small disturbances in fluid-saturated media may be derived easily from the energy functional E with these definitions of stresses and pressures. With time dependence of the form $\exp(-i\omega t)$, Biot's equations of poroelasticity are, using the later notation of Biot [10],

$$\mu\nabla^2\mathbf{u} + (H - \mu)\nabla e - C\nabla\zeta + \omega^2(\rho\mathbf{u} + \rho_f\mathbf{w}) = 0, \tag{7}$$

$$C\nabla e - M\nabla\zeta + \omega^2(\rho_f\mathbf{u} + q\mathbf{w}) = 0, \tag{8}$$

where

$$\rho = \phi\rho_f + (1 - \phi)\rho_m \tag{9}$$

and

$$q(\omega) = \rho_f[\tau/\phi + iF(\xi)\eta/\kappa\omega]. \tag{10}$$

Tortuosity $\tau \geq 1$ is a pure number related to frame inertia which has been measured [12,19] and can also be estimated theoretically [1]. Kinematic viscosity of the saturating fluid is η; permeability of the porous frame is κ; $F(\xi)$ is a dynamic viscosity factor proposed by Biot [9], where $\xi = (\omega h^2/\eta)^{\frac{1}{2}}$. The dynamic parameter h is a characteristic length generally associated with steady-flow hydraulic radius of the pores, or with a typical pore size.

The coupled equations (7) and (8) give rise to three distinct modes of wave propagation: two compressional waves (with speeds v_+ and v_- for the faster and slower of the two wave speeds, respectively) and a single shear wave (with speed v_s having two polarizations).

Coefficients appearing in Biot's equations of poroelasticity must be known before quantitative predictions can be made. Results of Brown and Korringa [13] may be used to show that these coefficients are given for general isotropic porous media by

$$H = K_u + \frac{4}{3}\mu, \tag{11}$$

where (using the definitions that follow) the undrained modulus K_u is determined by

$$\frac{K_s K_u}{K_s - K_u} = \frac{K_s K_d}{K_s - K_d} + \frac{K_\phi K_f}{\phi(K_\phi - K_f)}, \tag{12}$$

while the coefficients C and M are given by

$$C = (K_u - K_d)/\alpha \quad \text{and} \quad M = C/\alpha, \tag{13}$$

with

$$\alpha = 1 - K_d/K_s. \tag{14}$$

The three bulk moduli characteristic of the drained porous frame are defined by Brown and Korringa through the expressions:

$$\frac{1}{K_d} = -\frac{1}{V}\Big(\frac{\partial V}{\partial p_d}\Big)_{p_f}, \tag{15}$$

$$\frac{1}{K_s} = -\frac{1}{V}\Big(\frac{\partial V}{\partial p_f}\Big)_{p_d}, \tag{16}$$

and

$$\frac{1}{K_\phi} = -\frac{1}{V_\phi}\Big(\frac{\partial V_\phi}{\partial p_f}\Big)_{p_d}, \tag{17}$$

where V is total sample volume, $V_\phi = \phi V$ is pore volume, $p_c = -\frac{1}{3}Tr(\tau) = -\frac{1}{3}(\tau_{xx} + \tau_{yy} + \tau_{zz})$ is external (confining) pressure, p_f is pore pressure, and $p_d = p_c - p_f$ is differential pressure. Brown and Korringa [13] state that, although these three bulk moduli have simple physical interpretations, this "does not necessarily help in knowing their values." Observing the change in pore volume (an internal variable) required by (17) is clearly more difficult than observing the change in total sample volume (an external variable) required by the other two moduli. Nevertheless, all three moduli may, in principle, be measured in quasi-static measurements, thus determining their values directly. However, it is commonly the case that acoustic measurements are made on compressional and shear wave speeds in fluid saturated porous media. It would therefore be beneficial to have a means of deducing these constants from wave data.

Constant K_d is the (jacketed) bulk modulus of the drained porous frame, and is commonly measured since it is precisely what one would think of as the bulk modulus of the dry (or drained) rock in nonporous elasticity. However, values of the two remaining constants K_s (the unjacketed modulus) and K_ϕ (the unjacketed pore modulus) are not generally known or measured unless the porous frame is homogeneous on the microscopic scale. For this special circumstance (which is also the only one considered explicitly by Gassmann [14]) with a single type of elastic solid composing the frame, these two moduli both equal the bulk modulus K_m of the single granular constituent

$$K_s = K_\phi = K_m. \tag{18}$$

Thus, Gassmann's equation is equivalent to

$$\frac{1}{M} = \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_m}, \quad \alpha = 1 - K_d/K_m, \tag{19}$$

while Brown and Korringa's more general result (12) may be rearranged to show that

$$\frac{1}{M} = \frac{\phi}{K_f} + \frac{\alpha}{K_s} - \frac{\phi}{K_\phi}, \quad \alpha = 1 - K_d/K_s. \tag{20}$$

Note the important fact (to be used later) that the theory clearly shows $1/M$ is always a linear function of the fluid compressibility $\beta_f = 1/K_f$. Gassmann's result (19) has also been derived within the context of Biot's theory of poroelasticity by Biot and Willis [11] and by Geertsma [15]. Geertsma and Smit [16] discuss practical aspects of applications of the theory to rocks. Rice [24] and Rice and Cleary [25] also obtain a general result essentially equivalent to that of Brown and Korringa (20).

The more general constants of Brown and Korringa, K_s and K_ϕ, must somehow be related to material properties of multiple solid constituents of the porous frame. Equation (20) provides a possible method of determining the constants as we will show in Section 3: by saturating the same porous medium with different pore fluids, it should therefore be possible to deduce the values of the frame moduli from the variations in M with fluid compressibility.

Before attacking the inversion problem, we first show an example illustrating the success of the theory in predicting the results of certain wave propagation experiments.

Example: Predictions for fluid-saturated porous glass

Plona [22] observed two distinct compressional waves in a water-saturated, porous structure made from sintered glass beads. The speeds predicted by Biot's equations of poroelasticity are compared to the values observed by Plona shown in Figure 1.

The input parameters to the model are $K_s = 40.7$ GPa, $\mu_s = 29.7$ GPa, $\rho_s = 2.48$ g/cc, $K_f = 2.2$ GPa, $\rho_f = 1.00$ g/cc, $\nu = 1.00$ centistoke, and $\omega = 2\pi \times 500$ kHz. The drained frame moduli K_d and μ were calculated assuming spherically shaped glass particles and needle-shaped inclusions of voids, following Berryman [3]. We use $\tau = \phi^{-\frac{1}{2}}$ for the tortuosity. The permeability variation with porosity was taken to obey the Kozeny-Carman relation

$$\kappa = const \times \phi^3/(1-\phi)^2, \tag{21}$$

which has been shown empirically to provide a reasonable estimate of the porosity variation of permeability. We choose $\kappa_0 = 9.1 \times 10^{-8}$ cm^2 ($\simeq 9.1$ D) at $\phi_0 = 0.283$ and

Figure 1: Ultrasonic velocities (slow compressional – v_-, shear – v_s, fast compressional – v_+) in water-saturated porous glass. Theoretical curves from Biot's theory as described in the text. Data from Plona [22], Johnson and Plona [18], and Plona and Johnson [23].

then use (21) to compute the value of κ for all other porosities considered. No entirely satisfactory model for the characteristic length h has been found. However, dimensional analysis suggests that h^2 must be comparable to κ, so we have taken

$$h^2/\kappa = h_0^2/\kappa_0 = const. \tag{22}$$

At $\phi_0 = 0.283$, we choose $h_0 = 0.02$ mm corresponding to an average pore radius $\frac{1}{5}$ to $\frac{1}{7}$ of the grain radius (the glass beads in Plona's samples were 0.21–0.29 mm in diameter before sintering).

The theoretical results for the fast compressional wave and the shear wave agree with Plona's measurements within the experimental error ($\pm 3\%$ relative error in measured speeds and and an absolute error of ± 0.005 in measured porosity), showing that the Biot-Gassmann equations agree very well with data for these simple synthetic rocks.

3 Inverting ultrasonic data

At low frequencies, the fast compressional and shear wave velocities in poroelasticity are given by

$$v_+^2 = H/\rho \quad \text{and} \quad v_s^2 = \mu/\rho. \tag{23}$$

We will assume that the dispersion (dependence of velocities on frequency) is negligible over the range of frequencies considered. Then, the adiabatic moduli can be determined from measured sound velocities according to the standard formulas (from linear elasticity)

$$\mu = \rho v_s^2 \quad \text{and} \quad K_u = \rho(v_+^2 - \tfrac{4}{3}v_s^2). \tag{24}$$

When acoustic measurements are made on the porous medium while drained of fluid, we can also find the drained modulus K_d from the same formulas. The theory says that the shear modulus should be unaffected by the presence or absence of the fluid. Thus, changes in shear velocity should be due to the increase in density ρ when a fluid fills the pores, but experience has shown [17,21] this is not always true for low porosity rocks with microcracks.

Dividing (20) through by α^2, we find

$$\frac{1}{K_u - K_d} = a + b\beta_f, \tag{25}$$

where the fluid compressibility is $\beta_f = 1/K_f$ and the intercept a and slope b are given, respectively, by

$$a = (\alpha/K_s - \phi/K_\phi)/\alpha^2 \tag{26}$$

and

$$b = \frac{\phi}{\alpha^2}. \tag{27}$$

The inversion problem then reduces to fitting modulus data to a straight line according to (25), and then computing K_s and K_ϕ from the slope and intercept formulas (27) and (26).

TABLE 1. Measured values of adiabatic bulk moduli obtained from the ultrasonic data of Wang [29].

Saturant		*Sandstone*	
Pore Fluid	*Compressibility* β_f (1/GPa)	*Beaver* K_u (GPa)	*Boise* K_u (GPa)
water	0.4444	30.72	14.62
$C_{18}H_{36}$	0.6766	30.04	13.96
$C_{14}H_{28}$	0.7429	29.87	13.75
$C_{10}H_{20}$	0.8681	29.09	13.52
		K_d (GPa)	K_d (GPa)
air	–	22.12	9.94

Fluid-saturated sandstones

Wang [29] measured the wave velocities of Boise and Beaver sandstones when saturated with four different hydrocarbons, as well as when saturated with water and when air dry. The four hydrocarbons used were n-heptane (C_7H_{16}), 1-decene ($C_{10}H_{20}$), 1-tetradecene ($C_{14}H_{28}$), and 1-octadecene ($C_{18}H_{36}$). n-Heptane is an alkane, while the other three hydrocarbons are alkenes. The results of these ultrasonic measurements (made with wideband transducers having a center frequency of 2.25 MHz) are summarized in Table 1. We excluded one of the hydrocarbons (the alkane) from this analysis because it had virtually the same compressibility as that of 1-octadecene.

The Boise sandstone has high porosity ($\phi = 0.27$) and high permeability ($\kappa = 910$mD), while the Beaver sandstone has lower porosity ($\phi = 0.07$) and also much lower permeability (not measured for this sample, but other samples of Beaver sandstone have $\kappa \simeq 6$mD).

Using the data from Table 1, we plot $1/(K_u - K_d)$ versus β_f in Figure 2 to check whether the data fall on a straight line as predicted by Eq. (25). It is encouraging to see that the data do indeed seem to fall on a straight line. However, when we take a closer look and consider the values of the slopes of these lines, we find that the slope for Boise sandstone is $b \simeq 0.155$, while that for Beaver is $b \simeq 0.064$. Since Eq. (27) shows that $b = \phi/\alpha^2$ and it is well-known that $\alpha \leq 1$, then recalling that the porosities of these samples were measured to be $\phi \simeq 0.27$ and $\phi \simeq 0.07$ respectively, we find the paradoxical result that the data fall on a straight line, *but it is the wrong straight line!* The theory shows that we should expect these slopes to be somewhat larger than the measured porosities, but instead they are either substantially smaller or somewhat smaller in these two examples. Possible resolutions of this paradox will be the subject of the following discussion.

Discussion

There are various ways of trying to resolve the apparent paradox found in our attempt to invert ultrasonic data to find Biot-Gassmann parameters. Space limitations prevent us from elaborating on these possibilities at length in the present paper, but we can give a brief discussion of the main points. The three most likely sources of the apparent discrepancies are these: (1) The assumption of negligible velocity dispersion is invalid. (2) The "effective porosity" at high frequencies is smaller than the measured porosity. (3) The appropriate value of the drained modulus K_d is not that of the air-dry porous material, but another value that must also be determined from the data.

Since the theory does very well at predicting velocities in glass bead samples at ultrasonic frequencies (see Figure 1), it seems unlikely that any small effect of velocity dispersion we have neglected is likely to explain the observed large anomaly.

For analysis of fluid flow through porous media, it is known that the effective porosity is less than true porosity. When a porous sample has been drained of water, it is possible to measure the *irreducible water saturation* S_{wi} [28]. Then, the effective porosity is given by $\phi_{eff} = \phi(1 - S_{wi})$. However, the change in effective porosity obtained has generally been shown [28] to be substantially less than what would be needed to explain the anomalous results obtained by our inversion method. The irreducible water saturation S_{wi} thus provides a measure of the *static* effective porosity. The *dynamic* effective porosity we seek presumably depends on the magnitude of the fluid permeability and is also presumably less than the static effective porosity, but it is difficult to estimate at present how much

Figure 2: Showing the linear fit as indicated in Eq. (28) to data on Beaver and Boise sandstones saturated with various fluids.

less. So, while we believe that it must be the case that the effective porosity is less than that measured, we think it is unlikely that the entire observed anomaly is due to a reduction in effective porosity.

Of the three possibilities we are considering, the one remaining is that the value of K_d found by both draining and subsequently drying the samples is not the appropriate value to be used in Gassmann's equation for some types of porous materials. It has often been noted that the presence of water in a rock containing clay minerals can have a strong effect on the mechanical properties of the rock. The effect can either strengthen or weaken the rock depending on details of the microstructure and the actual fluid saturant. Clay tends to soften and expand in the presence of a polar fluid like water. Thus, the shear modulus might be expected to decrease in the absence of confining pressure; however, in the presence of confining pressure, the fact that the clay expands may dominate the softening effect and tend to strengthen the sample overall (in the same way that an increase in pore pressure tends to strengthen a rock). Gregory [17] and Mavko and Jizba [21] also show examples of anomalous behavior in low porosity rocks and attribute the anomalies to the presence of microcracks that are open at low confining pressure, but closed at high confining pressure. While the microcracks remain open, they introduce nonlinear behavior into the rock (at low confining stress) since the crack closure process dominates the mechanical response until a critical pressure is achieved above which essentially all of the cracks remain closed. However, for fixed confining pressure (as is normally provided in ultrasonic experiments), the main contribution of the microcracks should appear as a small reduction in the effective porosity from the ϕ value measured in the unconfined state. We conclude that there can be dramatic differences between the measured K_d for air-dry samples and the effective K_d for a saturated sample. We consider this effect to be the most likely explanation of the observed anomaly.

Space limitations prevent us from exploring these ideas in greater depth here. However, we have elsewhere shown that it is possible to obtain sensible values of Biot-Gassmann parameters by treating the drained modulus K_d as an independent parameter, computing the slope b and intercept a for each assumed value of K_d, and then solving for ϕ_{eff} and K_s assuming that the unjacketed pore modulus satisfies $K_\phi \simeq K_s$. We find that both ϕ_{eff} and K_s are then monotonically increasing functions of K_d. Other physical constraints (such as the previously measured values of ϕ, upper bounds on the possible range of K_s from the known mineralogy of the rock, and the fact that the intercept satisfies $a \geq 0$ [5]) provide sufficient information that we can find the range of values that the various physical constants can take consistent with the ultrasonic data. We will present a more thorough discussion of these issues elsewhere.

4 Conclusions

The inverse problem for inhomogeneous frames in the Brown and Korringa picture is complicated by the presence of three bulk moduli characterizing the porous frame — all three of which must be determined from the same data. Measurements of the three ultrasonic velocities of poroelasticity (slow compressional, shear, and fast compressional) may be used to determine pore-space tortuosity, shear modulus, and saturated bulk modulus, respectively, for a single fluid saturant. In principle, three more constants (the drained frame modulus and the two additional bulk moduli of Brown and Korringa) can be determined by repeating the ultrasonic measurements on the same porous sample when it is saturated with different fluids (*e.g.*, air and hydrocarbons). The theory suggests therefore that, in principle, the inversion problem can be solved. However, available real ultrasonic data on rocks [29] shows paradoxically (but conclusively) either that the effective porosity at ultrasonic frequencies is substantially smaller than the measured porosity or that the effective drained bulk modulus is larger than the measured air-dry modulus. Thus, either the effective porosity or the effective drained modulus must also be determined from the data, giving us one too many unknowns. The effective porosity might be interpreted as resulting from a type of high-frequency dispersion caused by the inability of the entire mass of pore fluid to respond instantaneously to acoustically induced pressure changes. Such an interpretation would allow us to estimate an effective porosity as a function of frequency and thereby resolve ambiguities in the inverse problem. However, to establish this mechanism as the correct one, we had to establish first that there were no static effects that could be the true cause. It is at least equally likely that the effective drained modulus of a rock differs from the air-dry modulus when the rock has complex mineralogy. These considerations show that the process of inverting ultrasonic data for Biot-Gassmann parameters must incorporate other physical constraints and then produces a range of possible values for each coefficient.

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