SEISMIC WAVE ATTENUATION IN FLUID-SATURATED POROUS MEDIA

James G. Berryman
Lawrence Livermore National Laboratory
P. O. Box 808 L-156
Livermore, CA 94550

ABSTRACT

Contrary to the traditional view, seismic attenuation in Biot's theory of fluid-saturated porous media is due to viscous damping of local (*not* global) pore-fluid motion. Since substantial inhomogeneities in fluid permeability of porous geological materials are to be expected, the regions of highest local permeability contribute most to the wave energy dissipation while those of lowest permeability dominate the fluid flow rate if they are uniformly distributed. This dichotomy can explain some of the observed discrepancies between computed and measured attenuation of compressional and shear waves in porous earth. One unfortunate consequence of this result is the fact that measured seismic wave attenuation in fluid-filled geological materials cannot be used directly as a diagnostic of the global fluid-flow permeability.

INTRODUCTION

The low frequency limit of Biot's theory of fluid-saturated porous media (Biot, 1956ab) predicts that the coefficients for viscous attenuation of shear waves and of the fast compressional wave are proportional to the fluid permeability. Although the observed attenuation is generally in qualitative agreement with the theory, the magnitude of the observed attenuation coefficient in rocks is often more than an order of magnitude higher than expected. This apparent dilemma can be resolved without invoking other attenuation mechanisms if the intrinsic permeability of the rock is inhomogeneous and varies widely in magnitude. A simple calculation (Berryman, 1986b) and a qualitative argument (presented here) both show conclusively that local flow effects dominate the wave dissipation. For layered inhomogeneous porous materials composed of rocks or porous earth, it is shown that the pertinent permeability for seismic wave attenuation is the mean of the constituent permeabilities while the pertinent permeability for fluid flow is the harmonic mean. These two averages can differ dramatically when the range of variation in the local permeability is one or more orders of magnitude. Thus, this difference in the physically appropriate averaging method can easily explain some of the observed discrepancies.

Biot's theory of elastic wave propagation in fluid-saturated porous media (Biot 1956ab) has correctly predicted the existence of the slow compressional wave at ultrasonic frequencies (Plona 1980, Berryman 1980) and has provided accurate estimates of wave speeds in a variety of porous materials (Murphy 1984, Murphy 1985). The theory has also correctly predicted the magnitude and frequency dependence of the wave attenuation in porous glass bead samples (Hovem 1980, Salin and Schön 1981, Ogushwitz 1985, Chin et al. 1985). In applications to marine sediments (Stoll 1974, Hovem and Ingram 1979), the theory was again successful at explaining the frequency dependence of the measured attenuation and, if the sediment permeability is used as a free parameter, the magnitude of the attenuation can also be fit. Even in fractured granites, the theory makes qualitative predictions of the relationship between attenuation and permeability which have been confirmed by experiment (Berryman et al. 1983). However, in the low frequency range of interest in seismology (*i.e.*, 1 – 100 Hz), the theory has so far been unsuccessful at predicting the magnitude of the attenuation coefficient for fluid-saturated and partially saturated laboratory rock specimens for which measurements of the fluid-flow permeability have also been made – being as much as order of magnitude or more lower than the measured values (Murphy 1982, Mochizuki 1982, Murphy 1984, Murphy 1985).

Various other mechanisms for the observed variations of attenuation in fully and partially saturated porous media have been proposed, including: (1) viscous shear relaxation (Walsh 1969), (2) intercrack squirt flow (Mavko and Nur 1975, O'Connell and Budiansky 1977, Tittmann et al. 1984), (3) macroscopic flow between regions of liquid saturation and regions of gas saturation (White 1975, Dutta and Odé 1979ab, Dutta and Seriff 1979), (4) effects of wetting on grain boundary friction (Johnston et al. 1979), and (5) intracrack flow (Mavko and Nur 1979). In this list of theories, the ones which are successful at explaining the observed effects quantitatively are generally aided in doing so by the presence of various phenomenological parameters which are difficult or impossible to measure independently. None of these theories has been able to produce quantitative, first principles predictions of both speed and attenuation in agreement with experiments on partially saturated samples in the frequency range of interest.

Although Biot's theory has been thoroughly studied during the last 30 years, it is only within the last few years that many of the subtle features of this theory have become understood. The point of the present paper is to stress the fact that prior applications of the theory to viscous attenuation in rocks have been carried through while incorrectly assuming that the value of permeability to be used in the equations is the global effective permeability of the entire "homogeneous" porous specimen. However, it is well known that the local permeability of rocks can vary widely. Thus, the spatial scale to which Biot's theory for homogeneous porous materials applies may be very much smaller than has been previously supposed. The necessity of this conclusion is nicely illustrated in Figure 1. Suppose that a shear wave or a fast compressional wave is incident on a layered material with alternating permeable and impermeable layers. If the impermeable layers are very thin and have an acoustic impedance closely matching that of the permeable layers, their presence has a negligible effect on the propagating wave. The viscous attenuation of the wave occurs solely in the permeable layers and the magnitude of that attenuation is completely determined by the permeability of these layers. By contrast, the global permeability of this material in the direction normal to the layering vanishes identically. If this null value were used in our predictions, the magnitude of the attenuation would be grossly under estimated. Although this choice of geometry is extreme, the example shows clearly that substantial errors in the estimates of attenuation will arise if the value of permeability for global flow is used.

Another convincing demonstration has been given by Mochizuki (1982) that, if we suppose global fluid-flow effects dominate the viscous dissipation, then Biot's theory of poroelasticity cannot explain the observed magnitude of wave attenuation in partially saturated rocks. Since the same theory explains the wave speeds quite well, it is reasonable to suppose that a small change in the theory may be adequate to repair this flaw. Many explanations are possible of course, but within the context of Biot's theory the simplest postulate is to suppose that local – rather than global – fluid-flow effects dominate the dissipation (Berryman 1986b). In the following sections, we will distinguish two related issues summarized in the following questions: (i) Does the physics of wave propagation require that the value of the permeability κ appearing in the Biot's equations should be that for global flow or for local flow? Then, if we can show that the value should be that for local flow, (ii) does this change in the interpretation make enough difference so that the theory can explain the correct magnitude of the attenuation? We will also discuss the consequences of these conclusions.

1. LOCAL-FLOW BIOT THEORY

To address the first question, we explore the consequences of supposing that Biot's theory should be applied at the local flow level rather than at the global flow level. (The length scale for local flow will be taken to be comparable to the size of a typical grain.) This assessment is easily done by examining the dispersion relations. When the Fourier time dependence is $e^{-i\omega t}$ with angular frequency ω sufficiently low, Biot's theory predicts the dispersion relations for the shear (s) wave and for the fast (+) and slow (–) compressional modes in any homogeneous porous material to be

$$k_s^2 \simeq \frac{\omega^2}{v_s^2}[1 + i\omega\frac{\rho_f^2}{\rho q_0}], \tag{1}$$

$$k_+^2 \simeq \frac{\omega^2}{v_+^2}[1 + i\omega\frac{\rho_f^2}{\rho q_0}(1 - v_0^2/v_+^2)^2], \tag{2}$$

and

$$k_-^2 \simeq \frac{i\omega q_0 H}{MH - C^2} \tag{3}$$

where

$$v_+^2 = H/\rho, v_s^2 = \mu/\rho, v_0^2 = C/\rho_f, q_0 = \rho_f\eta/\kappa. \tag{4}$$

The fraction of the total volume occupied by the fluid is the void volume fraction or porosity ϕ, which is assumed to be uniform. The bulk modulus and density of the fluid are K_f and ρ_f. The bulk and shear moduli of the drained porous frame are K and μ. For simplicity, we assume the frame is composed of a single constituent whose bulk and shear moduli and density are K_m, μ_m, and ρ_m. Then the elastic coefficients H, C, and M (Biot 1962, Stoll 1974, Berryman 1986a) are given by

$$H = K + \frac{4}{3}\mu + \sigma C, \tag{5}$$

$$C = [((\sigma - \phi)/K_m + \phi/K_f)/\sigma]^{-1}, \tag{6}$$

and

$$M = C/\sigma \tag{7}$$

where

$$\sigma = 1 - K/K_m. \tag{8}$$

The overall density is

$$\rho = \phi\rho_f + (1-\phi)\rho_m. \tag{9}$$

The kinematic viscosity of the fluid is η, which is assumed to be a fixed constant over the entire region of interest. The permeability of the porous frame is κ. The appropriate value to be used for κ is our main question.

Defining the quality factors for the shear wave Q_s and for the fast compressional wave Q_+ by

$$k_s^2 = \frac{\omega^2}{v_s^2}[1 + i/Q_s], \quad k_+^2 = \frac{\omega^2}{v_+^2}[1 + i/Q_+], \tag{10}$$

we find that Q_s and Q_+ are given by

$$1/Q_s = \omega\frac{\kappa\rho_f}{\eta\rho}, \quad 1/Q_+ = \omega\frac{\kappa\rho_f}{\eta\rho}(1 - v_0^2/v_+^2)^2. \tag{11}$$

Since $1/Q_s$ and $1/Q_+$ are both directly proportional to the permeability, the attenuation is necessarily greatest in regions of high permeability. Thus, we might say that the regions of high permeability control the attenuation.

In the very low frequency limit, the slow compressional mode is known to reduce to Darcy flow with slowly changing magnitude and direction as the driving potential gradient oscillates sinusoidally (Johnson et al. 1986). Now consider a layered porous material (whose constants depend only on depth z) with constituents having identical physical constants except for the permeability κ which varies widely from layer to layer but which has a constant value κ_n within the n-th layer (lying in the range $z_{n-1} \le z \le z_n$ with $z_0 = 0$). Thus, the permeability is a piecewise constant function of z. The thickness of the n-th layer is given by $l_n = z_n - z_{n-1}$. If we impose a potential gradient along the z-direction in such a layered material, it is well-known that the effective permeability for fluid flow is found by taking the harmonic mean of the constituent permeabilities, *i.e.*,

$$1/\kappa_f = \frac{1}{L}\sum_n \frac{l_n}{\kappa_n}, \tag{12}$$

where the total sample length L is given by the sum of the layer thicknesses

$$L = \sum_n l_n. \tag{13}$$

From (12), we can conclude that the regions of lowest permeability dominate the effective overall permeability for fluid flow through a porous layered medium. Figure 1 provides an extreme example of this result. Thus, we might say that the regions of low permeability control the fluid flow – at least for this special choice of geometry.

The apparent attenuation of a fast compressional mode at normal incidence on such a structure has two distinct components: (*i*) Reflection and mode conversion at layer interfaces will have a tendency to degrade the fast wave, but this effect will be quite small at low frequencies for the model structure we are considering. (*ii*) The attenuation within a layer is determined by the quality factor for that layer, as shown by Eq. (10). Assuming the attenuation is small enough, we may approximate (10) within any layer by $k(z) = \frac{\omega}{v}[1+i/2Q(z)]$, where the functions $k(z)$ and $Q(z)$ take the piecewise constant values appropriate for the depth argument z. Neglecting the small effects of reflection and mode conversion, the behavior of the fast compressional wave at normal incidence is then easily seen to be given by

$$A_+ exp[i\int_0^z dz k_+(z) - i\omega t] \simeq A_+ exp[i\frac{\omega}{v_+}z - i\omega t - \frac{\omega}{v_+}\int_0^z dz \frac{1}{2Q_+(z)}], \tag{14}$$

where A_+ is the amplitude of the wave at $z = 0$. A similar formula applies to the shear wave. In writing (14), we have used the piecewise constant property of the functions. The integral in the exponent is given by

$$\int_0^z dz \frac{1}{2Q_+(z)} = \frac{\omega\rho_f}{2\eta\rho}(1 - v_0^2/v_+^2)^2 \int_0^z dz \kappa(z). \tag{15}$$

At the $z = L$ boundary of the material, we have

$$\int_0^L dz\kappa(z) = \sum_n l_n \kappa_n. \tag{16}$$

If the layering is periodic with period much less than either z or L or if it is statistically homogeneous on this length scale, then we may approximate the integral in the exponent of (14) using (15) and

$$\int_0^z dz\kappa(z) \simeq \kappa_a z, \tag{17}$$

where the effective permeability for attenuation measurements is given by the mean

$$\kappa_a = \frac{1}{L}\sum_n l_n \kappa_n. \tag{18}$$

It is well-known that the mean is always greater than or equal to the harmonic mean of any function; thus,

$$\kappa_f \leq \kappa_a. \tag{19}$$

The answer to our first question is therefore that the physics of wave propagation dictates that local-flow effects must dominate the attenuation of the fast compressional wave. Furthermore, Eq. (19) shows that, if the other parameter values have been chosen correctly, then using the global flow permeability to predict attenuation will *always* underestimate its magnitude.

2. MAGNITUDE OF THE ATTENUATION COEFFICIENT

Now, can the theory predict the correct magnitude for the attenuation even with this change in the interpretation of the permeability factor? To use our conclusions in a predictive mode – *i.e.*, to predict the wave attenuation from measurements of permeability – we need some independent means of measuring the local permeability distribution. Normal laboratory flow experiments will not suffice, because they necessarily measure the global permeability. One promising method of estimating the local permeability uses image processing techniques to measure pertinent statistical properties of rock topology from pictures of cross sections (Berryman and Blair 1986; Berryman and Blair 1987). This approach is still under development and we will not attempt to describe it in detail here.

Another approach, which is ultimately much less satisfactory than the image processing method but much easier to use at present, is to suppose that we can obtain reasonable estimates of the local permeability κ_L from the known values of the global permeability κ_G, the porosity ϕ, the electrical formation factor F, and the tortuosity $\tau = (\phi F)^{\frac{1}{2}}$. To do so, we require an approximate formula, and we will use a form of the Kozeny-Carman relation derived recently by Walsh and Brace (1984). For tubes of arbitrary ellipsoidal (major and minor axes a,b) cross-section, the effective permeability of straight sections of such tubes is given by $\kappa = (\pi/4A)[a^3b^3/(a^2+b^2)]$. The porosity for an ellipsoidal tube is $\phi = \pi ab/A$ and the specific surface area is well approximated by $s \simeq 2\pi[(a^2+b^2)/2]^{\frac{1}{2}}/A$. Then, a Kozeny-Carman relation satisfied by κ, ϕ, and s can be shown to be

$$\kappa = \frac{\phi^3}{2s^2} \tag{20}$$

for the effective permeability of a single tube oriented along the pressure gradient. If the tube is at an angle θ to the this gradient, then it is not hard to show (Walsh and Brace 1984) that

$$\kappa = \frac{\phi^3}{2s^2\tau^2}, \tag{21}$$

where $\tau = 1/cos\theta$. If we suppose that (20) and (21) are fairly representative of the material of interest, then (20) describes the maximum local permeability κ_L and (21) the effective global permeability κ_G. We then conclude that

$$\kappa_L = \tau^2\kappa_G = \phi F \kappa_G. \tag{22}$$

The tortuosity τ has been measured for many sandstones; the values for samples studied by Simmons et al. (1982, 1983) lie in the range $1.5 \leq \tau \leq 5$, with most values $\tau \simeq 2$. To obtain estimates of attenuation close to experiment (Murphy 1982), we need to increase the value of permeability used in Mochizuki's calculations (Mochizuki 1982) by a factor of $\tau^2 \simeq 10$. This requirement implies a tortuosity of $\tau \simeq 3$, which is clearly well within the established experimental bounds. This argument provides strong evidence for the plausibility of a local-flow explanation of the observed discrepancies. However, a completely satisfying demonstration must await the collection of the required data on local-flow permeability.

3. CONCLUSIONS AND DISCUSSION

We conclude that the high degree of local heterogeneity in rock properties causes different parts of the rock to play the leading roles in the processes of wave attenuation and fluid flow: The regions of high permeability produce the majority of the attenuation, while the regions of low permeability tend to limit the flux and dominate the fluid flow. These ideas could be tested experimentally by constructing a layered porous material (possibly using glass bead samples) with uniform elastic properties but nonuniform local permeability.

The results presented here are based on the low frequency asymptotic limits of the solutions to Biot's theory of fluid-saturated porous media. The asymptotic behavior of the theory is very different in the high frequency limit; thus, these results do not apply to ultrasonic experiments and our conclusions do not affect the very successful comparisons between theory and experiment that have been published previously (Plona 1980, Berryman 1980, Salin and Schön 1981, Johnson et al. 1982). The present results require a reinterpretation of the appropriate permeability for calculations of shear wave and fast compressional wave attenuation in the low frequency limit, but they also show that there is little need for a reinterpretation of the permeability for applications to the slow compressional wave in its low frequency diffusive mode. The dispersion relation (3) shows clearly that the slow wave's dependence on permeability is through an inverse proportionality – thus, making the regions of lowest permeability the dominant ones. It follows that, in an inhomogeneous medium, the slow wave behavior is dominated by the same regions that dominate the behavior of the global fluid flow. We conclude that the present results also do not affect the successful comparison between theory and experiment at very low frequency by Chandler (1981) showing the correspondence between Biot's slow wave and the transient streaming potential used for measurement of permeability.

One unfortunate consequence of the observation that local permeability controls attenuation is that measured attenuation in wet geological materials cannot be used directly as a diagnostic of the global fluid-flow permeability. A further complication can arise if the pore fluid viscosity varies within the region of interest due to the presence of thermal gradients. Then, it will be difficult to make unambiguous inferences about permeability from attenuation measurements. However, if the viscosity is uniform as assumed here, then the effective permeability obtained from attenuation measurements can be used to provide an upper limit on the desired global value. This result follows from the preceding analysis since the mean of the local permeabilities will always be greater than the global permeability regardless of the actual spatial distribution of the constituent κ's.

ACKNOWLEDGMENTS

I thank S. C. Blair, B. P. Bonner, and L. Thigpen for helpful conversations. This work was performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48 and supported specifically by the Nuclear Test Containment Program.

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FIGURE CAPTION

Fig. 1. Illustration of a simple experiment to prove that the attenuation of a shear wave or fast compressional wave depends on the local – not the global – value of permeability κ. A wave field incident normal to the impermeable partitions will experience a small but finite attenuation even though the global permeability in this direction vanishes identically.