ELASTIC WAVE ATTENUATION IN ROCKS CONTAINING FLUIDS

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ABSTRACT

The low frequency limit of Biot's theory of fluid-saturated porous media predicts that the coefficients for viscous attenuation of shear waves and of the fast compressional wave are proportional to the fluid permeability. Although the observed attenuation is generally in qualitative agreement with the theory, the magnitude of the observed attenuation coefficient in rocks is often more than an order of magnitude higher than expected. This apparent dilemma can be resolved without invoking other attenuation mechanisms if the intrinsic permeability of the rock is inhomogeneous and varies widely in magnitude. A simple calculation of the overall behavior of a layered porous material using local-flow Biot theory shows that the effective permeability for attenuation is the mean of the constituent permeabilities while the effective permeability for fluid flow is the harmonic mean. When the range of variation in the local permeability is one or more orders of magnitude, this difference in averaging method can easily explain some of the observed discrepancies.

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Biot's theory of elastic wave propagation in fluid-saturated porous media[1,2] has correctly predicted the existence of the slow compressional wave[3,4] and has provided accurate estimates of wave speeds in a variety of porous materials.[5,6] The theory has also correctly predicted the magnitude and frequency dependence of the wave attenuation in porous glass bead samples.[7–10] In applications to marine sediments,[11,12] the theory is again successful at explaining the frequency dependence of the measured attenuation and, if the sediment permeability is used as a free parameter, the magnitude of the attenuation can also be fit. Even in fractured granites, the theory makes qualitative predictions of the relationship between attenuation and permeability which have been confirmed by experiment.[13] However, the theory has so far been unsuccessful at predicting the magnitude of the attenuation coefficient for fluid-saturated and partially saturated laboratory rock specimens for which measurements of the fluid-flow permeability have also been made – being as much as order of magnitude or more lower than the measured values.[5,6,14,15]

Various other mechanisms for the observed variations of attenuation in fully and partially saturated porous media have been proposed, including: (1) viscous shear relaxation,[16] (2) intercrack squirt flow,[17–19] (3) macroscopic flow between regions of liquid saturation and regions of gas saturation,[20–23] (4) effects of wetting on grain boundary friction,[24] and (5) intracrack flow.[25] In this list of theories, the ones which are successful at explaining the observed effects quantitatively are aided in doing so by the presence of various phenomenological parameters which are difficult or impossible to measure independently. None of these theories has been able to produce quantitative, first principles predictions of both speed and attenuation in agreement with experiments on partially saturated samples.

Although Biot's theory has been thoroughly studied during the last 30 years, it is only within the last few years that many of the subtle features of this theory have become understood. The point of the present paper is to stress the fact that prior applications of the theory to viscous attenuation in rocks have been carried through by incorrectly assuming that the value of permeability to be used in the equations is the global effective permeability of the entire "homogeneous" porous specimen. However, it is well known that the local permeability of rocks can vary widely. Thus, the spatial scale to which Biot's theory for homogeneous porous materials applies may be very much smaller than has been previously supposed.

The consequences of assuming that Biot's theory should be applied at the local flow level rather than at the global flow level can be easily assessed through an examination of the dispersion relations. When the Fourier time dependence is $e^{-i\omega t}$ with angular frequency ω sufficiently low, Biot's theory predicts the dispersion relations for the fast (+) and slow

(−) compressional modes in any homogeneous porous material to be

$$k_+^2 \simeq \frac{\omega^2}{v_+^2}[1 + i\omega\frac{\rho_f^2}{\rho q_0}(1 - v_0^2/v_+^2)^2] \tag{1}$$

and

$$k_-^2 \simeq \frac{i\omega q_0 H}{MH - C^2} \tag{2}$$

where

$$v_+^2 = H/\rho, v_0^2 = C/\rho_f, q_0 = \rho_f\eta/\kappa. \tag{3}$$

The fraction of the total volume occupied by the fluid is the void volume fraction or porosity ϕ, which is assumed to be uniform. The bulk modulus and density of the fluid are K_f and ρ_f. The bulk and shear moduli of the drained porous frame are K and μ. For simplicity[26] we assume the frame is composed of a single constituent whose bulk and shear moduli and density are K_m, μ_m, and ρ_m. Then the coefficients H, C, and M are given by

$$H = K + \frac{4}{3}\mu + \sigma C, \tag{4}$$

$$C = [((\sigma - \phi)/K_m + \phi/K_f)/\sigma]^{-1}, \tag{5}$$

and

$$M = C/\sigma \tag{6}$$

where

$$\sigma = 1 - K/K_m. \tag{7}$$

The overall density is

$$\rho = \phi\rho_f + (1-\phi)\rho_m. \tag{8}$$

The kinematic viscosity of the fluid is η and the permeability of the porous frame is κ.

Defining the quality factor for the fast compressional wave Q_+ by

$$k_+^2 = \frac{\omega^2}{v_+^2}[1 + i/Q_+], \tag{9}$$

we find that Q_+ is given by

$$1/Q_+ = \omega\frac{\kappa\rho_f}{\eta\rho}(1 - v_0^2/v_+^2)^2. \tag{10}$$

Since $1/Q_+$ is proportional to the permeability, the attenuation is therefore greatest in regions of high permeability. Thus, we might say that the regions of high permeability control the attenuation.

In the very low frequency limit, the slow compressional mode is known to reduce to Darcy flow with slowly changing magnitude and direction as the driving potential gradient oscillates sinusoidally.[27] Now consider a layered porous material (whose constants depend only on depth z) with constituents having identical physical constants except for the permeability κ which is constant in each layer but varies widely from layer to layer. Thus, the permeability is a piece-wise constant function of z. If we impose a potential gradient along the z-direction in such a layered material, it is well-known that the effective permeability for fluid flow is found by taking the harmonic mean of the constituent permeabilities, *i.e.*,

$$1/\kappa_f = \frac{1}{L}\sum_n \frac{l_n}{\kappa_n}, \tag{11}$$

where the total sample length L is given by the sum of the layer thicknesses

$$L = \sum_n l_n. \tag{12}$$

From (11), we can conclude that the regions of lowest permeability dominate the effective overall permeability for fluid flow through a porous layered medium. Thus, we might say that the regions of low permeability control the fluid flow – at least for this special choice of geometry.

The behavior of the fast compressional mode at normal incidence on such a structure is given by

$$A_+exp[i\int_0^z dzk_+(z) - i\omega t] \simeq A_+exp\,[i\frac{\omega}{v_+}z - i\omega t - \frac{\omega}{v_+}\int_0^z dz\frac{1}{2Q_+(z)}], \tag{13}$$

where A_+ is the amplitude of the wave at $z=0$. The integral in the exponent is given by

$$\int_0^z dz\frac{1}{2Q_+(z)} = \frac{\omega\rho_f}{2\eta\rho}(1 - v_0^2/v_+^2)^2\int_0^z dz\kappa(z). \tag{14}$$

At the $z = L$ boundary of the material, we have

$$\int_0^L dz\kappa(z) = \sum_n l_n\kappa_n. \tag{15}$$

If the layering is periodic with period much less than either z or L or if it is statistically homogeneous on this length scale, then we may approximate the integral in the exponent of (13) using (14) and

$$\int_0^z dz\kappa(z) \simeq \kappa_a z, \tag{16}$$

where the effective permeability for attenuation measurements is given by the mean

$$\kappa_a = \frac{1}{L}\sum_n l_n \kappa_n. \tag{17}$$

It is well-known that the mean is always greater than or equal to the harmonic mean of any function; thus,

$$\kappa_f \leq \kappa_a. \tag{18}$$

We therefore expect the measured attenuation to be greater than that predicted using the effective fluid-flow permeability. This conclusion provides at least a qualitative explanation of the observed discrepancies between predicted and measured attenuation in porous materials. One unfortunate consequence of this result is that measured attenuation in wet rocks cannot be used as a direct measure of the fluid-flow permeability. On the other hand, since the mean of the local permeabilities will always be greater than the true fluid-flow permeability regardless of the actual spatial distribution of the constituent κ's, the effective permeability obtained from attenuation measurements can be used to provide an upper bound on the desired fluid-flow permeability.

To provide a simple quantitative example, consider a layered Berea sandstone composed of two types of homogeneous Bereas[27,28] – one with $\kappa = 23mD$ and the other with $\kappa = 383mD$. Then, with equal amounts of the two constituents, the values of the effective permeabilities are $\kappa_f = 43mD$ and $\kappa_a = 203mD$. The numbers used in this example are taken from actual fluid flow experiments on samples of Berea sandstone and result in a factor of about 5 difference in the effective permeabilities. If there are small regions of much higher permeability in such samples, those regions have not been properly accounted for in this calculation and the observed discrepancy could be much larger. In general, the present result will be difficult to test on naturally occurring samples unless a method of estimating the local permeability from images of rock cross-sections can be perfected.[30,31]

We conclude that the high degree of local heterogeneity in rock properties causes different parts of the rock to play the leading roles in the processes of attenuation and fluid flow: The regions of high permeability produce the majority of the attenuation, while the regions of low permeability tend to limit the flux and dominate the fluid flow. These ideas could be tested experimentally by constructing a layered porous material (possibly using glass bead samples) with uniform elastic properties but nonuniform local permeability.

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