On Constitutive Equations and Effective Stress for Deformable, Double Porosity Media

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Abstract

Tuncay and Corapcioglu [1995] used volume averaging methods to derive an effective stress principle for the bulk volumetric strain in a deformable double-porosity medium. The coefficients of the matrix pore pressure and fracture pore pressure in their equation for the effective stress are shown to be identical to those which can be obtained from the constitutive equation approach of Berryman and Wang (1995). Representative values for a fractured Berea sandstone show that a change in pore pressure within matrix blocks produces about 10% of the volume change due to an equal but opposite change in confining pressure, whereas the same change in pore pressure within fractures is about 90% as effective. A similar result is true for Westerly granite.

1 Introduction

The theory of poroelasticity lies at the intersection of mechanics of porous media and fluid flow within porous media. As a consequence, papers dealing with poroelasticity appear in journals that appeal primarily to one or the other discipline. Therefore, it might be worthwhile to show the commonality of results that were arrived at independently and which appeared in different journals. Tuncay and Corapcioglu [1995] derived an effective stress principle for a deformable double-porosity medium using volume averaging techniques which have also been used recently by Pride et al. [1995]. The purpose of this note is to point out that Tuncay and Corapcioglu's expressions for the coefficients of the matrix pore pressure, $p_f^{(1)}$ and fracture pore pressure, $p_f^{(2)}$ are identical to those obtained from the analysis of the physical meaning of the constitutive equations by Berryman and Wang [1995]. The constitutive equations for a double-porosity medium are a straightforward generalization of Biot's [1941] equations for a single-porosity medium.

2 Derivation

Berryman and Wang expressed the constitutive equations in matrix form as

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}. \tag{1}$$

where e is volumetric strain, p_c is confining pressure (equal to the negative of the mean stress with the convention that extensional stresses are positive), $\zeta^{(1)}$ is the increment of fluid content in the matrix, and $\zeta^{(2)}$ is the increment of fluid content in the fracture. The coefficient matrix is symmetric. The coefficients a_{12} and a_{13} can be interpreted as *poroelastic expansion coefficients*, analogous to thermal expansion. The submatrix elements a_{22}, a_{33}, and a_{23}

are *storage coefficients* of the matrix. Berryman and Wang argued that the cross-storage coefficient a_{23} may often be considered to be negligible.

In general, many effective stress principles are possible, depending upon which property is under consideration [Berryman, 1992]. The effective stress principle discussed by Tuncay and Corapcioglu is the one for volumetric strain. Their result can be obtained by first writing out the top row of (1) as

$$\delta e = a_{11}(-\delta p_c) + a_{12}(-\delta p_f^{(1)}) + a_{13}(-\delta p_f^{(2)}). \tag{2}$$

The definition of effective confining pressure p_c^e is that changes in volumetric strain are determined solely by changes in p_c^e, i.e.,

$$\delta e = a_{11}(-\delta p_c^e). \tag{3}$$

From (2) the change in effective confining pressure δp_c^e is the linear combination of confining pressure and pore pressure changes given by

$$-\delta p_c^e = -\delta p_c + \frac{a_{12}}{a_{11}}(-\delta p_f^{(1)}) + \frac{a_{13}}{a_{11}}(-\delta p_f^{(2)}). \tag{4}$$

Berryman and Wang [1995] derived expressions for a_{11}, a_{12}, and a_{13} in terms of the bulk modulus of the porous matrix ($K^{(1)}$), the bulk modulus of the fracture phase ($K^{(2)}$), and the bulk modulus of the solid grains (K_s):

$$a_{11} = 1/K \tag{5}$$
$$a_{12} = -v^{(1)}\alpha^{(1)}/K^{(1)} \tag{6}$$
$$a_{13} = -v^{(2)}/K^{(2)} \tag{7}$$

where $\alpha^{(1)} = 1 - K^{(1)}/K_s$. Inserting these expressions into (4) leads to the same results obtained by Tuncay and Corapcioglu for the effective stress coefficients β_p of $p_f^{(1)}$ and β_f of $p_f^{(2)}$ (their Eqs. (23) and (24)), where their K_{fr} is K, K_{fr}^m is $K^{(1)}$, and α_f is $v^{(2)}$. Explicitly,

$$\beta_p = v^{(1)}\alpha^{(1)}\frac{K}{K^{(1)}} \tag{8}$$
$$\beta_f = 1 - v^{(1)}\frac{K}{K^{(1)}} \tag{9}$$

3 Examples

Representative values from Tables 1 and 2 of Berryman and Wang give $\beta_p = 0.096$ and $\beta_f = 0.877$ for a fractured Berea sandstone, and $\beta_p = 0.112$

and $\beta_f = 0.750$ for a fractured Westerly granite. Therefore, a change in pore pressure within matrix blocks of Berea sandstone produces only 9.6% of the volume change due to an equal but opposite change in confining pressure, whereas the same change in pore pressure within fractures is 87.7% as effective. A similar result is true for Westerly granite. A change in matrix pore pressure is only 11.2% as effective as a similar change in confining pressure, whereas a change in fracture pore pressure is 75% as effective.

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