Effective-Stress Rules for Pore-Fluid Transport in Rocks Containing Two Minerals

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Technical Note

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INTRODUCTION

When an oil field is being pumped down, the in situ hydrostatic pressure decreases and permeability also decreases in response. Possibility of controlling this process depends on knowing the correct effective-stress rule for permeability in the field. Although some effective-stress analysis for permeability has been published previously [1], the work summarized here is the first to use scaling rules to establish general results.

Data on the pore-pressure dependence of fluid permeability for some rocks cannot be explained using any equivalent homogeneous porous medium. However, deformation measurements on both high porosity sandstones and low porosity granites have been explained [2] adequately in terms of an equivalent two-constituent model of porous rocks, for which exact results relating constituent moduli to macroscopic effective moduli have been discovered [3].

EXACT EFFECTIVE STRESS RULES

Let V be the total sample volume, $V_\phi = \phi V$ the pore volume, $p_c = -\frac{1}{3}\mathrm{Tr}(\tau) = -\frac{1}{3}(\tau_{xx} + \tau_{yy} + \tau_{zz})$ the confining (external) pressure, p_f the fluid (pore) pressure, and $p_d = p_c - p_f$ the differential pressure. Using these definitions, the isotropic-stress/volume-strain relations become

$$-\frac{\delta V}{V} = \frac{\delta p_d}{K} + \frac{\delta p_f}{K_s} = \frac{1}{K}\left(\delta p_c - \alpha \delta p_f\right) \tag{1}$$

for the total volume strain and

$$-\frac{\delta V_\phi}{V_\phi} = \frac{\delta p_d}{K_p} + \frac{\delta p_f}{K_\phi} = \frac{1}{K_p}\left(\delta p_c - \beta \delta p_f\right) \tag{2}$$

for the pore volume strain, where $\alpha = 1 - K/K_s$ and $\beta = 1 - K_p/K_\phi$ are the corresponding effective-stress coefficients. The constants appearing in the equations are the jacketed modulus K, the unjacketed modulus K_s, the jacketed pore modulus K_p, and the unjacketed pore modulus K_ϕ. Reciprocity [4] shows that $K_p = \phi K/\alpha$.

Based on these definitions, Berryman [2] has done a thorough analysis of the various possible effective-stress rules and the corresponding coefficients. The results of this analysis are summarized in Table 1.

POROUS FRAME CONTAINING A SINGLE MINERAL

One elementary result of the analysis for a porous frame composed of just one mineral is the fact that the effective-stress coefficient χ for the porosity ϕ is unity. However, Fatt [5] used results of Brandt [6] to show that $\chi \simeq 0.85$ in some sandstone reservoir rocks, showing clearly that some data are inconsistent with the result $\chi = 1$. The implication of the empirical result $\chi < 1$ is that no equivalent microhomogeneous rock or set of rocks can be used to explain these effective-stress data. It will not be surprising then if data for other effective-stress coefficients also contradict the "equivalent homogeneous rock" model.

Electrical conductivity

Treating the grains in a porous rock as insulators and injecting a conducting fluid into the pores, the overall conductivity of the saturated medium is of the form $g = g_f G$, where g_f is the conductivity of the pore fluid. The function G depends only on relative positioning of the insulating grains and is therefore scale invariant. The porosity ϕ is also scale invariant, so in general we may consider expressing G as a function of porosity.

Archie's law [7] for the electrical conductivity g of a brine saturated porous medium is

$$g = \frac{g_f}{F} = g_f \phi^m, \tag{3}$$

where g_f is the electrical conductivity of the saturating fluid, $F \equiv g_f/g = 1/G$ is the formation factor, ϕ is the porosity of the porous formation, and m is Archie's cementation exponent (generally in the range $1 < m \le 2$). With a single mineral constituent,

$$\frac{\delta G}{G} \simeq m\frac{\delta\phi}{\phi} = -m\left(\frac{\alpha-\phi}{\phi K}\right)\delta p_d. \tag{4}$$

Neglecting the pressure dependence of g_f, the effective pressure for g is just the differential pressure. Effective stress for the electrical conductivity is $\delta p_c - \epsilon\delta p_f = \delta p_d$, where $\epsilon = 1$ for microhomogeneous frames.

Fluid permeability

Darcy's constant k has the dimensions of length squared, so a uniform swelling or shrinking of the isotropic porous medium changes the value of the permeability proportional to $V^{2/3}$ (since V has dimensions of length cubed). The dependence of the permeability on geometry may therefore be expressed in general as

$$k = H \times V^{2/3}, \tag{5}$$

where H depends only on the relative positioning of the grains and is therefore scale invariant. H behaves much like G. In analogy with the arguments leading to (4), we suppose that

$$\frac{\delta H}{H} \simeq n\frac{\delta\phi}{\phi} = -n\left(\frac{\alpha-\phi}{\phi K}\right)\delta p_d. \tag{6}$$

The constant n may be related to Archie's cementation exponent m through the Kozeny-Carman relation [8]

$$k \simeq \frac{\phi^2}{2s^2F}, \tag{7}$$

where s is a measure of the specific surface area (for an equivalent smooth-walled pore) and therefore $s^{-2} = const \times V^{2/3}$. Thus, $H \simeq \phi^2/F = \phi^{2+m}$ and $n \simeq 2+m$. A nominal value of $n \simeq 4$ is consistent with experimental evidence.

The effective-stress coefficient for permeability is determined by

$$\begin{aligned}\frac{\delta k}{k} &= \frac{\delta H}{H} + \tfrac{2}{3}\frac{\delta V}{V} \\ &= -\left[n\left(\frac{\alpha-\phi}{\phi K}\right) + \frac{2}{3K}\right](\delta p_c - \kappa\delta p_f),\end{aligned} \tag{8}$$

where the effective-stress coefficient for permeability is

$$\kappa = 1 - \frac{2\phi(1-\alpha)}{3n(\alpha-\phi)+2\phi} \le 1. \tag{9}$$

The inequality follows from the facts that $\phi \le \alpha \le 1$ for a microhomogeneous frame and that the denominator is always positive as long as $\phi > 0$.

If any porous rock can be approximated by an "equivalent homogeneous rock," then (9) predicts that the corresponding effective-stress coefficient κ is less than unity. However, (9) conflicts with some experimental results [9,10], showing that the effective-stress coefficient κ for fluid permeability can be significantly greater than unity (as high as 7 in some cases) for various sandstones containing clay. Thus, it is impossible to explain such behavior of clay-bearing porous rocks under stress in terms of an equivalent homogeneous frame. Therefore, a more rigorous analysis of porous media containing two or more minerals is required.

TWO MINERALS WITH PORES

For rocks with two minerals, Berryman and Milton [3] have shown that there exists a linear combination of macroscopic pressure increments $\delta p_c - \theta\delta p_f \equiv 0$ so that the relative change in the volumes of each constituent (and therefore of the composite) is the same. Thus, the composite porous medium undergoes a uniform swelling or shrinking so the shapes and relative positions of all the porous constituents remain fixed while the overall size increases or decreases. Thus, θ is another effective-stress coefficient and Berryman [2] shows how to compute it exactly for two component rocks.

The integrability conditions required to guarantee reversibility [*i.e.*, so that the total volume is a function $V = V(p_c, p_f)$ independent of the particular stress-strain path used to achieve the final pressures p_c and p_f] may also be used to find θ. The Euler conditions for integrability show that

$$\theta = 1 - \frac{\partial}{\partial p_c}(1/K_s) / \frac{\partial}{\partial p_c}(1/K). \tag{10}$$

This result requires no assumptions about the model material and also shows how stress-strain data on K and K_s determine θ. Examples have been computed by Berryman [2] from Coyner's [11] data on rocks, and show that θ is close to but generally slightly less than unity.

Now consider the regional volume fractions

$$v_A \equiv \frac{V^{(1)}}{V} \quad \text{and} \quad v_B \equiv \frac{V^{(2)}}{V}, \tag{11}$$

as functions of applied pressure. Since v_A and v_B are geometrical variables, we postulate effective-stress rules

$$\begin{aligned} \delta v_A &\equiv \frac{1}{K_A}(\delta p_c - \theta \delta p_f), \\ \delta v_B &\equiv \frac{1}{K_B}(\delta p_c - \theta \delta p_f), \end{aligned} \tag{12}$$

where $\frac{1}{K_A} \simeq v_A\left(\frac{1}{K} - \frac{1}{K^{(1)}}\right)$ and $\frac{1}{K_B} \simeq v_B\left(\frac{1}{K} - \frac{1}{K^{(2)}}\right)$.

In the "clayey-sandstone model," one constituent (say 2) has no porosity, so $K^{(2)} = K_m^{(2)}$ and $\alpha^{(2)} = \phi^{(2)} = 0$, while the other constituent has a very soft frame modulus. Thus, $K^{(1)} \to 0$ and $\alpha^{(1)} \to 1$. Substituting these limits into the exact formulas gives $\alpha \simeq 1 - \frac{K}{K_m^{(2)}}$ and $\theta \to \alpha^{(1)} \to 1$. Then, $K_s \simeq K_m^{(2)}$ and from [3] we have $\frac{1}{K_\phi} \simeq \frac{1}{K_m^{(1)}} + \frac{1}{\phi^{(1)}}\left(\frac{1}{K_m^{(2)}} - \frac{1}{K_m^{(1)}}\right)$, showing that K_ϕ can take a wide range of values. Then, $\chi \simeq 1 + \left(\frac{v_A - \phi}{\alpha - \phi}\right)\left(\frac{1}{K_m^{(1)}} - \frac{1}{K_m^{(2)}}\right)K$, while $\frac{1}{v_A K_A} \simeq -\frac{1}{K^{(1)}}$ and $\frac{1}{v_B K_B} \simeq \frac{\alpha}{K}$. The porosity is given by $\phi = v_A \phi^{(1)}$ initially.

TRANSPORT PROPERTIES

Results of the last section will now be used to analyze electrical conductivity and fluid permeability in a two-mineral porous medium.

Electrical conductivity

Considering a clayey sandstone, let $g_2 = 0$ represent the conductivity of the insulating sand grains. Then, let the clay be composed of insulating particles of a single microhomogeneous material. If g_f is the conductivity of the pore fluid, the corresponding effective conductivity of the saturated clay is $g_1 = \frac{g_f}{F_1}$, where F_1 is the formation factor of the porous clay and satisfies $F_1 = \phi_1^{-m_1}$ with ϕ_1 being the porosity of the clay. Finally, the effective conductivity of the clayey sandstone is

$$g = \frac{g_f}{F_1 F_A} = g_f \phi_1^{m_1} v_A^{m_A} = g_f \phi^{m_1} v_A^{m_A - m_1}, \tag{13}$$

recalling that the total porosity for this model is $\phi = \phi_1 v_A$. We see that (13) will show some deviations from Archie's law for the composite only if the exponents m_1 and m_A differ significantly. Otherwise, $m_1 \simeq m_A \simeq m$ will give $g \simeq g_f \phi^m$.

Combining the results, (13) shows that the stress rule for effective conductivity is

$$\frac{\delta g}{g} = -\left[m_1\left(\frac{\alpha - \phi}{\phi K}\right) + \frac{m_1 - m_A}{v_A K_A}\right](\delta p_c - \epsilon \delta p_f), \tag{14}$$

where

$$\epsilon = 1 - \frac{m_1(\alpha - \phi)(1 - \chi) + (m_1 - m_A)\phi K(1 - \theta)/v_A K_A}{m_1(\alpha - \phi) + (m_1 - m_A)\phi K / v_A K_A}. \tag{15}$$

For microhomogeneous frames, $\epsilon = 1$ as expected. If $m_1 = m_A$, then $\epsilon = \chi$ and the value of θ does not affect the conductivity. If $m_1 \neq m_A$, then the values of both θ and K_A are important in determining ϵ. Thus, (15) is a prediction of this analysis suggesting some new experiments.

For the clayey-sandstone model, if $m_1 \simeq m_A$ then $\epsilon \simeq \chi$, but if $m_1 \neq m_A$ then $\epsilon \simeq \theta \simeq 1$. Since $\theta \leq \chi$ for this model, the general conclusion is that $\theta \leq \epsilon \leq \chi$, but these bounds are only approximate. Evaluating the formulas for the clayey sandstone model, suppose that $m_1 = 2$, $m_A = 2.15$, $\alpha = 0.85$, $\phi^{(2)} = 0.8$, $v_A = 0.25$, $\phi = 0.2$, and $K/K^{(1)} = 10$. Then, $\chi > 0.99$ (assuming only that $K_m^{(1)} < \infty$) and $\epsilon \simeq 1.3\chi - 0.3$, showing that the effective-stress coefficient for electrical conductivity can be greater than unity if $\chi > 1$.

Fluid permeability

Since $V_1 = v_A V$, the effective-stress formula for the permeability is

$$\begin{aligned} \frac{\delta k}{k} &= n_1 \frac{\delta\phi}{\phi} - q\frac{\delta v_A}{v_A} + \tfrac{2}{3}\frac{\delta V}{V} \\ &= -\left[n_1\left(\tfrac{\alpha-\phi}{\phi K}\right) + \tfrac{2}{3K} + \tfrac{q}{v_A K_A}\right](\delta p_c - \kappa\delta p_f), \end{aligned} \tag{16}$$

where $\phi = \phi_1 v_A$, $q = n_1 - m_A - \frac{2}{3}$, and

$$\kappa = 1 - \tfrac{3n_1(\alpha-\phi)(1-\chi)+2\phi(1-\alpha)+3q\phi K(1-\theta)/v_A K_A}{3n_1(\alpha-\phi)+2\phi+3q\phi K/v_A K_A}. \tag{17}$$

Rigorous bounds on κ are difficult to obtain because the terms in the denominator of (17) do not always have the same sign. For microhomogeneous frames, (17) reduces correctly to (9).

In general, the porosity coefficient χ can have values either less than or greater than unity. For the two mineral problem, K_A will normally be negative, while θ is restricted by the empirical inequalities $\alpha \leq \theta \leq 1$. Thus, the expression (17) can take a wide variety of values because of the variability of χ and K_A. Considering the poorly consolidated limit where $K^{(1)} \to 0$ of the clayey-sandstone model, we find that $\kappa \to \theta \to 1$. However, if $K^{(1)}/K << 1$ but remains finite, some magnification occurs due to cancellations in the denominator of (17). The result for κ is given approximately by $\kappa \simeq \alpha + M(\chi - \theta)$, where the magnification factor $M \simeq 40$ if $n_1 = 4$, $\alpha = 0.85$, $v_A = 0.25$, $\phi = 0.2$, $q = 4/3$, and $K/K^{(1)} = 10$. Evaluating (17) using $\chi \simeq 1.1$, the result for the effective-stress coefficient is $\kappa \simeq 5$. This estimate agrees reasonably well with experimental results on Berea sandstone of Zoback and Byerlee [9] and Coyner [11]. Thus, if the effective grain modulus of the pore-filling material $K_m^{(1)}$ is sufficiently small compared to that of the sand grains $K_m^{(2)}$, both $\chi > 1$ and $\kappa > 1$ are possible. The theory shows that the effective-stress coefficient κ can be greater than unity as observed [9-11]. To obtain better agreement between theory and experiment, values of constants such as K_ϕ, β, and χ should be measured.

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TABLE 1. Definitions and relations among various effective-stress coefficients from [2].

Physical Quantity Preserved	*Effective Stress Coefficient*	*Definition*	*Single Mineral Restrictions*	*Multiple Mineral Restrictions*
Total volume V	α	$1 - K/K_s$	$\phi \le \alpha \le 1$	$\alpha \le 1$ $(\phi \le \alpha)$
Pore volume V_ϕ	β	$1 - K_p/K_\phi$	$\alpha \le \beta \le 1$	$\alpha \le \beta$
Fluid content ζ	γ	$\beta + K_p/K_f$	$\beta \le \gamma$	$\beta \le \gamma$
Porosity ϕ	χ	$\left(\frac{\beta - \phi}{\alpha - \phi}\right)\alpha$	$\chi = 1$	$(\alpha \le \chi)$
Solid volume $V_s = V - V_\phi$	σ	$\left(\frac{1-\beta}{1-\alpha}\right)\alpha$	$\sigma = \phi$	$\sigma \le \alpha$
Geometry	θ	$1 - \frac{\partial}{\partial p_c}(1/K_s)\,/\,\frac{\partial}{\partial p_c}(1/K)$	$\theta = 1$	$(\theta \le 1)$
Formation factor F	ϵ	*model dependent*	$\epsilon = 1$	*no restriction*
Permeability k	κ	*model dependent*	$\beta \le \kappa \le 1$	*no restriction*
				$0 \le (\alpha - \phi)(\chi - \alpha)$