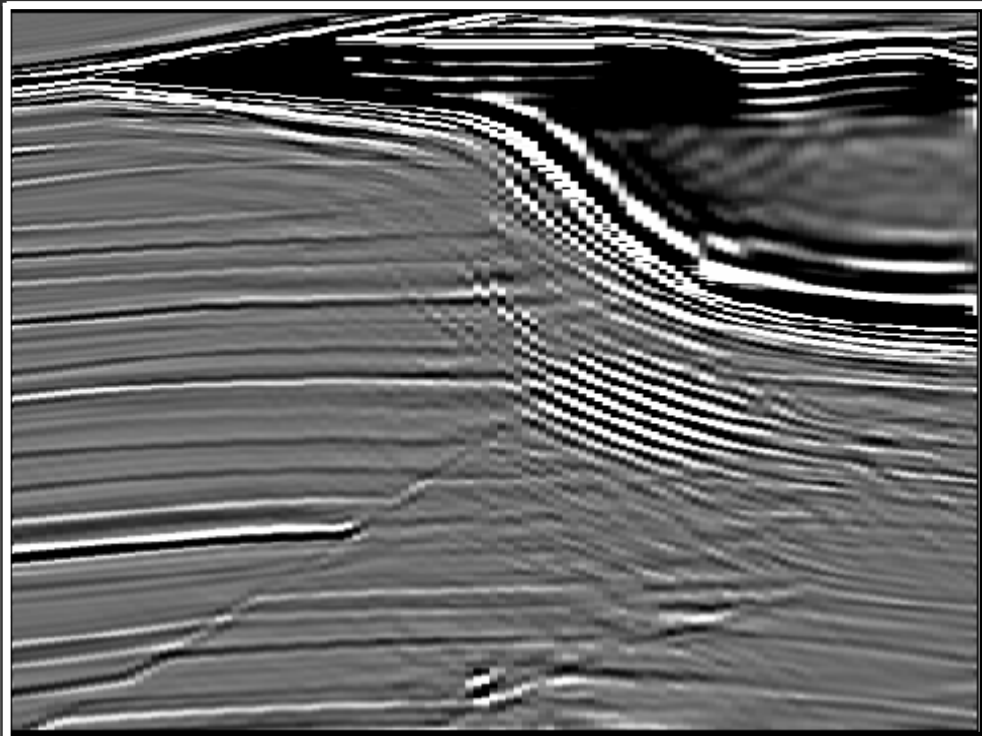


# Multiple attenuation prior to wave-equation inversion

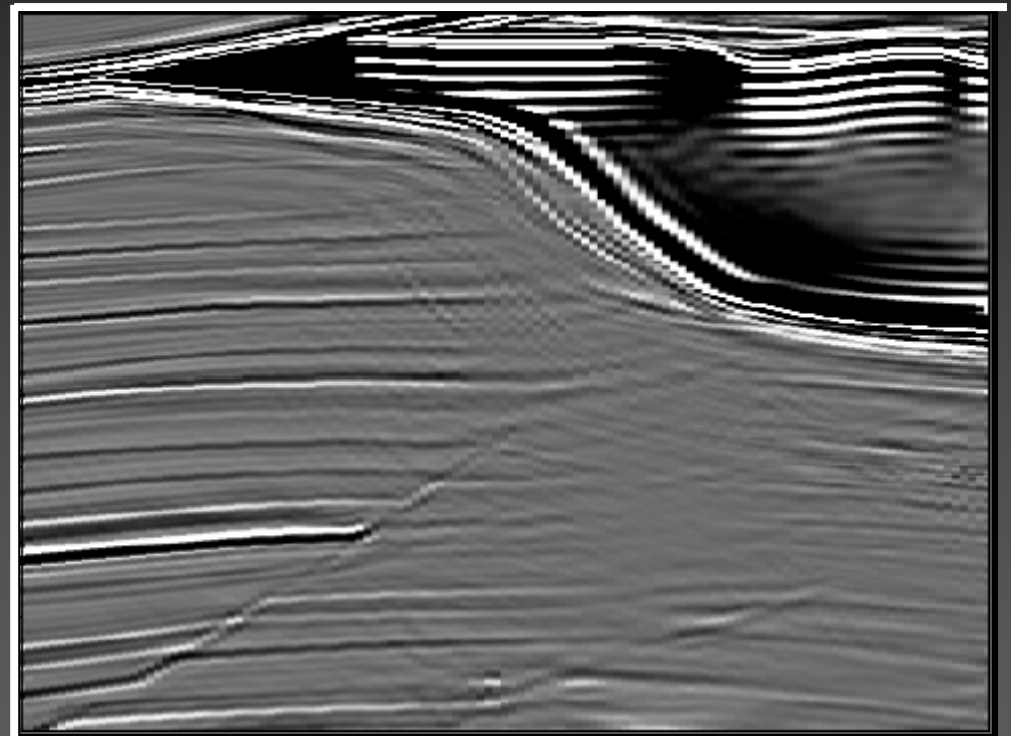
cmp



**Inversion with no multiple attenuation**

Claudio Guerra and Alejandro Valenciano

cmp



**Inversion with multiple attenuation**

SEP-134, p.25

# Initial considerations

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- **Inversion is sensitive to noise**
  - noise does not fit forward modeling
  - slows down convergence
  - dominates the residuals
- **Incorporate the modeling of the noise**
  - physics explains the data
- **Pre-processing**
  - data fits the physics
- **Multiples**
  - not modeled by the one-way wave equation

# Outline

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- **Linear least-squares wave-equation inversion**
- **Multiples in Sigsbee2b**
- **Results**
- **Conclusions**

# Linear least-squares wave-equation inversion

- Valenciano (2008)

$$\mathbf{S}(\mathbf{m}) = \frac{1}{2} \|\mathbf{L}\mathbf{m} - \mathbf{d}_{\text{obs}}\|^2$$

$$\hat{\mathbf{m}} = (\mathbf{L}^*\mathbf{L})^{-1}\mathbf{L}^*\mathbf{d}_{\text{obs}} = (\mathbf{L}^*\mathbf{L})^{-1}\mathbf{m}_{\text{mig}} \quad \partial^2\mathbf{S}(\mathbf{m})/\partial\mathbf{m}^2 = \mathbf{L}^*\mathbf{L} = \mathbf{H}$$

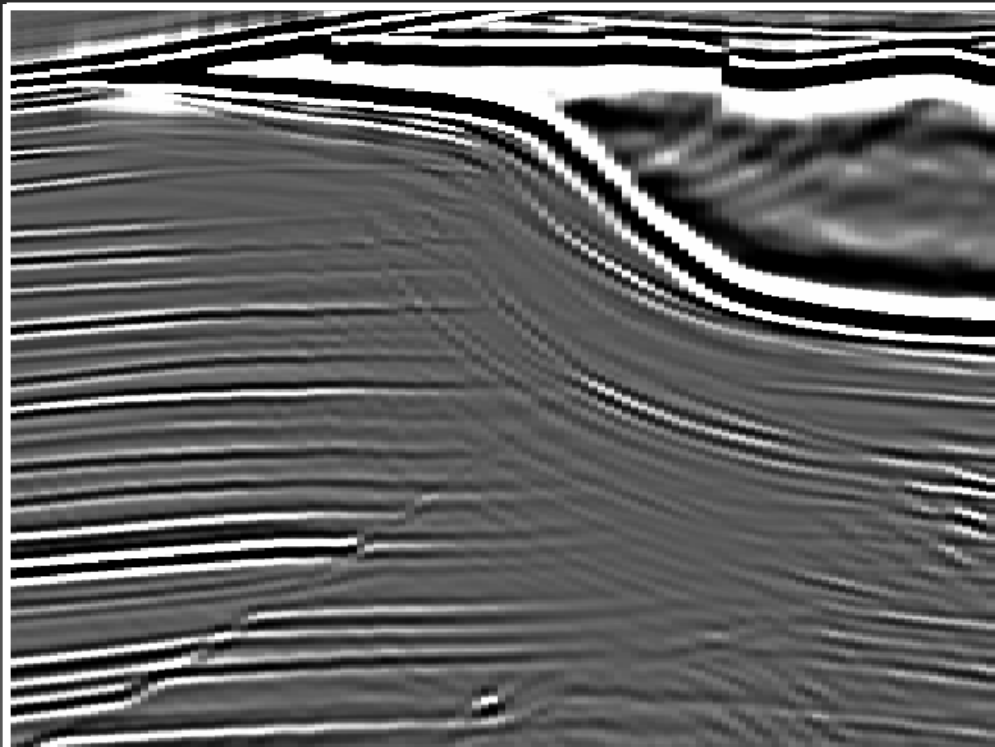
$$\mathbf{H}\hat{\mathbf{m}} = \mathbf{m}_{\text{mig}}$$

$\mathbf{L}$  – modeling operator  
 $\mathbf{S}(\mathbf{m})$  – cost function  
 $\mathbf{m}_{\text{mig}}$  – migrated image

$\mathbf{L}^*$  – migration  
 $\mathbf{H}$  – offset-domain Hessian  
 $\hat{\mathbf{m}}$  – inverse image

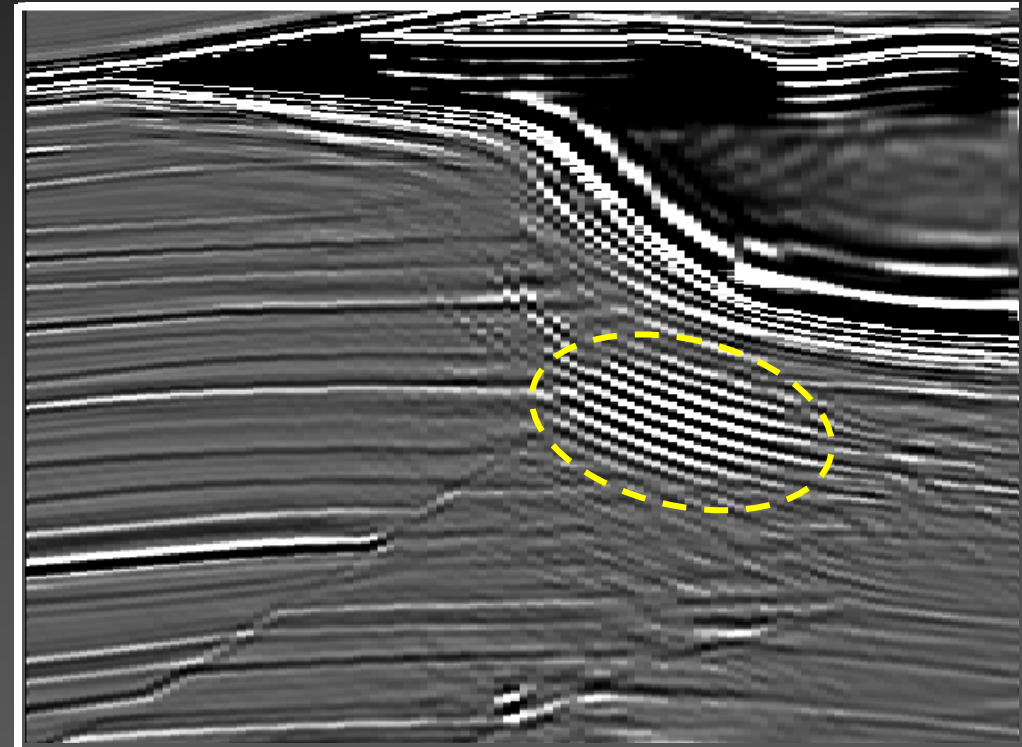
# Linear least-squares wave-equation inversion

cmp



Shot-profile migration

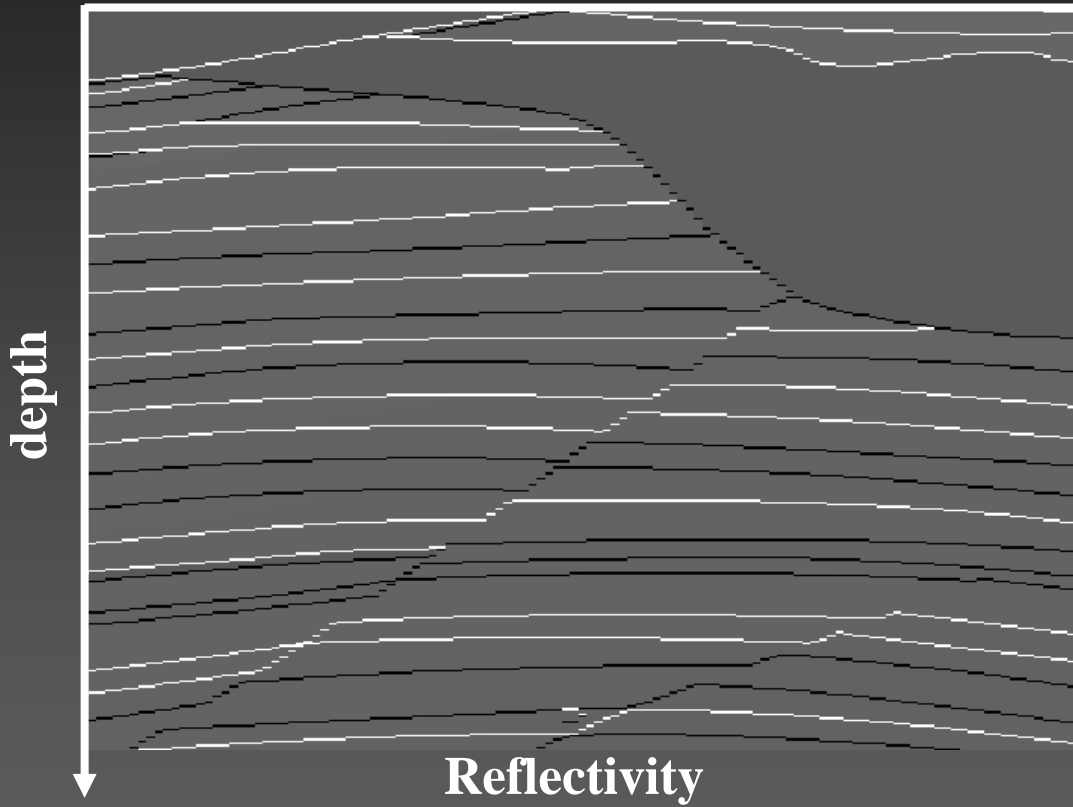
cmp



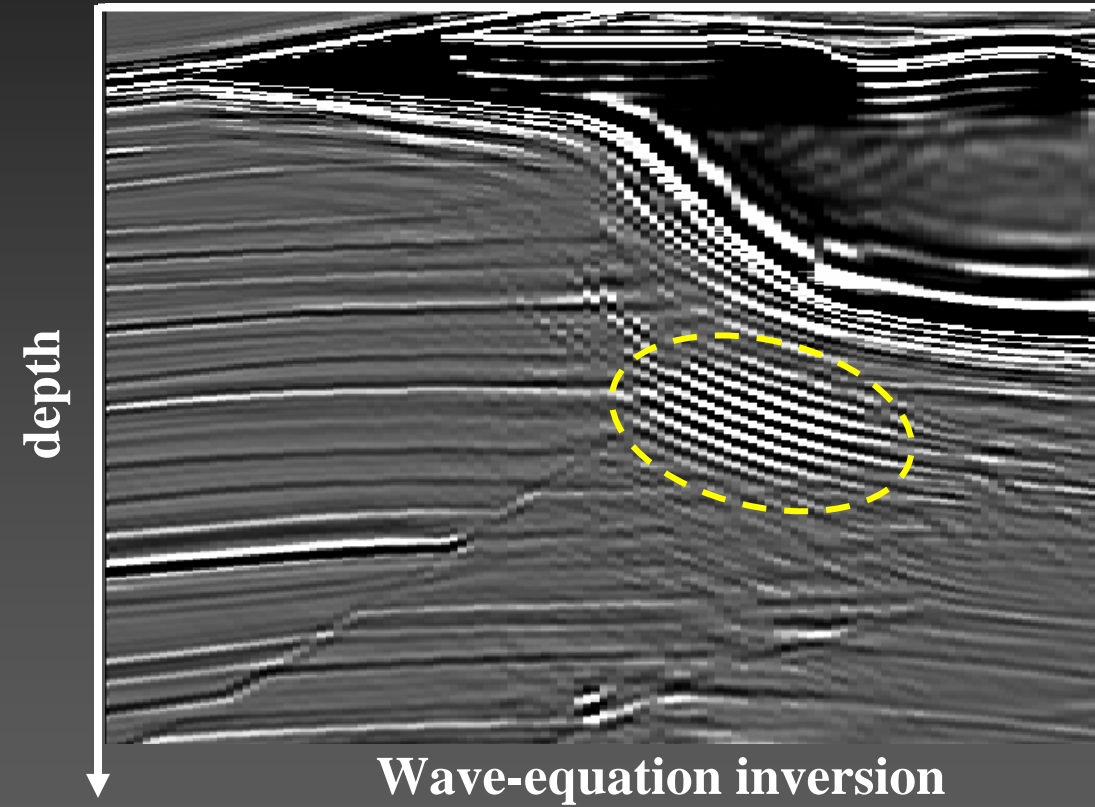
Wave-equation inversion

# Linear least-squares wave-equation inversion

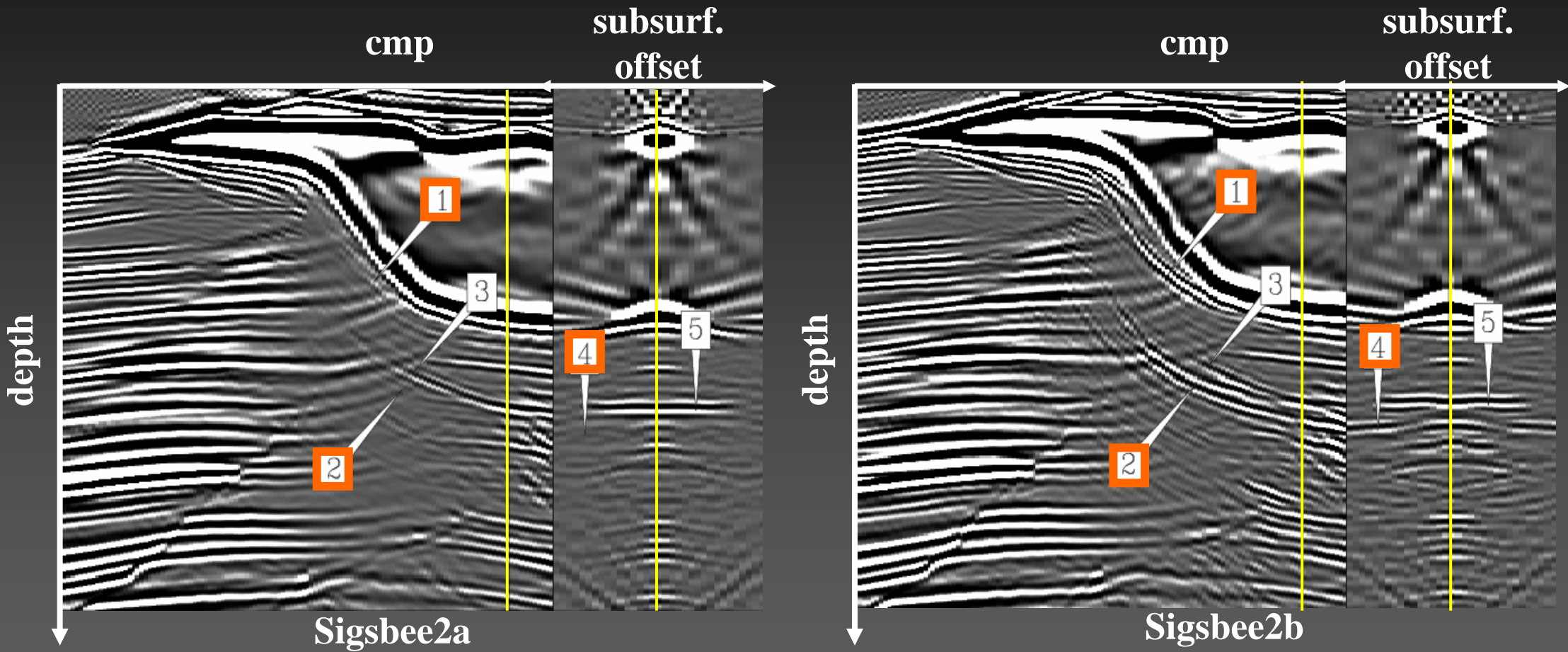
cmp



cmp

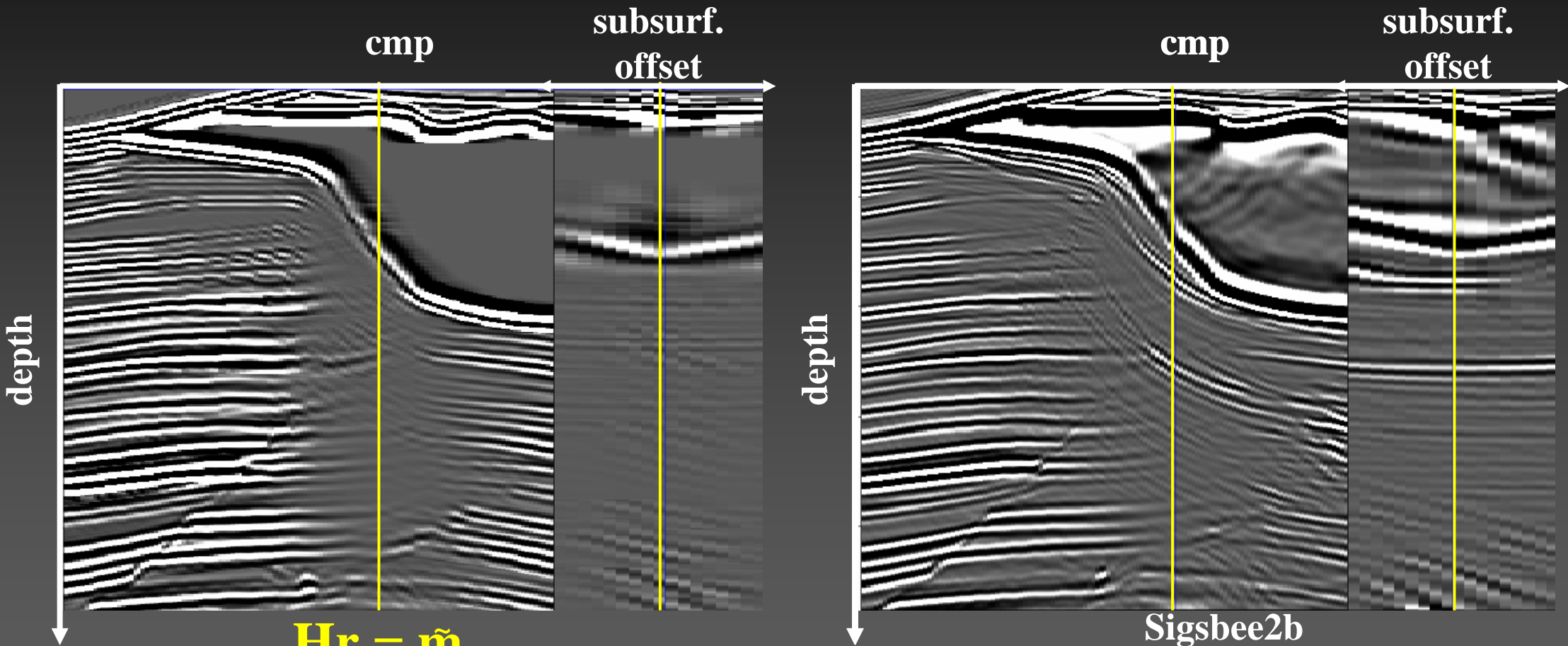


# Multiples in Sigsbee2b



1, 2 and 4 – peg-leg multiples  
3, 5 – internal multiples ...

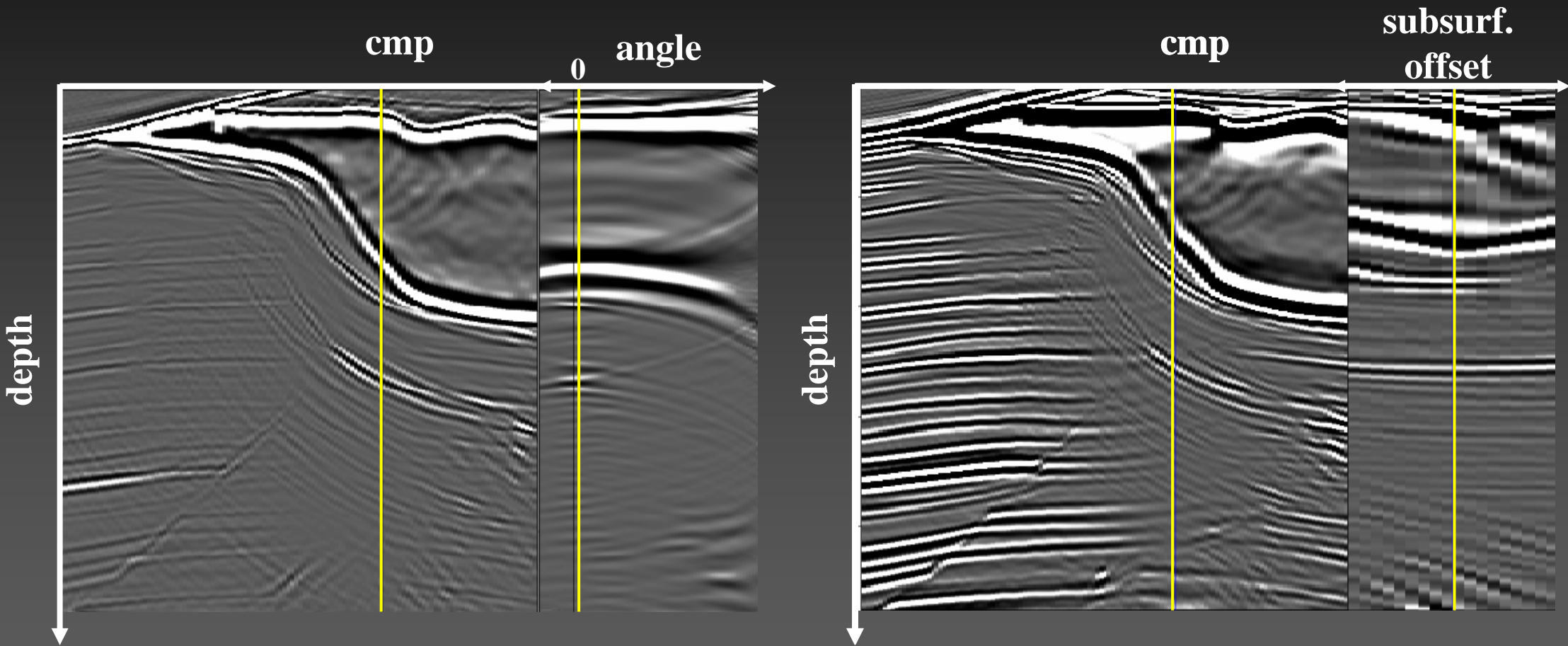
# Multiples in Sigsbee2b



$$Hr = \tilde{m}_{mig}$$

- 1, 2 and 4 – peg-leg multiples
- 3, 5 – internal multiples ...
- or migration artifacts (?)

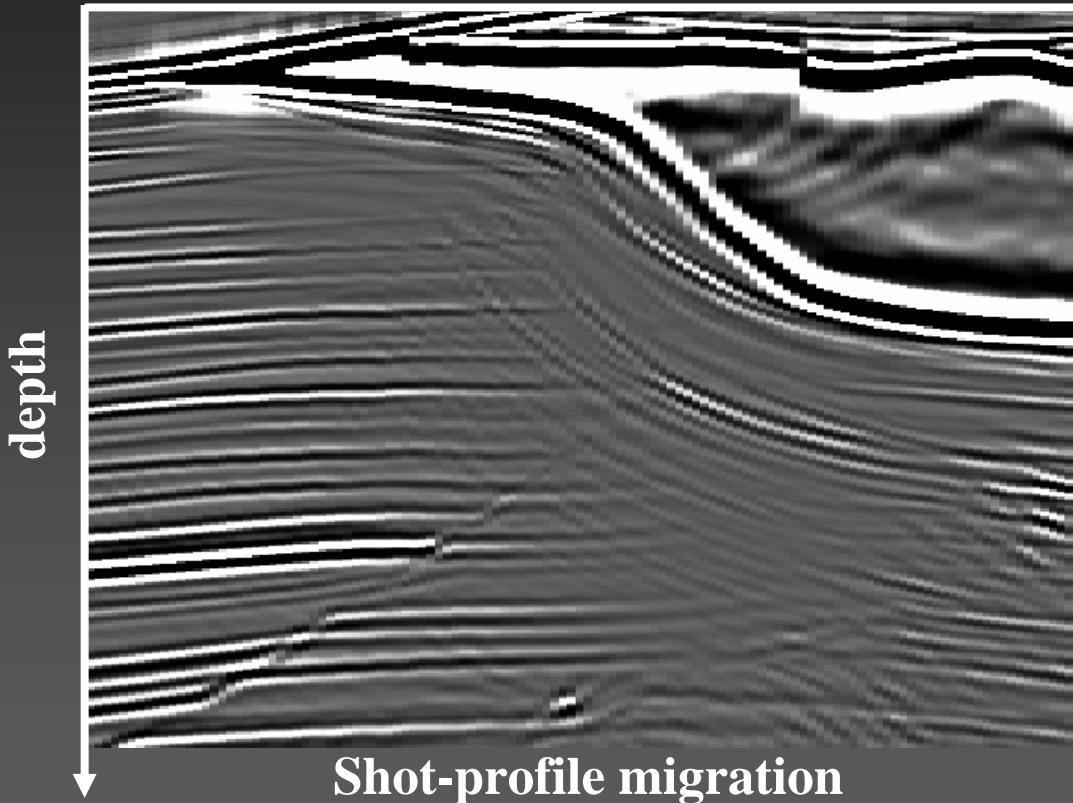
# Multiples in Sigsbee2b



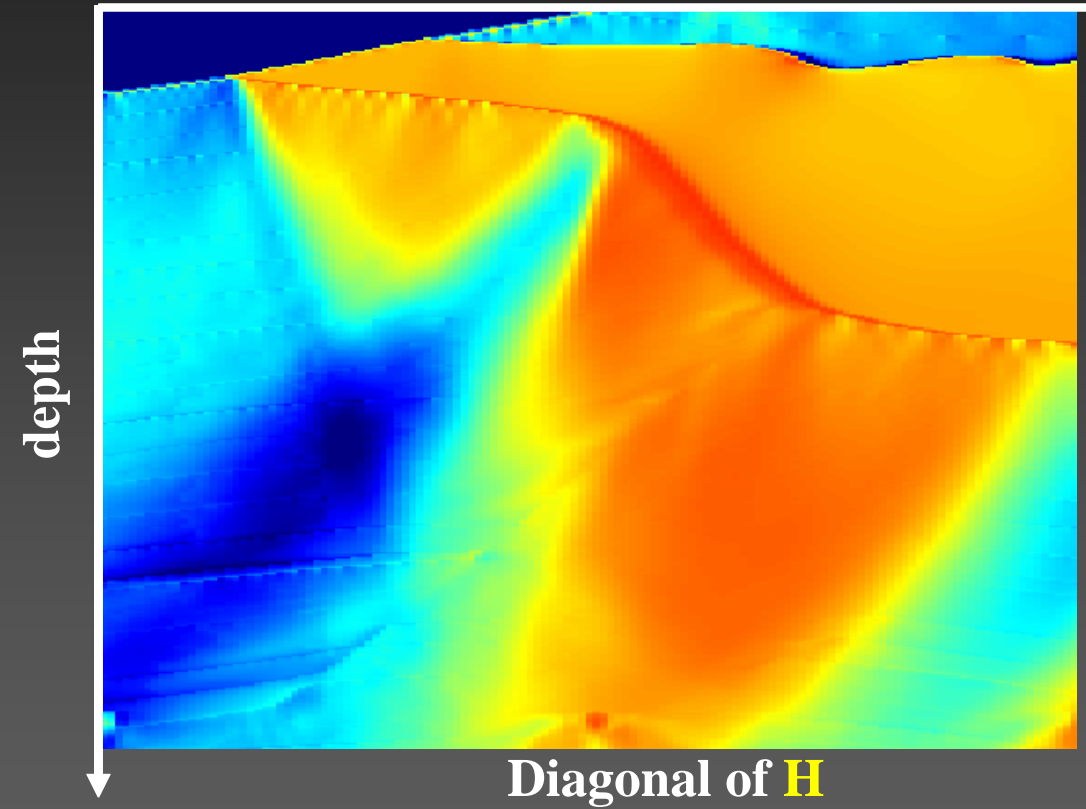
1, 2 and 4 – peg-leg multiples  
3, 5 – internal multiples ...  
or migration artifacts (?)

# Multiples in Sigsbee2b

cmp



cmp



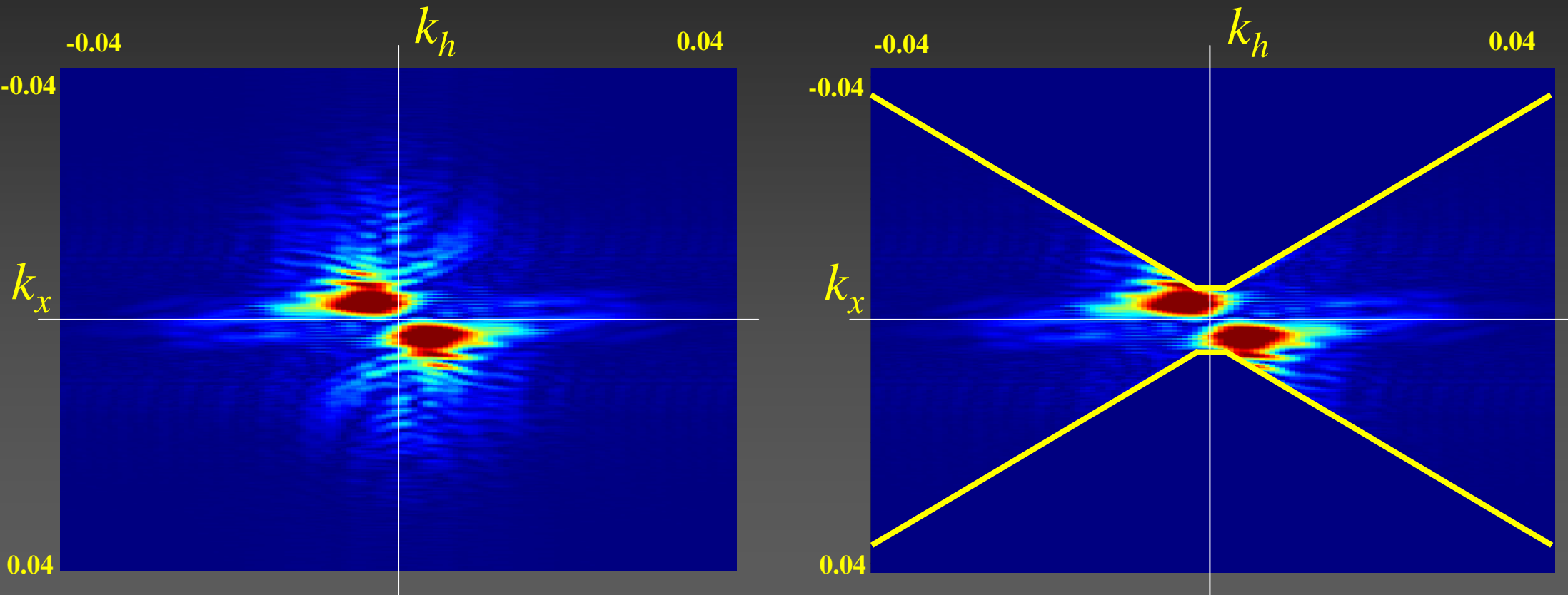
# Multiples in Sigsbee2b

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- Multiples are more pronounced in low illumination areas
  - so are the migration artifacts ...
- Multiples are flat in the subsurface-offset gathers (**low  $k_h$** ) and dipping in the common-subsurface-offset section (**high  $k_x$** )
- Primaries are dipping in the subsurface-offset gathers (**high  $k_h$** ) and dip in different directions in a common-subsurface-offset section

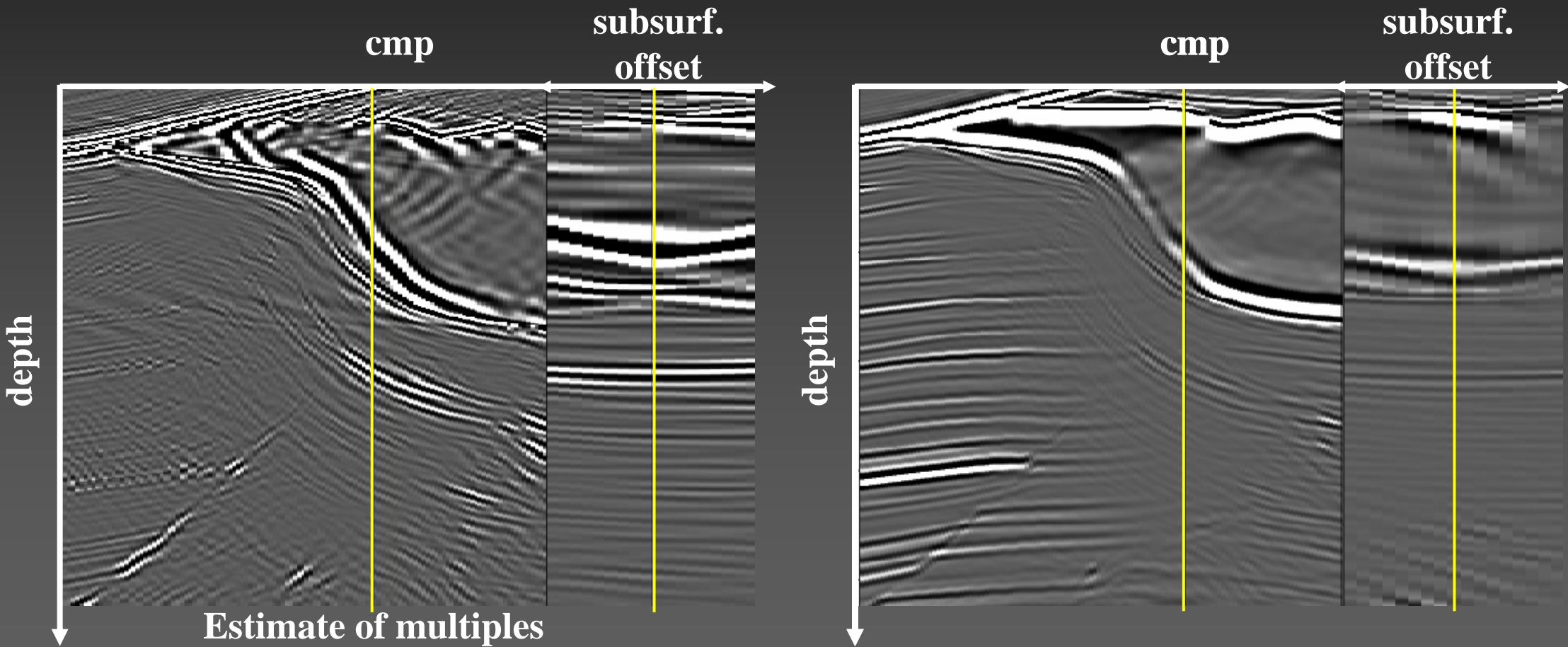
# Results

- Separate primaries and multiples in the  $k_x-k_h$  domain
  - generate an estimate for the multiples



# Results

- Separate primaries and multiples in the  $k_x-k_h$  domain
  - generate an estimate for the multiples



# Results

- Adapt the amplitude and phase of the estimated multiple (Alvarez and Guitton, 2006)

$$\begin{bmatrix} \mathbf{M} & \mu\mathbf{P} \end{bmatrix} \begin{bmatrix} \mathbf{f}_m \\ \mathbf{f}_p \end{bmatrix} \approx \mathbf{0}$$
$$\varepsilon\mathbf{A} \begin{bmatrix} \mathbf{f}_m \\ \mathbf{f}_p \end{bmatrix} \approx \mathbf{0}$$

$\mathbf{M}$  – estimated multiples

$\mathbf{P}$  – estimated primaries

$\mu$  – parameter to balance  $\mathbf{M}$  and  $\mathbf{P}$

$\varepsilon$  – amount of regularization

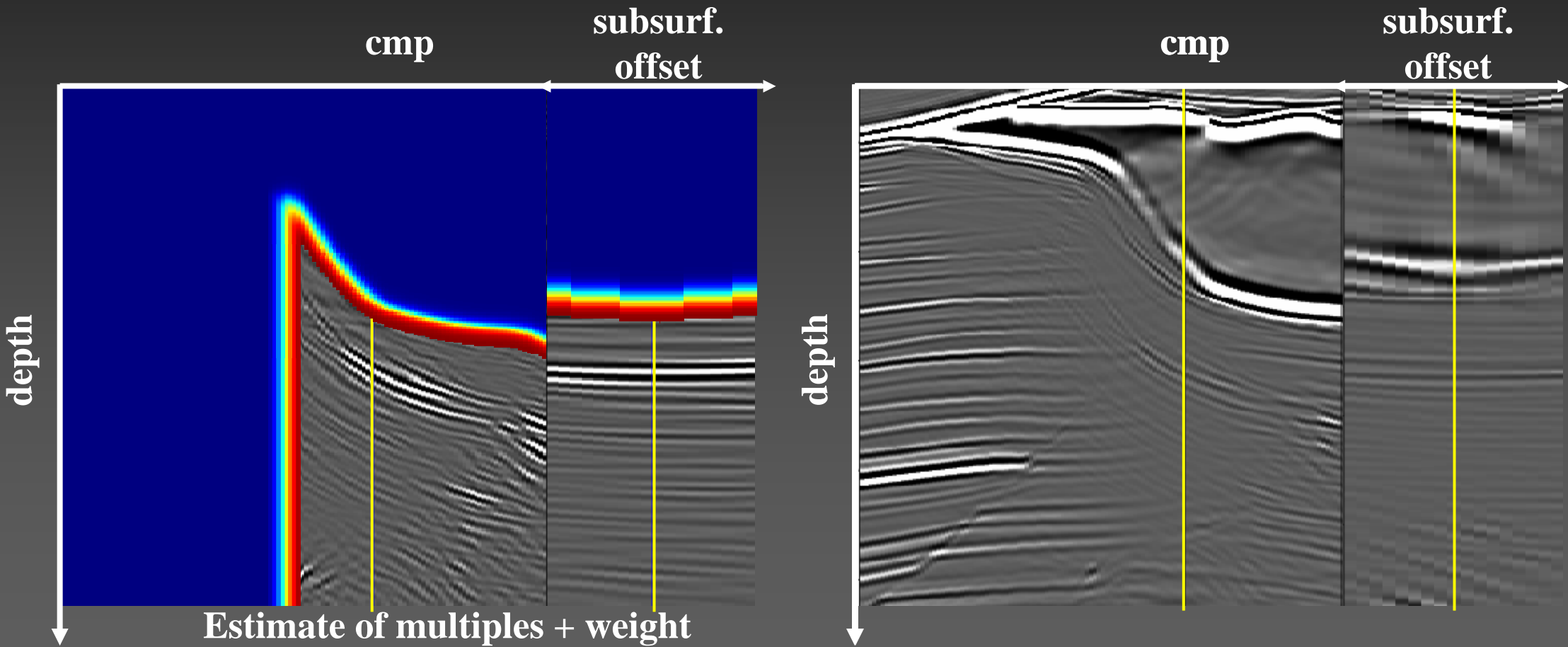
$\mathbf{f}_m$  – matching filter for the multiple

$\mathbf{f}_p$  – matching filter for the primary

$\mathbf{A}$  – Laplacian operator

# Results

- Adapt the amplitude and phase of the estimated multiple – using weights

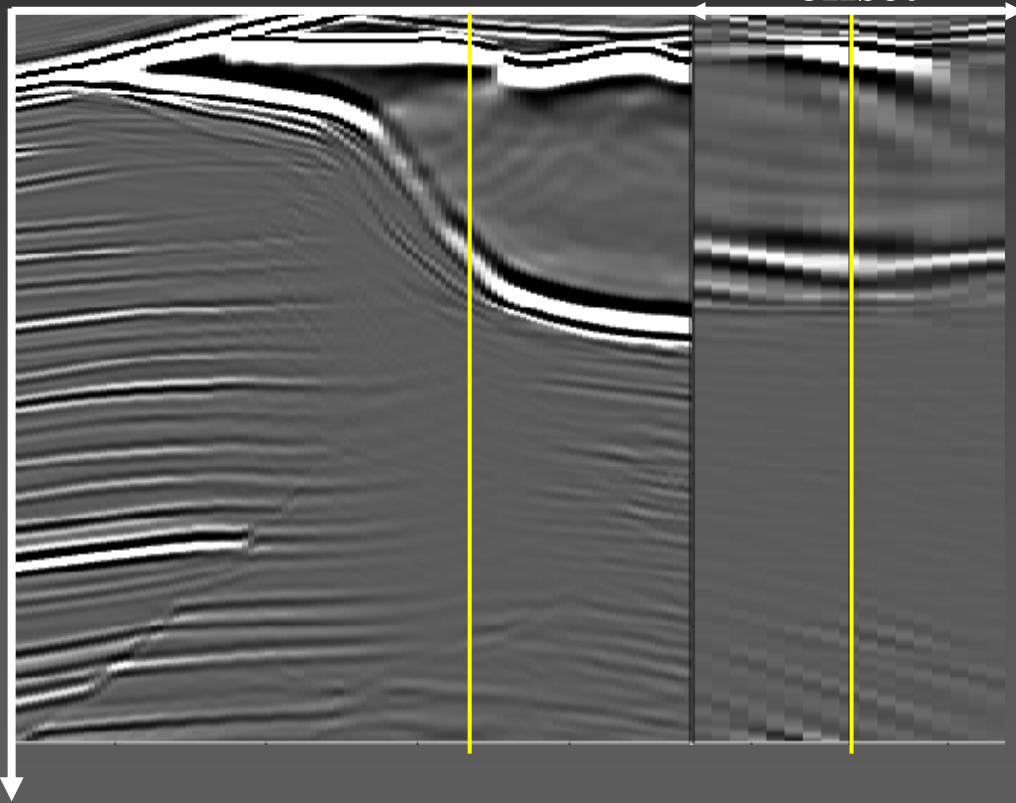


# Results

**Filtered**

cmp

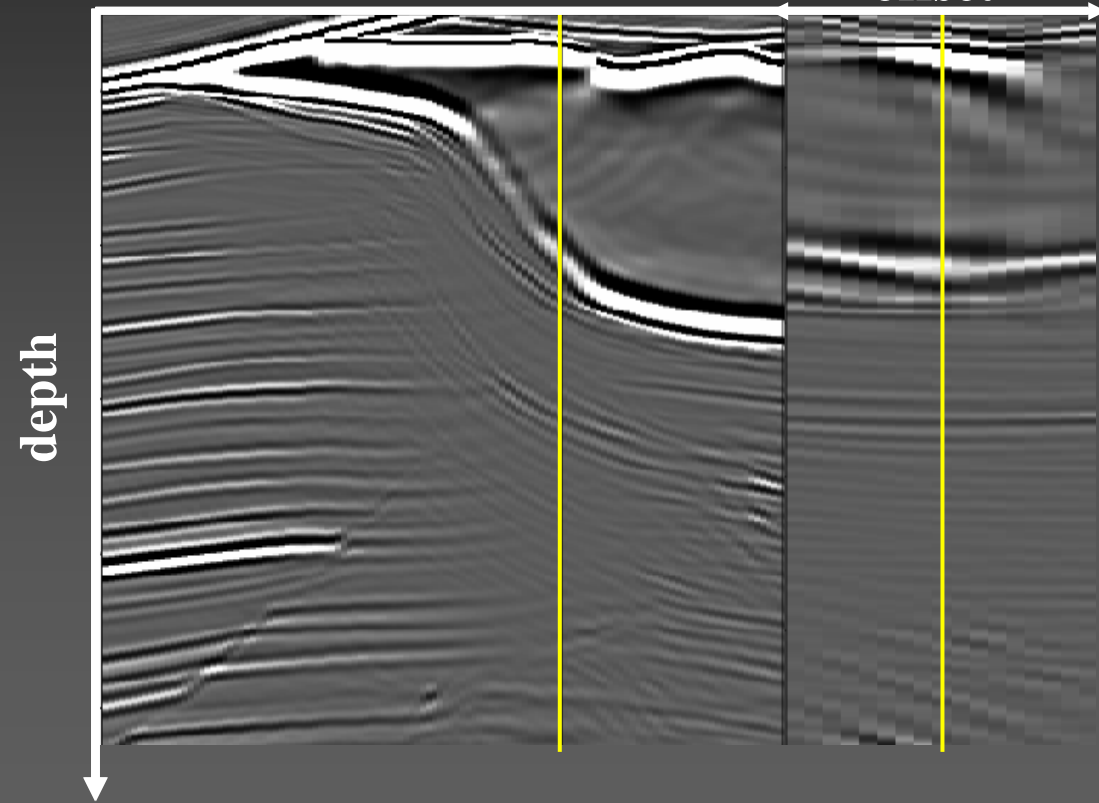
subsurf.  
offset



**Unfiltered**

cmp

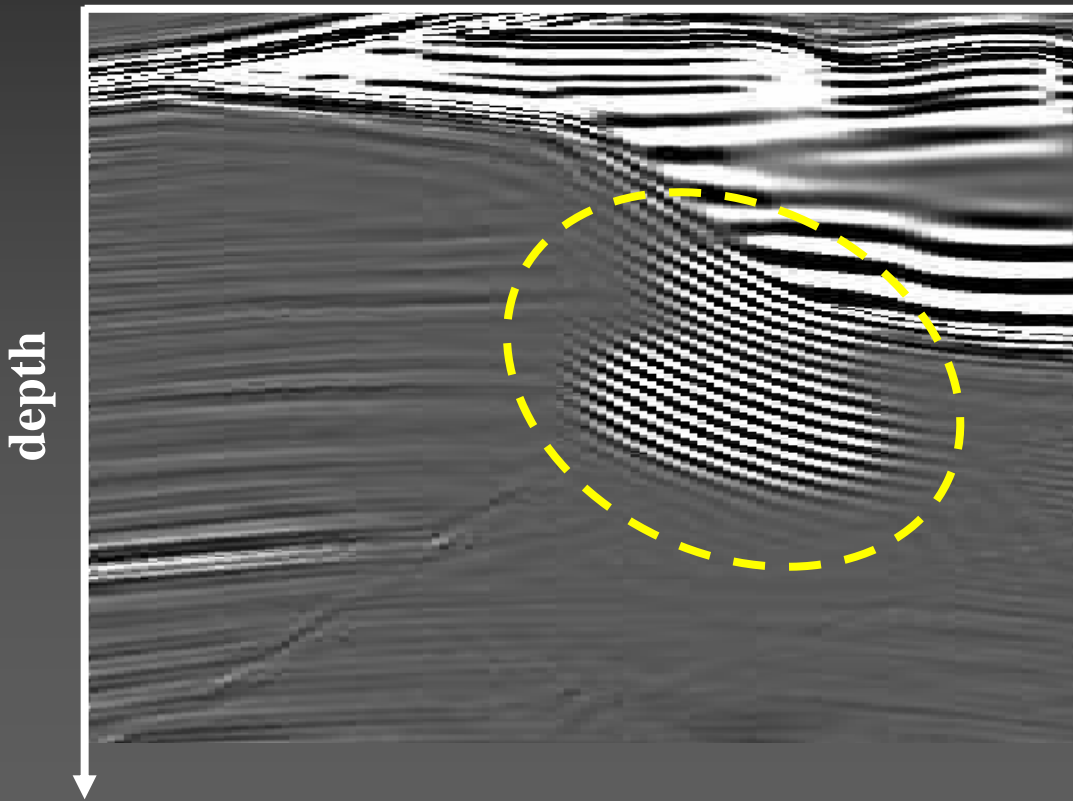
subsurf.  
offset



# Results

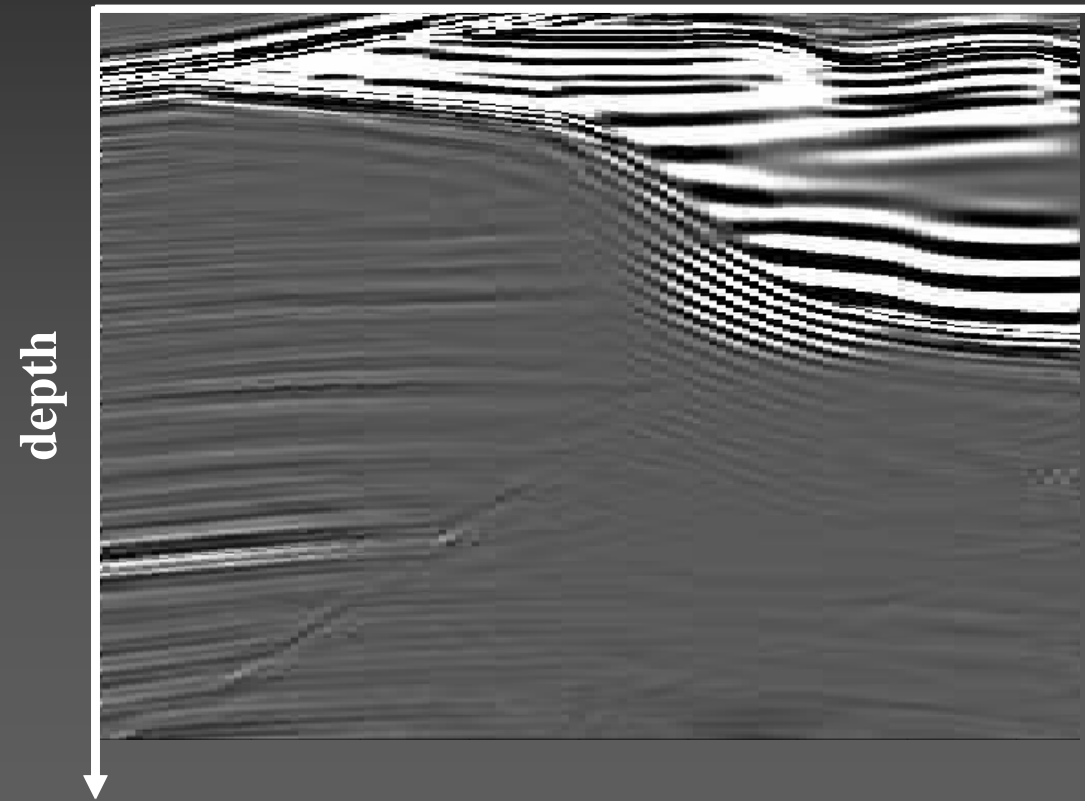
**Residuals of the  
unfiltered inversion**

cmp



**Residuals of the  
filtered inversion**

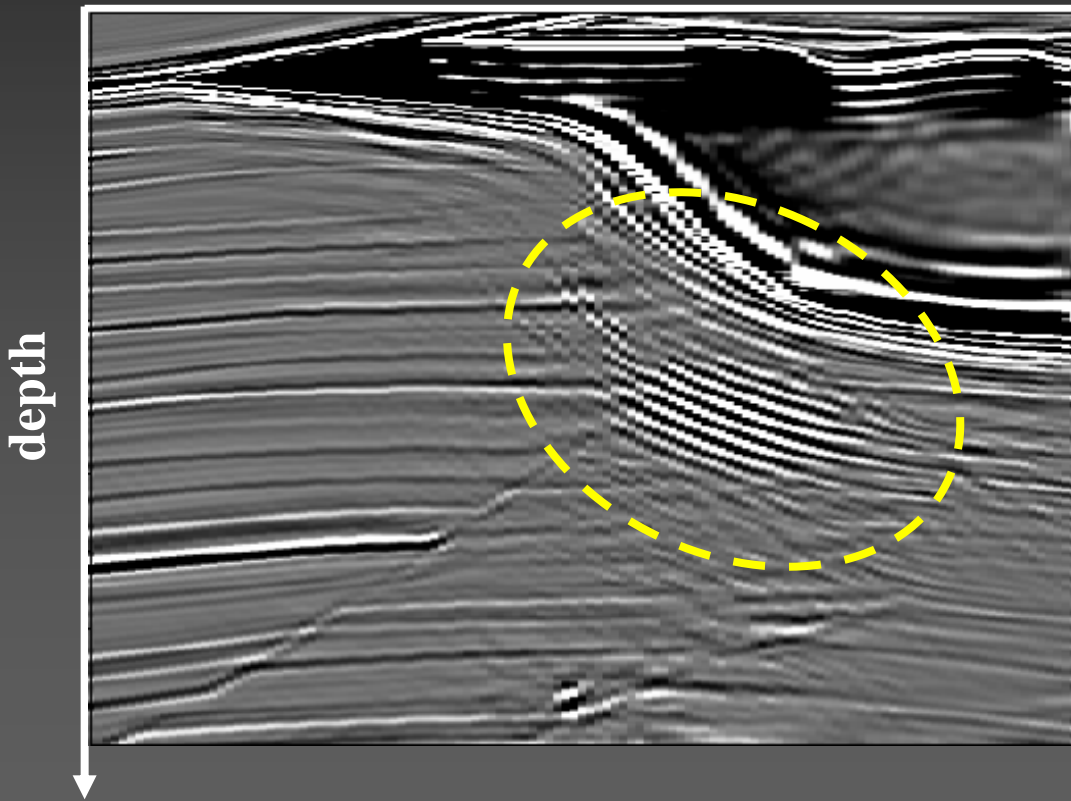
cmp



# Results

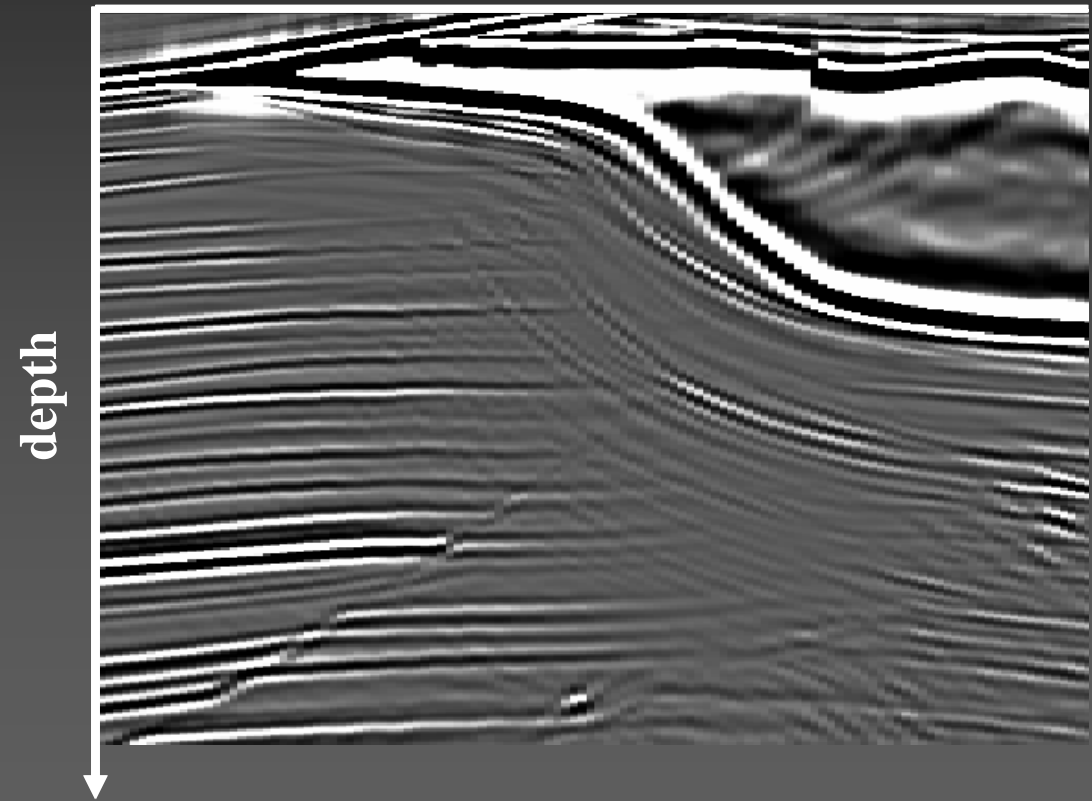
**Unfiltered  
inversion**

cmp



**Unfiltered  
migration**

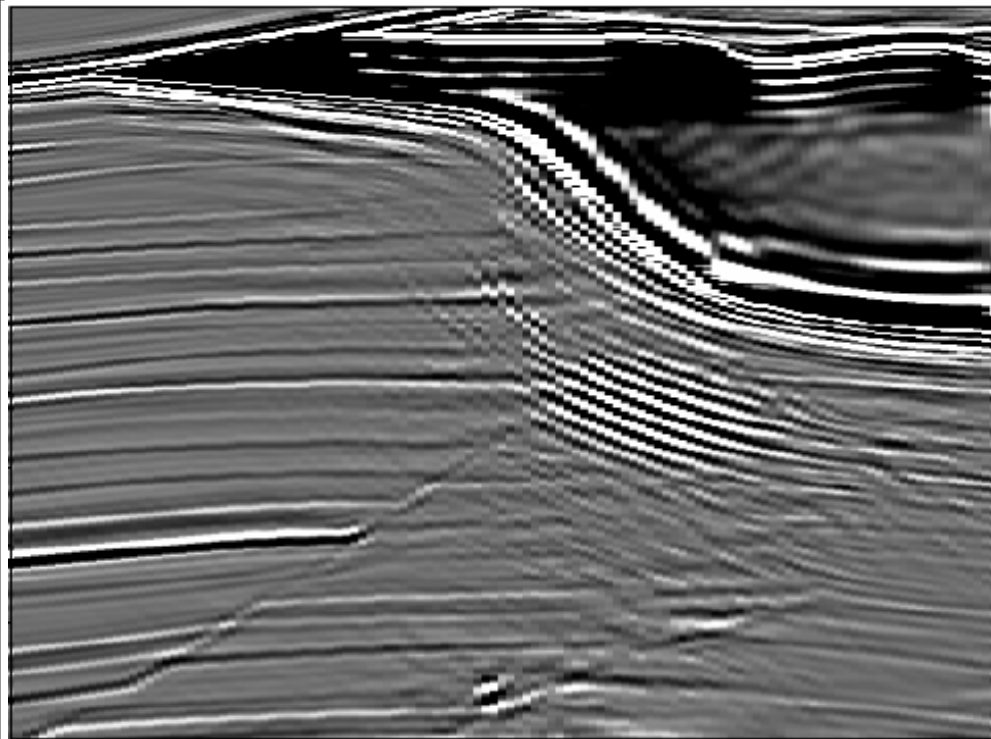
cmp



# Results

**Unfiltered  
inversion**

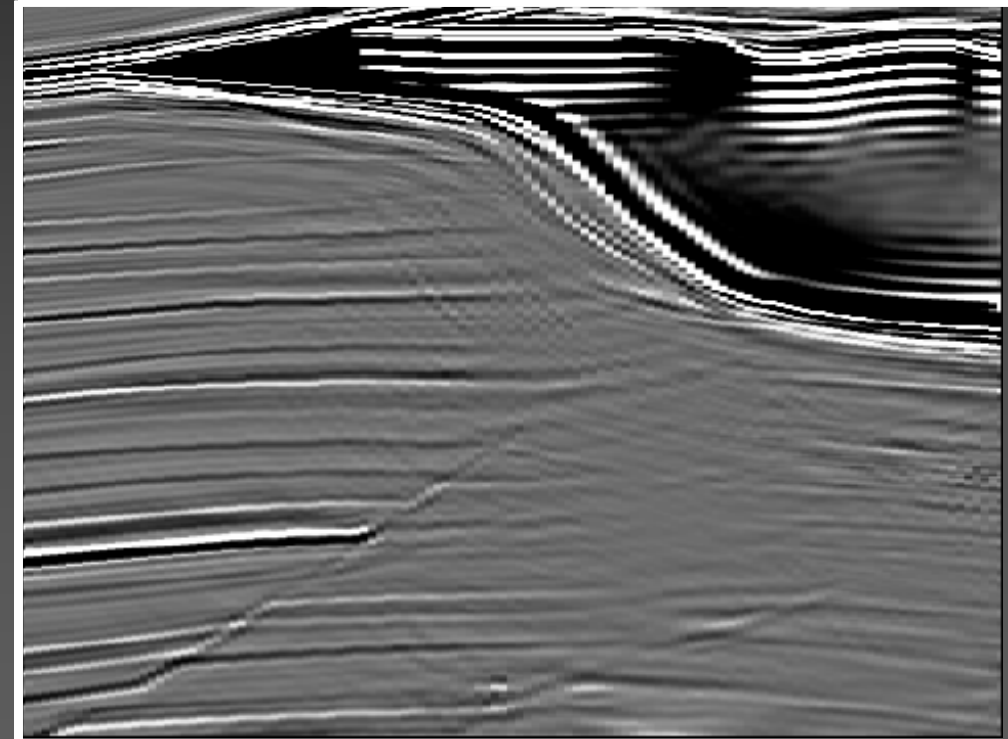
cmp



depth  
↓

**Filtered  
inversion**

cmp

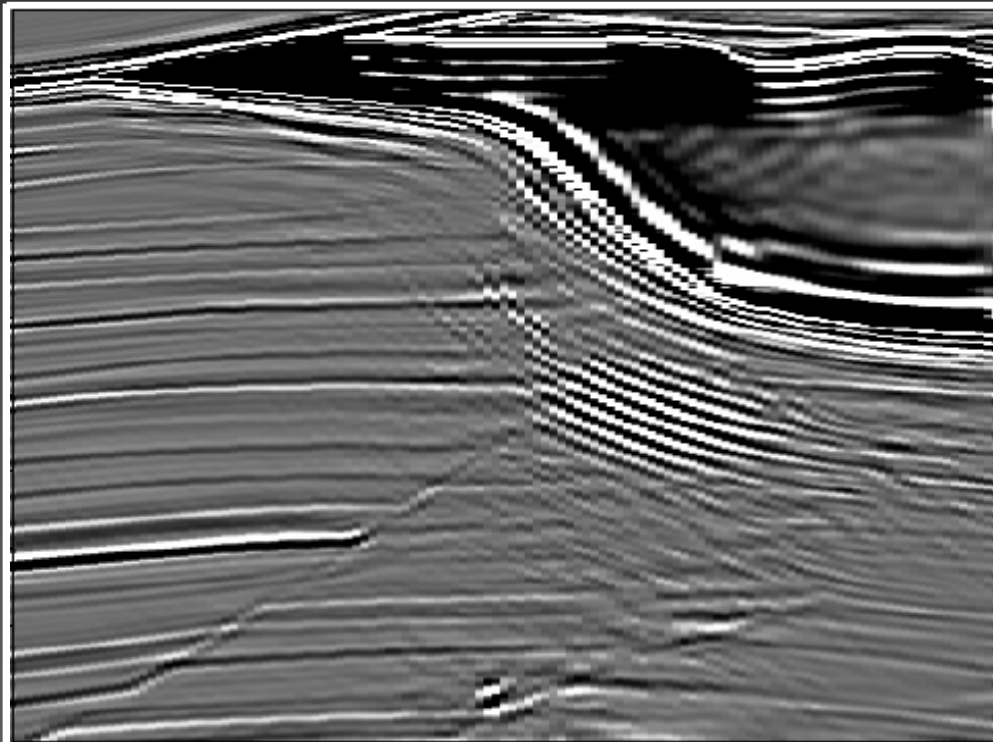


depth  
↓

# Results

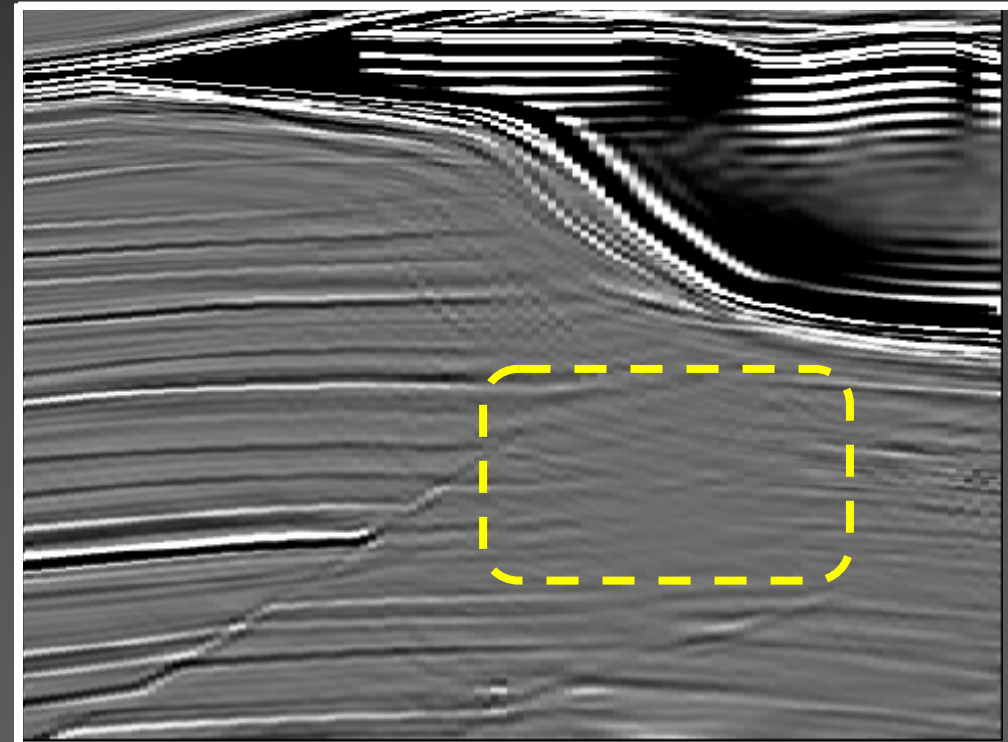
**Unfiltered  
inversion**

cmp



**Filtered  
inversion**

cmp



# Conclusions

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- Multiples were characterized in the subsurface-offset domain;
  - distinguishable from primaries using dip information
  - ... but not from illumination artifacts
- Multiples are not predicted by the one-way modeling;
- Preprocessing largely improved the inversion results; and
  - ... but also filtered useful energy for inversion
- Need for a more robust formulation.



# Multiples in Sigsbee2b

- Adapt the amplitude and phase of the estimated multiple – using weights

