Acoustic daylight imaging via spectral factorization: Helioseismology and reservoir monitoring

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The earth and the sun both have a noisy surface and seismic velocity that increases with depth. Acoustic noise generates some waves that dive into the sphere and emerge at all distances. A process that we call acoustic daylight imaging enables us to form pictures of the interior. (Note: The term *acoustic daylight imaging* is used in oceanography but refers to a different concept.)

The Stanford Exploration Project (SEP) became interested in acoustic daylight imaging many years ago when theory showed that under certain idealized conditions, the autocorrelation of an earthquake seismogram should mimic an exploration shot record. This leads to the following conjecture:

> By crosscorrelating noise traces recorded at two locations on the surface, we can construct the wavefield that would be recorded at one of the locations if there was a source at the other.

In 1987, Steve Cole tried to verify this conjecture on a passive 3-D survey, recorded with an array of 4096 geophones, donated by Amoco, that covered more than a half kilometer square on the Stanford campus.

Nobody expected the geophones to record plunging waves from great distances, but that is exactly what happened. Cole saw seismic waves apparently coming from the midwestern United States. Earthquake seismologists were surprised to learn that a dense array could receive seismic waves from so far at such high frequency (10 Hz) because they cannot, due to their small numbers of seismometers.

Unfortunately, he was not able to observe the much smaller-scale reflected waves that could be crosscorrelated within the array. Such waves would illuminate the area within drilling distance and so provide the proof of concept that would interest SEP's industrial sponsors.

There are several possible reasons why the passive seismic data recorded in this experiment could not be used for imaging. First, due to battery limitations, only 20 minutes of passive seismic data were recorded. Much longer time series may be needed to bring the signals of interest above the noise level. Second, the proximity of the San Andreas Fault makes the Stanford area structurally complex and difficult to analyze. Additionally, the experiment was troubled by poor coupling between the geophones and the dry summer soil.

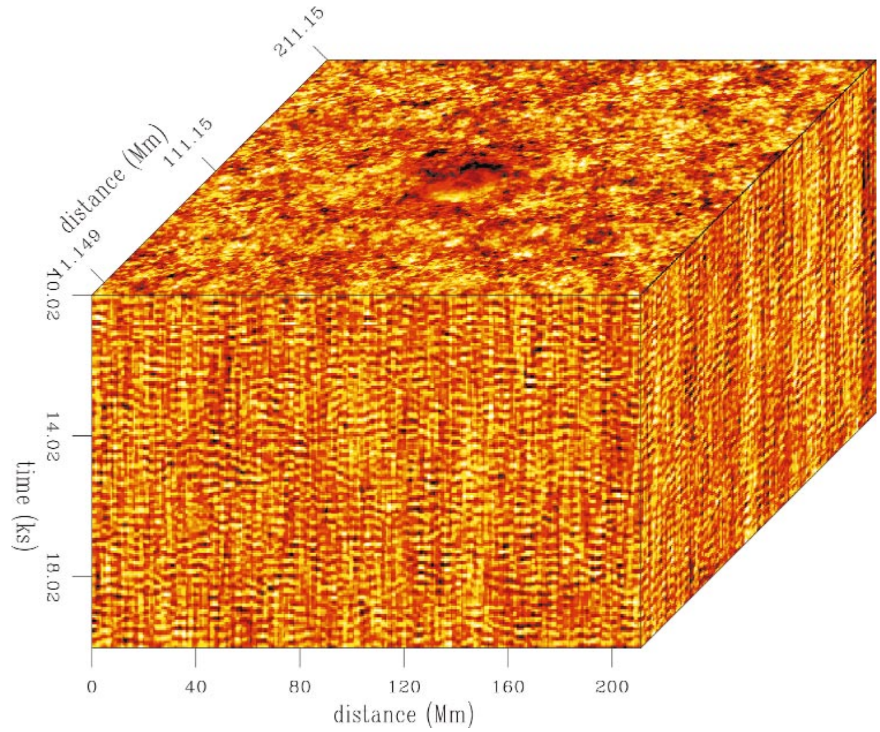

Figure 1. Cube of raw MDI data. The object in the center is a sunspot. Distances are megameters (thousands of km), and time units are kiloseconds.

Helioseismology. In 1995, solar physicists developed a new instrument for studying the sun—the Michelson Doppler Imager (MDI), which measures the Doppler shift of solar absorption lines formed in the lower part of the solar atmosphere. This provides line-of-sight velocity measurements for points on the sun's surface that can be used to study solar oscillations. MDI amounts to having a million (1024 x 1024) seismometers uniformly distributed on the surface of the sun. Furthermore, the solar seismologists are able to zoom their lens to reposition their million virtual seismometers to give a magnified view anywhere they choose.

Most helioseismology is done in the frequency domain with spherical harmonic functions. Spherical harmonics are excellent for studying the whole sun at one time. However, small-scale events are described only by harmonic modes of very high order. Spherical harmonic functions are, therefore, inefficient for studying small, localized areas of the sun's surface (e.g., around sunspots).

Solar seismologists have also come up with the idea of creating "time-distance" seismograms by crosscorrelating surface-noise observations to mimic impulsive sources on the solar surface. Convective flow in the outer third of the sun leads to a breakdown in reciprocity of time-distance seismograms derived by crosscorrelation. Helioseismologists have used this breakdown in reciprocity to estimate the 3-D structure of flow velocity structure in this region of the sun.

Helioseismologists were successful with acoustic daylight imaging, with real data in three dimensions, before geophysicists could do it on earth. Their results provide the proof of concept for the original conjecture and stand as a challenge to terrestrial geophysicists. If it works on the sun, can it work on the earth?

Permanent recording installations. Earthquake seismologists have installed many permanent seismometers around the world; however, these are typically point receivers or sparse arrays. Dense, exploration-style field arrays with many receiver stations have two critical advantages over point receivers for acoustic daylight imaging. First, array directivity can be controlled to remove energy trapped in the near surface. Second, their crosscorrelations will provide dense data coverage in the zone of interest.

The success of two modern exploration technologies, time-lapse seismic reservoir monitoring (often called 4-D seismic) and four-component (4-C) ocean-bottom seismometers, means that exploration companies are deploying an increasing number of permanent recording arrays throughout the world. Permanent arrays should be more repeatable than conventional surveys for time-lapse monitoring (and the initial extra financial outlay is partly compensated for by the lower costs for repeated surveys).

Typically, these receivers are turned on only when surveys are shot. The challenge is to leave them turned on and to take advantage of continual recording and acoustic daylight imaging to continually monitor reservoir production. Computing technology is cheap, so a PC could be left out in the field to crosscorrelate traces in real time.

MDI data. Although helioseismologists have provided the proof of concept for acoustic daylight imaging, they have only recently started to study time-distance seismograms. Exploration geophysicists, on the other hand, have been dealing with these data for many years. Consequently, the Solar Oscillations Investigation (*http://soi.stanford.edu/*) provided us with a cube of MDI data with the hope that our experience could expedite its analysis.

Figure 1 shows a cube of raw MDI data. It has been transformed to

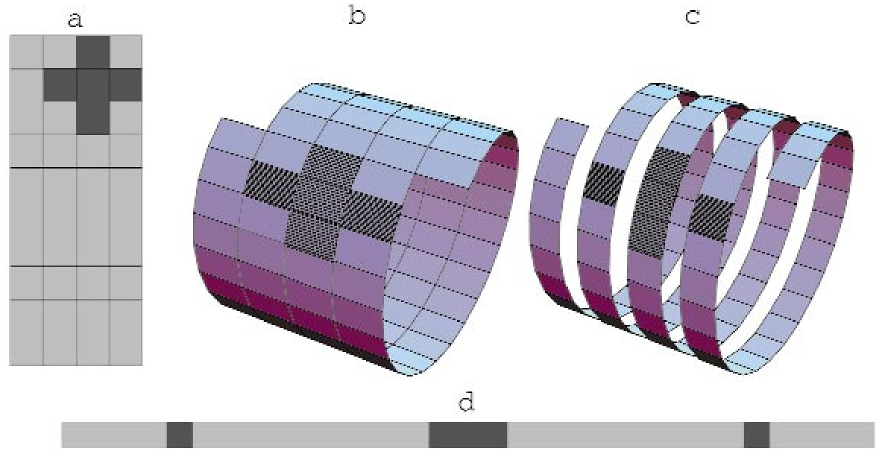

Figure 2. Helical boundary conditions mapping a 2-D function (a) onto a helix (b), and then unwrapping the helix (c) into an equivalent 1-D function (d). (Figure by Sergey Fomel.)

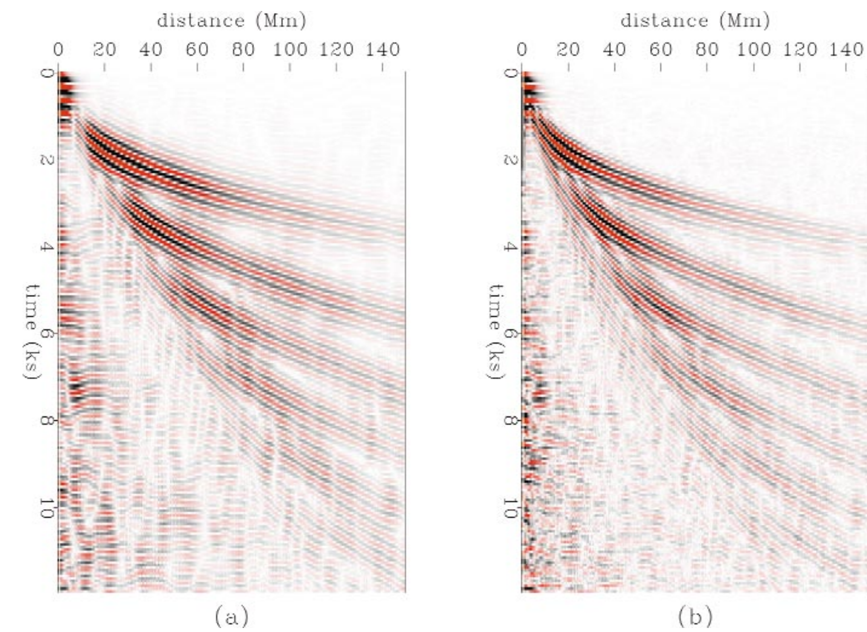

Figure 3. Two estimates of the solar impulse response. Panel (a) was derived in the conventional way by crosscorrelating traces. Panel (b) was derived by multidimensional Kolmogorov spectral factorization.

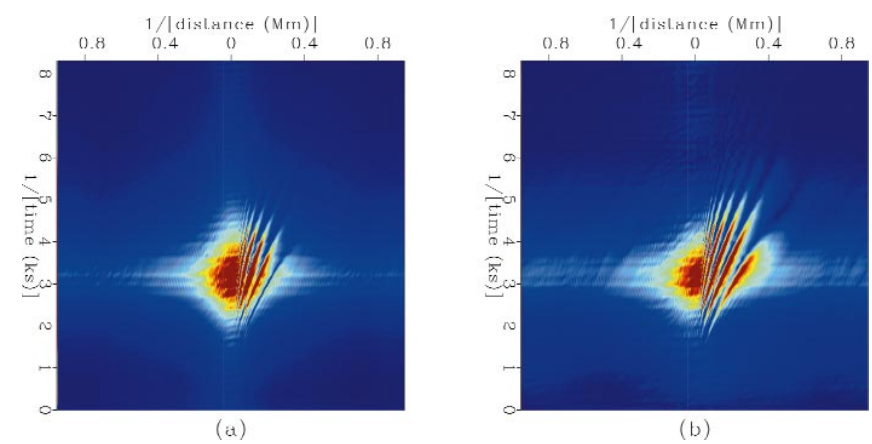

Figure 4. Amplitude spectra of the impulse responses in Figure 3. Panel (a) was derived by crosscorrelating traces. Panel (b) was derived by Kolmogorov spectral factorization.

Cartesian coordinates by projecting high-resolution data from an area approximately 18° square onto a tangent plane. The object in the center of the time slice at the top of the cube is a sunspot. The sampling interval is 1 minute on the time axis and approximately 68 km on the two spatial axes.

Time-varying features of Figure 1 fall into two distinct spectral windows. The low-frequency events (<1.5 mHz) are related to solar convection; higher-frequency events are related to acoustic wave propagation. We were interested in the latter so, as a preprocessing step, we applied a low-cut filter to the data.

3-D Kolmogorov spectral factorization. Solar time-distance curves are usually calculated by crosscorrelating noise traces. Our new idea was to obtain higher-resolution time-distance curves by estimating the minimum-phase impulse response directly from the data.

A simple linear model for the solar oscillations in Figure 1 consists of a convolution of a source function with the impulse response of the sun's surface. The source function is stochastic in nature and may be characterized as being spectrally white in time and space with random phase. The impulse response contains the spectral color, and we expect it to be minimum phase.

If this model holds true, then estimating the source function reduces to estimating a minimum-phase function with the same spectrum as the original data—or, equivalently, multidimensional spectral factorization.

Helical boundary conditions provide a framework for converting a multidimensional problem into an equivalent problem in only one dimension, allowing us to efficiently solve the 3-D spectral factorization. Figure 2 illustrates how helical boundary conditions may transform a 2-D function into one dimension.

We perform the spectral factorization rapidly in the frequency domain in three steps.

1) We transform the multidimensional signal to an equivalent 1-D signal using the helical boundary conditions. The concept is shown in Figure 2; however, rather than map a 2-D function, we map the entire 3-D MDI data into one dimension.
2) We perform a 1-D factorization with Kolmogorov's frequency-domain method.
3) We remap the impulse response back to 3-D space. We reduce wrap-around effects by padding the spatial axes.

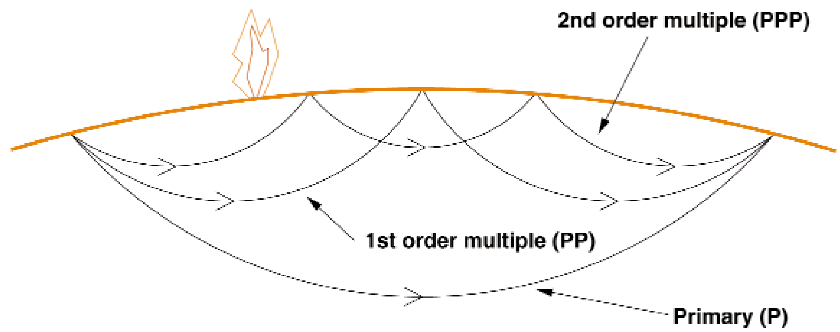

Figure 5. Diving waves of increasing order visible in the solar impulse response.

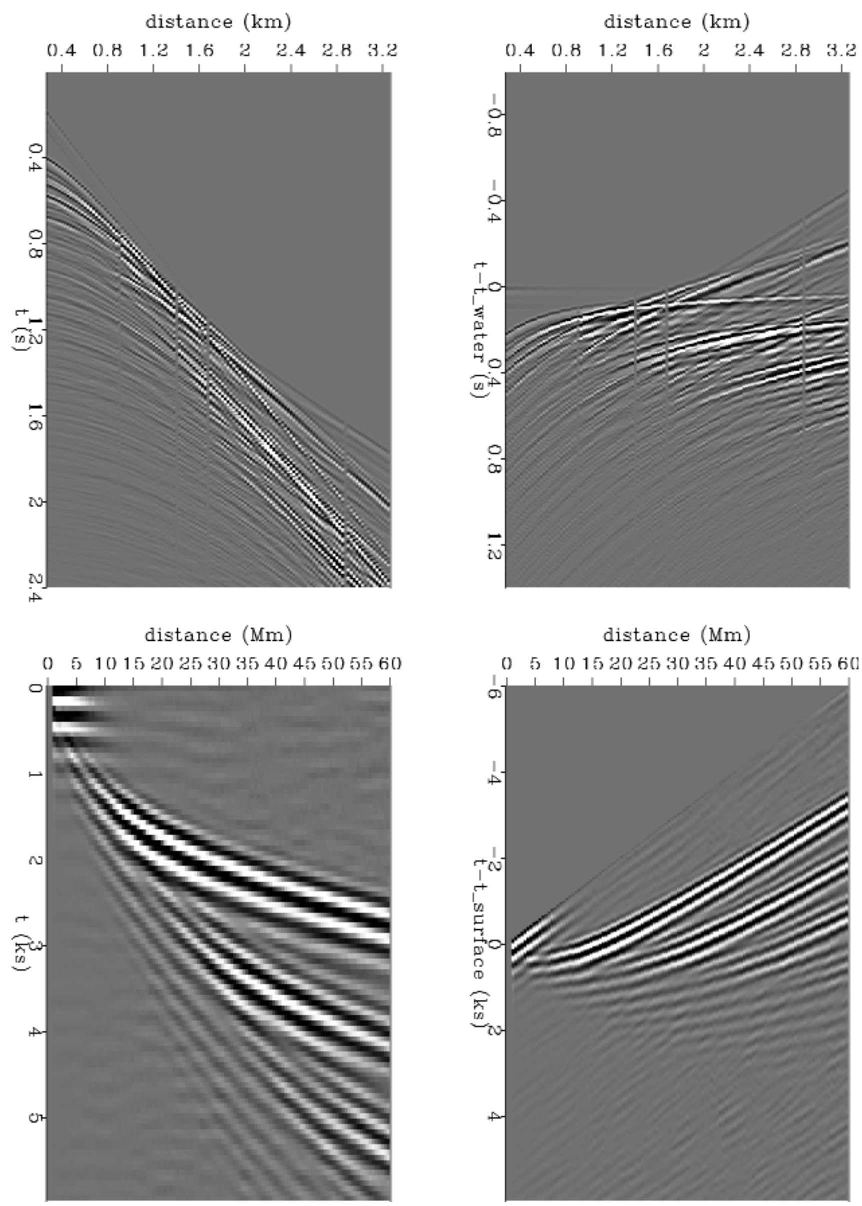

Figure 6. Shot gather from the Gulf of Mexico before (top left) and after (top right) linear moveout with the water velocity. Solar impulse response before (bottom left) and after (bottom right) linear moveout at 10 km/s.

Figure 3 compares the impulse response derived from Kolmogorov spectral factorization and the one derived by crosscorrelation. The Kolmogorov result has a much broader bandwidth. The Kolmogorov result also has the same amplitude spectrum as the original data, but the amplitude spectrum of the crosscorrelation impulse response equals the *power* spectrum of the original data. The raw MDI data have a narrow temporal bandwidth with most of its energy having a period of about five minutes; squaring the amplitude spectrum reduces this bandwidth even more, resulting in the monochromatic appearance of the left panel in Figure 3. Moreover, not just the temporal bandwidth is decreased by crosscorrelating traces; the spatial bandwidth is reduced as well. The steep dips clearly visible near the origin in the right panel of Figure 3 are very heavily attenuated in the crosscorrelation impulse response (left panel). This difference in spatial bandwidth can be also be seen in the amplitude spectra (Figure 4). The temporal bandwidth may be broadened relatively simply by conventional deconvolution, but recovering the full spatial bandwidth in the original data is more difficult.

Interpretation. The lack of sharp velocity contrasts mean there are no clearly observable reflections in the impulse response. The visible events are diving waves (refractions) of increasing order (Figure 5). The first arrival is the direct wave that would be characterized as the "P" arrival in terrestrial geophysics. The second arrival is equivalent to a "PP" event (or first-order multiple) having bounced once on the solar surface. About six distinct arrivals are visible in Figure 3, corresponding to multiples up to fifth order.

Since there are no distinct velocity interfaces (i.e., only a smooth velocity gradient), the head waves continually curve upward. Figure 6 shows the contrast between this behavior and head waves on a shot gather from the Gulf of Mexico. On the terrestrial gathers, distinct linear head waves are visible from several interfaces; on the solar seismogram, the first arrival is a single curved event.

Conclusions. Helioseismology validates a long-standing geophysical conjecture that the crosscorrelation of noise traces may provide impulse-response seismograms. This is an interesting result in itself, but our sponsors trust that we are looking toward matters that can have significant practical and financial implications. Not long ago, applying this concept to hydrocarbon exploration and production would have seemed far-fetched. But advances in electronics and communications have been revolutionizing seismology for decades. Technology now available may make it possible to continually record from permanent seismometers and process these data in real time with acoustic daylight imaging techniques and obtain low-cost continual reservoir monitoring.

Even if this terrestrial goal is still some years in the future, our work did contribute to the helioseismic community by showing that combining traditional 1-D Kolmogorov spectral factorization algorithms with helical boundary conditions allows calculation of improved acoustic impulse response with broader spatial and temporal bandwidth.

Suggestions for further reading. *Helioseismology Beginner's Bibliography* (*http://soi.stanfordedu/papers/heliobiblio.html*). "Passive seismic and drill-bit experiments using 2-D arrays" by Cole (1995 doctoral dissertation, Stanford University; *http://sepwww.stanford.edu/public/docs/sep86/*). *Fundamentals of Geophysical Data Processing* by Claerbout (Blackwell, 1976; *http://sepwww.stanford.edu/public/docs/fgdp/toc_html/*). "Multidimensional recursive filters via a helix" by Claerbout (GEOPHYSICS, 1998). "4-C/4-D at Teal South" by Ebrom et al. (*TLE* 1998).

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