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Implementation of model space weighing for Fourier domain Bidirectional deconvolution approach

Qiang Fu

ABSTRACT

Last time we see why we should use model space weight to compensate the divergence in bidirectional deconvolution. Now let see the implementation for this in Fourier domain Bidirectional deconvolution approach.

IMPLEMENTATION OF FOURIER DOMAIN BIDIRECTIONAL DECONVOLUTION

Claerbout et al. (2011) provide the complete derivation of Fourier domain Bidirectional deconvolution approach step by step. Let us see what need to be changed if we want to include a weighting function $c(t)$ in model domain to compensate the divergence in Fourier domain Bidirectional deconvolution approach.

original implementation without model space weighing

Our goal for Fourier domain Bidirectional deconvolution is to minimize the object function

$$J = \sum_t H(r_t). \tag{1}$$

And $H(r_t)$ is the hyperbolic penalty function

$$H(r_t) = \sqrt{r_t^2 + R_{thr}^2} - R_{thr}, \tag{2}$$

where R_{thr} is a constant threshold for determining the behavior the hyperbolic penalty function between $l1$ and $l2$ norm.

There are three key formulas for implementation.First of all, we need to know the gradient direction of the object function. From Claerbout et al. (2011), it is

$$\Delta u_\tau = \frac{\partial J}{\partial u_\tau} = \sum_t r_{t+\tau} H'(r_t). \tag{3}$$

And then we want to know the relation between the model update and the residual update.

$$\Delta \mathbf{r} = \mathbf{r} * \Delta \mathbf{u}. \tag{4}$$

Finally we also need the step length of each updating. In Claerbout et al. (2011), we use a Newton iterative method to solve this step length α. And in each iteration of this Newton search, the incremental step length α_{inc} is

$$\alpha_{inc} = -\frac{\sum_t \Delta r_t H'(r_t)}{\sum_t (\Delta r_t)^2 H''(r_t)}. \tag{5}$$

new implementation with model space weighing

Now we add the weight function $c(t)$ to compensate the divergence.

First the object function need to include the new weight function, so the 1 changes into

$$J = \sum_t H(c_t r_t). \tag{6}$$

Consequently, the new gradient direction 3 becomes

$$\Delta u_\tau = \frac{\partial J}{\partial u_\tau} = \sum_t c_t r_{t+\tau} H'(c_t r_t). \tag{7}$$

Here is eaxcltly the place where I made mistake at the first time. I thought the Foourier domain update should be

$$\Delta \mathbf{U} = \overline{\mathrm{FT}\,(\mathbf{r_{wt}})}\,\mathrm{FT}\,(\mathbf{H}'(\mathbf{r_{wt}})). \tag{8}$$

where $r_{wt}(t) = c(t) r(t)$.

But in fact it should be

$$\Delta \mathbf{U} = \overline{\mathrm{FT}\,(\mathbf{r})}\,\mathrm{FT}\,(\mathbf{H}'_{\mathbf{wt}}(\mathbf{r_{wt}})). \tag{9}$$

where $H'_{\mathbf{wt}}(r_{\mathbf{wt}}(t)) = c(t) H'(c(t) r(t))$

Or in words, the weighting function should be applied on the second derivative of the hyperbolic panelty function (softclip function) instead of on the residual function. Previously, I applied the weighting function on the residual and got not correct gradient direction. This is why the weighting did not help us to restore the dipole reflections.

However, due to the relation between the model update and the residual update has nothing to do with the new object function, it stays unchanged. In another word, 4 is still valid in the new implementation with weighing function.

The incremental step length α_{inc} in Newtow search for updating step length α depends on the object function, hence it also has to be changed into

$$\alpha_{inc} = -\frac{\sum_t \Delta(c_t^2 r_t H'(c_t r_t)}{\sum_t c_t^4 (\Delta r_t)^2 H''(c_t r_t)}. \tag{10}$$

Now we have all we need to implement model space weighint in Fourier domain Bidirectional deconvolution, if I did all the derivation above correctly

REFERENCES

Claerbout, J., Q. Fu, and Y. Shen, 2011, A log spectral approach to bidirectional deconvolution: SEP-Report, **143**, 297–300.