Due Date: 17:00, Fri, February 13, 2015
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Lab 4 - Missing data estimation

Your name[1]

ABSTRACT

We will look at a missing data estimation problem. You will code a linear interpolation routine and run a least squares optimization for a data-fitting equation. You will then code the convolution operator and create several filters. Finally, you will run regularized least squares optimizations using these filters and analyze the results.

LINEAR INTERPOLATION

A linear interpolation operator is a very simple operator that maps between a surveys coordinates and a regular mesh with linear interpolation. For this operator, the model space is the completed or filled data on a uniform mesh and the data space is the irregular and missing data. Although the operator seems trivial, the missing data problem is very common in many fields and has several challenges that emphasize many aspects of optimization properties and its limitations.

YOUR ASSIGNMENT

In this lab, we will work with a simple 1D dataset shown in Figure 1. The data and its coordinates can be found in the directory `Dat`. You are provided with modules for the operators, the dot product test, the stepper and the solver. All the modules (except the operator) are complete and do not need to be modified. The operators' modules have the initialization subroutine only and you need to complete the rest. Moreover, you are provided with two programs: one program that creates filters from provided parameters (which is complete) and another one which implements the linear interpolation operator (which you need to complete). You will code several programs that implement the modules to answer the questions and perform the required operations. Pay close attention to the provided codes since they show several steps that might be

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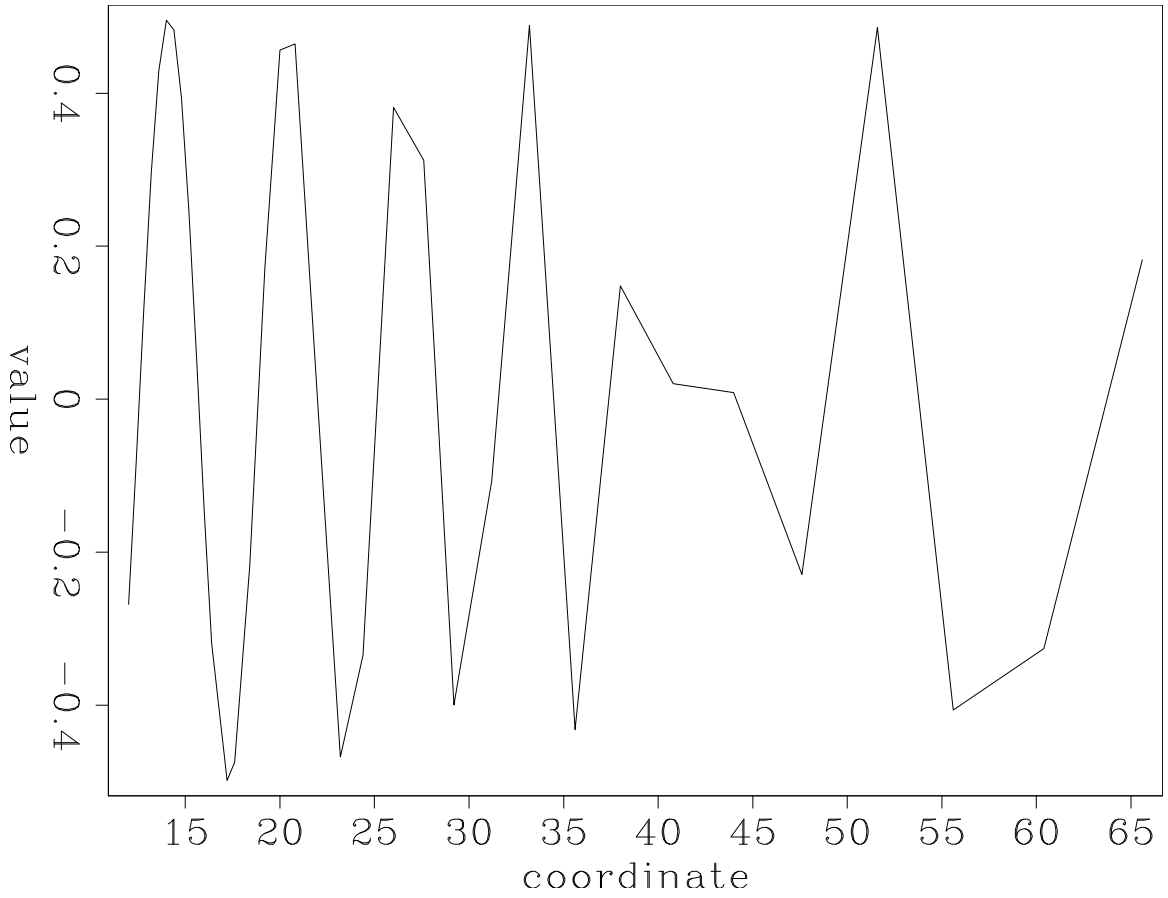

Figure 1: Observed data.

new to you (such as reading the history and binary of auxiliary files, using pointers in modules initializations, etc.). Modify the `Makefile` to add rules that generate the required tests, results and figures for each question. Make sure to include the correct and complete set of prerequisites for each `Makefile` rule (including the executable programs). Also, do not forget to add the PDF figures to the default target and include the figures in this paper.

Questions

Data-fitting optimization

1. Complete the linear interpolation module. Then, write a program that runs the dot product test on the operator using the provided coordinates and the parameters in `Par/lint.p`. What are your test results?

YOUR ANSWER.

2. Complete the linear interpolation program. Then, run the adjoint linear interpolation to get your first attempt of completing the missing data using the parameters in `Par/lint.p`. Plot the data and describe the effect of the adjoint operator.

YOUR ANSWER.

3. By comparing the adjoint results to the observed data, what do you notice about some values (especially at the beginning)? Why did this happen?

YOUR ANSWER.

4. Write a program that implements the data-fitting solver with the conjugate direction stepper and run it for many iterations. The solver will automatically stop if the updates or residuals are small. Plot both the inverted model and the reconstructed data. What did the optimization do regarding the values mentioned in the previous question?

YOUR ANSWER.

5. Without actually doing so, how would you code a weighting operator that improves the adjoint? What would this operator need? Is it going to be linear?

YOUR ANSWER.

6. Do we have a model null space for the data-fitting optimization? How can you test this? Write a second optimization program that illustrates your claim. Plot both the inverted model and the reconstructed data.

YOUR ANSWER.

Regularized optimization

7. Use the `MakeFilter` program to create a first derivative filter. Then, complete the transient convolution module and write a program that runs the dot product test on the operator using the filter you created. What are your test results?

YOUR ANSWER.

8. Write a program that implements the regularized solver with the conjugate direction stepper. Obviously, you want to use the convolution operator for regularization where the filter is supplied from an auxiliary file. Use the derivative filter and run the optimization for many iterations with a small epsilon (around 0.1). Plot both the inverted model and the reconstructed data. What is the effect of regularization by a first derivative?

YOUR ANSWER.

9. Is the regularization goal conflicting with the data-fitting goal? How can you verify your answer?

YOUR ANSWER.

10. Create a second derivative filter and run another optimization with this filter (You should be able to use the same program from the previous question if you coded it correctly). Plot both the inverted model and the reconstructed data. What is the effect of regularization by a second derivative?

YOUR ANSWER.

11. Make one more filter of your choice and run the optimization with this filter. Plot both the inverted model and the reconstructed data. Why did you chose this filter? How does it affect the results?

YOUR ANSWER.

12. Of all the optimizations we ran, which inverted model do you think is best? Why?

YOUR ANSWER.

Extra Credit

1. What does it mean to regularize with the identity operator?

YOUR ANSWER.

2. Does regularization always remove the null space? be specific.

YOUR ANSWER.

3. If an operator does not have a model null space, do you still need regularization? Why?

YOUR ANSWER.

DONE

When you are all finished modifying the source files, `Makefile` and latex file, make sure that will paper will compile correctly from a cleaned directory using the default targets only. In other words, the following sequence of commands should produce your a PDF version of your paper without any problems: `make burn`; `scons -c`; `make`; `scons`.

Once you make sure everything is working properly, clean up your directory by typing `make burn`; `scons -c`. Then, compress the lab directory using the command `tar cvzf Lab4.gzip Lab4` and submit the compressed file to your TA.