Logarithm Bidirectional Method With Regularization

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When we consider regularization in the logarithm bidirectional method, we need to change our objective function into

$$J = \mathrm{hyp}(\mathbf{r}) + \varepsilon \left\| \mathbf{u} \right\| = \sum_t H(r_t) + \varepsilon \sum_n u_n^2 \tag{1}$$

Take the gradient of the penalty function assuming there is only one variable, u_3 giving a single regression equation:

$$0 \approx \frac{\partial J}{\partial u_3} = \sum_t \frac{\partial H}{\partial r} \frac{\partial r}{\partial u_3} + 2\varepsilon u_3 \tag{2}$$

Then the gradient for all nonzero lags is:

$$0 \approx \Delta \mathbf{u_{new}} = \sum_t r_{t+\tau} H'(r_t) + 2\varepsilon \mathbf{u} \tag{3}$$

Although the gradient is changed, we still want to keep Δr unchanged:

$$\Delta \mathbf{u_{old}} = \sum_t r_{t+\tau} H'(r_t) \tag{4}$$

$$\Delta \mathbf{r} = \mathbf{r} * \Delta \mathbf{u_{old}} \tag{5}$$

Now let us figure out how to find the scalar factor α. By taylor expansion,

$$J = \sum_t (H_t + \alpha \Delta r_t H_t' + \alpha^2 \Delta r_t^2 H_t'') + \varepsilon \sum_n ((u_{new} + \alpha \Delta u_{new})^2)_n \tag{6}$$

Setting $\frac{\partial J}{\partial \alpha} = 0$ we get,

$$\alpha = -\frac{\sum\limits_t \Delta r_t H_t' + 2\varepsilon \sum\limits_n (u_{new} \Delta u_{new})_n}{\sum\limits_t (\Delta r_t)^2 H_t'' + 2\varepsilon \sum\limits_n ((\Delta u_{new})^2)_n} \tag{7}$$