Logarithm Bidirectional Deconvolution Method With Regularization

Yi Shen, Qiang Fu and Jon Claerbout

ABSTRACT

In this paper, we introduce two ways of constraining the model on logarithm bidirectional deconvolution proposed by Claerbout et al. (2011): masking δu and regularization. First, we theoretically illustrate the regularization. Then we test the logarithm method with two constrainings on the field data. The results show that the precursors with masking are more severe than the ones without maksing. But the anti-causal part of the estimated wavelet with masking is cleaner than the one without masking. However, regularization does not perform well. The iterative approach is diverging because of the overshooting in steepest descent. Hence, we need some tricks in the code to prevent a big step during searching the global minimum.

INTRODUCTION

Bidirectional deconvolution is an ill-posed and highly non-linear problem with many unexpected traps-local minima. When we test logarithm bidirectional deconvolution proposed by Claerbout et al. (2011) on one set of Common Offset gather, we notice the anti-causal part of the estimated wavelet is unexpectedly strong and the results contain high frequency noise. Thus, introducing additional information is needed in order to solve ill-posed problem and guide the result away from an unwelcome local minimum. Regularization, providing such information as restrictions for smoothness or bounds on the vector space norm becomes an important tool in bidirectional deconvolution. Therefore, we propose regularization be applied on logarithm bidirectional deconvolution method in order to reduce the strong precursors and noise.

THEORY

When we consider regularization in the logarithm bidirectional method, we need to change our objective function into

$$J = \text{hyp}(\mathbf{r}) + \varepsilon \left\| \mathbf{W} \cdot \mathbf{u} \right\| = \sum_t H(r_t) + \varepsilon \sum_n W_n^2 u_n^2 \tag{1}$$

where u is in time domain and its Fourier domain formulation is the logarithm of the mixed-phase filter, and W is weighting function. The hybrid norm is applied to residual r. Take the gradient of the penalty function assuming there is only one variable, u_3 giving a single regression equation:

$$0 \approx \frac{\partial J}{\partial u_3} = \sum_t \frac{\partial H}{\partial r}\frac{\partial r}{\partial u_3} + 2\varepsilon W_3^2 u_3 \tag{2}$$

Then the gradient for all nonzero lags is:

$$0 \approx \Delta\mathbf{u} = \sum_t r_{t+\tau} H'(r_t) + 2\varepsilon(..., W_{-1}^2 u_{-1}, 0, W_1^2 u_1, ...) \tag{3}$$

With this new gradient, we can have $\Delta\mathbf{r}$ as $\Delta\mathbf{r} = \mathbf{r} * \Delta\mathbf{u}$. Now let us figure out how to find the scalar factor α. By Taylor expansion,

$$J = \sum_t (H_t + \alpha\Delta r_t H_t' + \alpha^2 \Delta r_t^2 H_t'') + \varepsilon \sum_n W_n^2 (u_n + \alpha\Delta u_n)^2 \tag{4}$$

Setting $\frac{\partial J}{\partial \alpha} = 0$ we get,

$$\alpha = -\frac{\sum\limits_t \Delta r_t H_t' + 2\varepsilon \sum\limits_n W_n^2 (u_n \Delta u_n)}{\sum\limits_t (\Delta r_t)^2 H_t'' + 2\varepsilon \sum\limits_n W_n^2 (\Delta u_n)^2} \tag{5}$$

ALGORITHM

Here we fill in more details of the algorithm.

```
D(omega,x) = FT d(t,x)
u=0;
iteration {
        U = FT(u)
        remove mean from U(omega)
        exp(U(Z))
        dU = 0
        for all x
                r(t,x) = IFT( D exp(U) )
                softclip( r )
                dU += conjg(FT(r)) * FT(softclip)        #  "*" means multiply
                du=IFT(dU)
                for all du
                   du_i+=2eps (W_i)^2 u_i
                dU=FT(du)
        remove mean from dU(omega)
        for all x
                dR = FT(r) * dU                          #  "*" means multiply
                dr = IFT(dR)
        iterate {
```

```
        alpha = - ((Sum_i H'(r_i) dr_i) +2eps (Sum_i (W_i)^2 u_i du_i))&&
                / ((Sum H''(r_i) dr_i^2)+2eps (Sum_i (W_i)^2 (du_i)^2))
        r = r + alpha dr
        alpha_sum+=alpha
         }
u = u + alpha_sum du
}
```

RESULT

Constrain δu

The weighting function on δu is shown in figure 1.

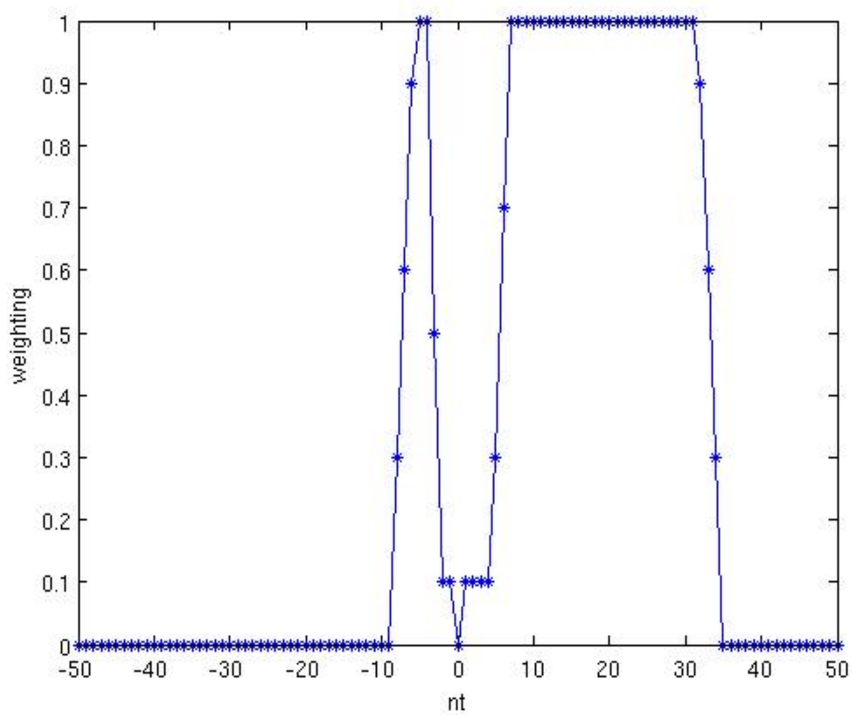
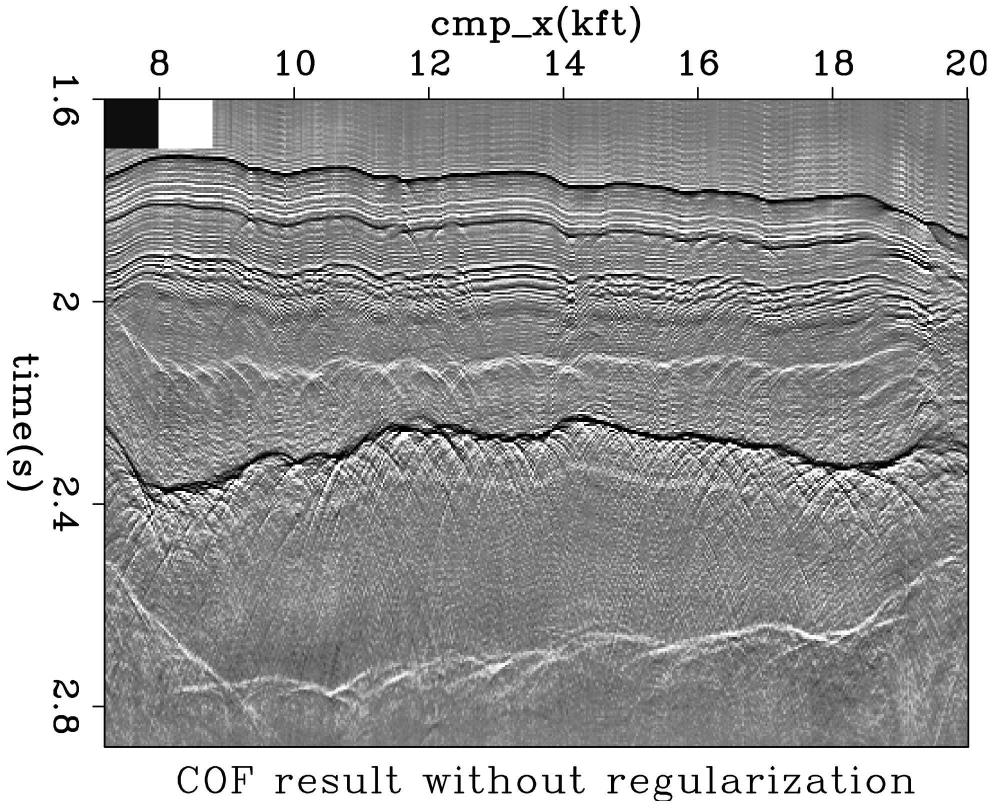
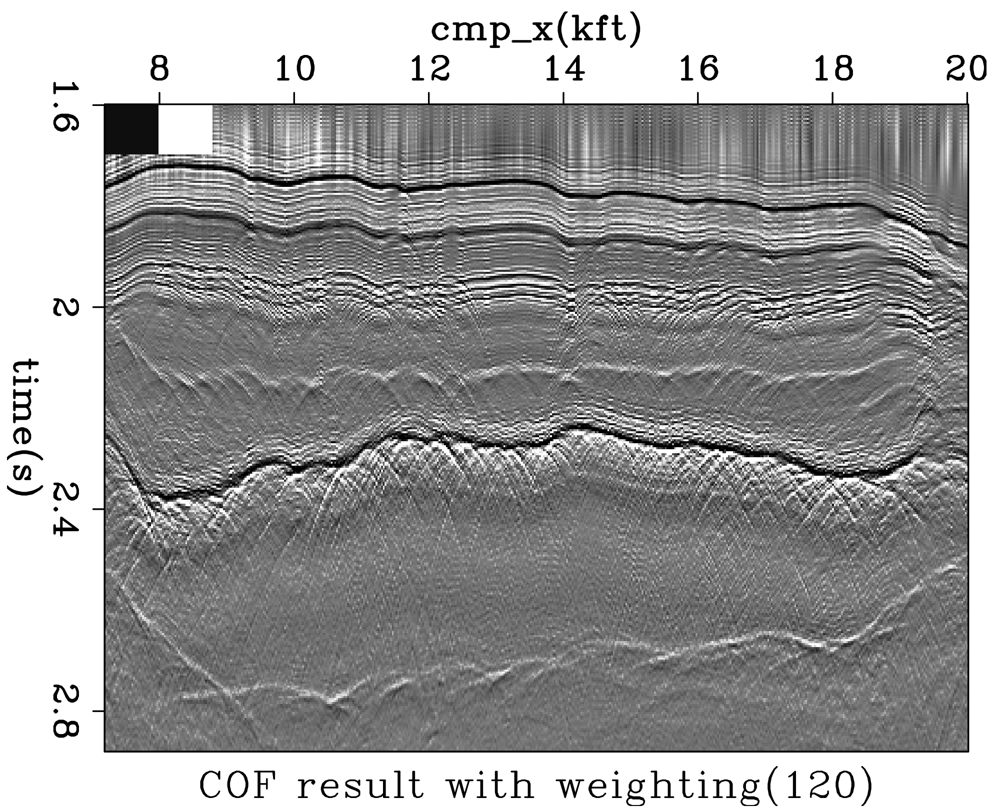


Figure 1: Weighting function on δu

The deconvolution results are shown in figure 2(a) and 2(b). We notice the precursors with masking are more severe than the ones without maksing.However, the estimated wavelets which are shown in figure 3 and 4 show the different conclusion. The anti-causal part of the wavelet with masking is cleaner than the one without masking.



(a)



(b)

Figure 2: (a) reflectivity model retrieved without masking; (b) reflectivity model retrieved with masking. [**ER**]

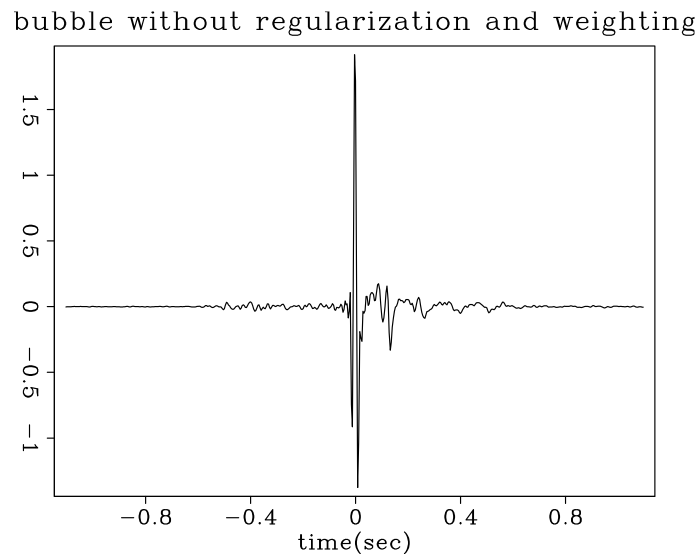


Figure 3: Deconvolution result without masking. [**ER**]

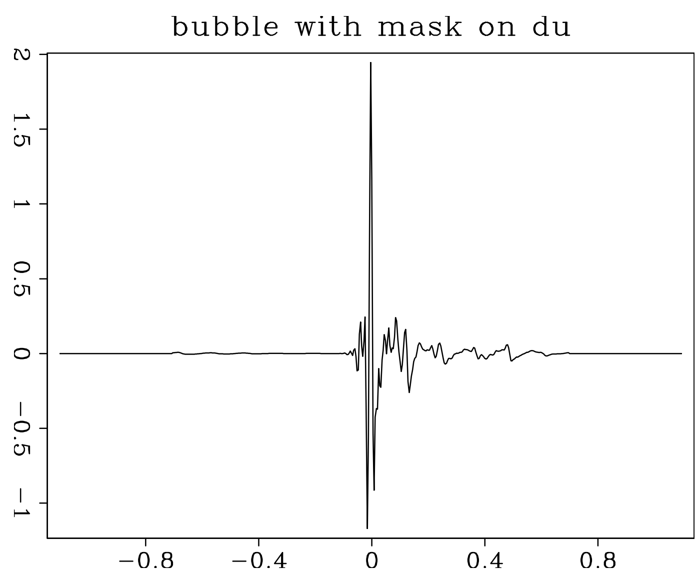


Figure 4: Deconvolution result with masking. [**ER**]

Regularize u

The bidecon result is shown in figure 5 and estimated wavelet is shown figure 6. After applying the regularization, The iterative approach is always diverging because of the overshooting in steepest descent. We need to incorparate Qiang's trick to prevent a big step during searching the global minimum.

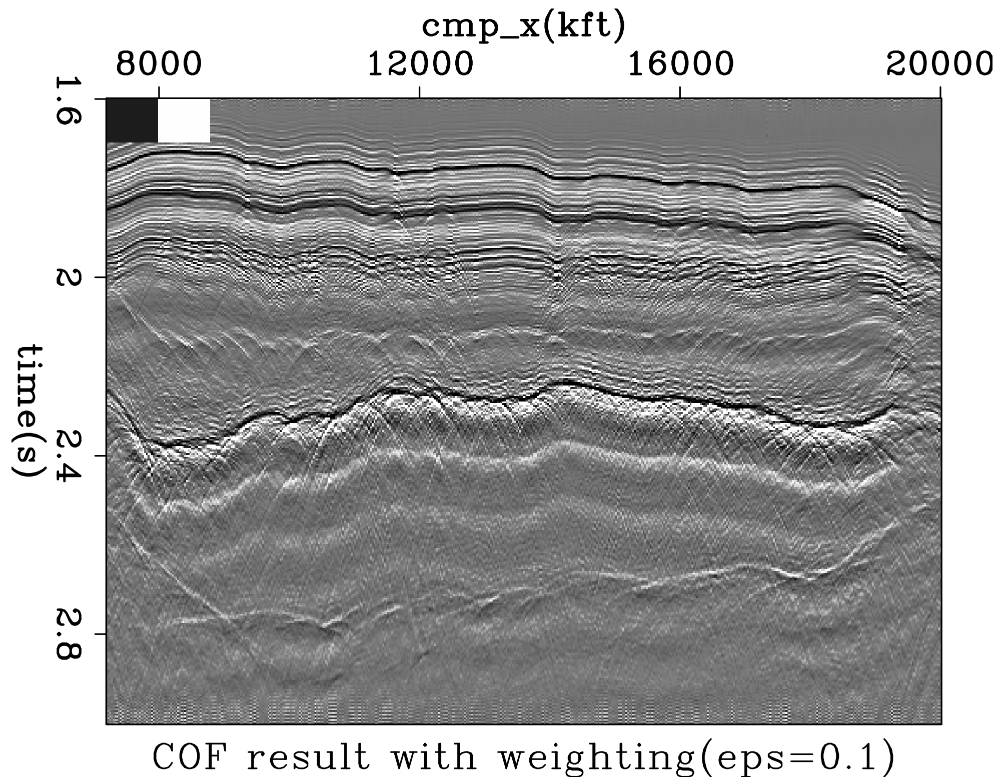


Figure 5: Bidirectional deconvolution withregularization

REFERENCES

Claerbout, J., Q. Fu, and Y. Shen, 2011, A log spectral approach to bidirectional deconvolution: SEP-Report, **143**, 295–298.

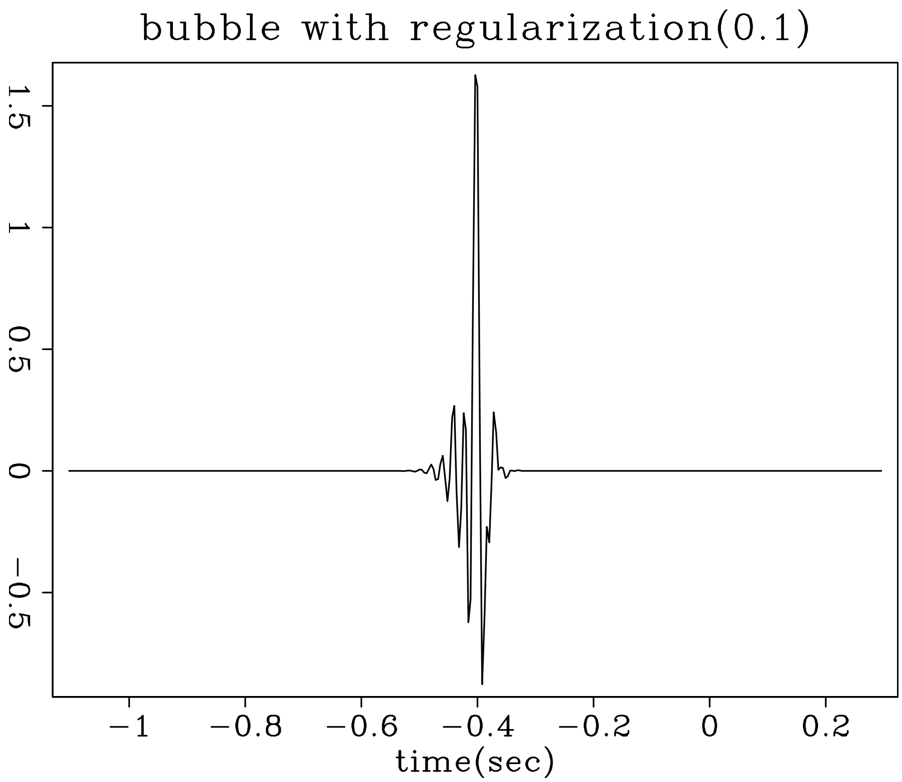


Figure 6: wavelet with regularization