Logarithm Bidirectional Deconvolution Method With Regularization

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ABSTRACT

In this paper, we theoretically discuss about regularization on logarithm bidirectional deconvolution proposed by Claerbout et al. (2011). We hope model fitting helps reduce the unwelcome strong precursors and noise in the bidirectional deconvolution results.

INTRODUCTION

Bidirectional deconvolution is an ill-posed and highly non-linear problem with many unexpected traps-local minima. When we test logarithm bidirectional deconvolution proposed by Claerbout et al. (2011) on one set of Common Offset gather, we notice the anti-causal part of the estimated wavelet is unexpectedly strong and the results contain high frequency noise. Thus, introducing additional information is needed in order to solve ill-posed problem and guide the result away from an unwelcome local minimum. Regularization, providing such information as restrictions for smoothness or bounds on the vector space norm becomes an important tool in bidirectional deconvolution. Therefore, we propose regularization be applied on logarithm bidirectional deconvolution method in order to reduce the strong precursors and noise.

THEORY

When we consider regularization in the logarithm bidirectional method, we need to change our objective function into

$$J = \mathrm{hyp}(\mathbf{r}) + \varepsilon \left\| \mathbf{W} \cdot \mathbf{u} \right\| = \sum_t H(r_t) + \varepsilon \sum_n W_n^2 u_n^2 \tag{1}$$

where u is in time domain and its Fourier domain formulation is the logarithm of the mixed-phase filter, and W is weighting function. The hybrid norm is applied to residual r. Take the gradient of the penalty function assuming there is only one variable, u_3 giving a single regression equation:

$$0 \approx \frac{\partial J}{\partial u_3} = \sum_t \frac{\partial H}{\partial r} \frac{\partial r}{\partial u_3} + 2\varepsilon W_3^2 u_3 \tag{2}$$

Then the gradient for all nonzero lags is:

$$0 \approx \Delta \mathbf{u} = \sum_t r_{t+\tau} H'(r_t) + 2\varepsilon(..., W_{-1}^2 u_{-1}, 0, W_1^2 u_1, ...) \tag{3}$$

With this new gradient, we can have $\Delta \mathbf{r}$ as $\Delta \mathbf{r} = \mathbf{r} * \Delta \mathbf{u}$. Now let us figure out how to find the scalar factor α. By Taylor expansion,

$$J = \sum_t (H_t + \alpha \Delta r_t H'_t + \alpha^2 \Delta r_t^2 H''_t) + \varepsilon \sum_n W_n^2 (u_n + \alpha \Delta u_n)^2 \tag{4}$$

Setting $\frac{\partial J}{\partial \alpha} = 0$ we get,

$$\alpha = -\frac{\sum\limits_t \Delta r_t H'_t + 2\varepsilon \sum\limits_n W_n^2 (u_n \Delta u_n)}{\sum\limits_t (\Delta r_t)^2 H''_t + 2\varepsilon \sum\limits_n W_n^2 (\Delta u_n)^2} \tag{5}$$

ALGORITHM

Here we fill in more details of the algorithm.

```
D(omega,x) = FT d(t,x)
u=0;
iteration {
        U = FT(u)
        remove mean from U(omega)
        exp(U(Z))
        dU = 0
        for all x
                r(t,x) = IFT( D exp(U) )
                softclip( r )
                dU += conjg(FT(r)) * FT(softclip)        #  "*" means multiply
                du=IFT(dU)
                for all du
                   du_i+=2eps (W_i)^2 u_i
                dU=FT(du)
        remove mean from dU(omega)
        for all x
                dR = FT(r) * dU                          #  "*" means multiply
                dr = IFT(dR)
        iterate {
                alpha = - ((Sum_i H'(r_i) dr_i) +2eps (Sum_i (W_i)^2 u_i du_i))&&
                        / ((Sum H''(r_i) dr_i^2)+2eps (Sum_i (W_i)^2 (du_i)^2))
                r = r + alpha dr
                alpha_sum+=alpha
                 }
        u = u + alpha_sum du
        }
```

CONCLUSION

After applying the logarithm method with regularization to one set of Common Offset gather, when ε is small, the results show few changes in comparison with those without regularization; when ε is large,the results are even worse with strong precursors. Some possible explanations to such phenomenon are: this piece of data is not ill-posed; there are unidentified bugs in the code; or the weighting function we use is not appropriate. It is too early to draw any conclusion concerning whether or not regularization on logarithm method helps improve the result. One thing for sure is that the basic theory of regularization should be helful for our future work.

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REFERENCES

Claerbout, J., Q. Fu, and Y. Shen, 2011, A log spectral approach to bidirectional deconvolution: SEP-Report, **143**, 295–298.